Hydrodynamics

TALENT 7:

Nuclear Theory for Astrophysics

Fluid Dynamics

Fluid Dynamics, subsuming both hydrodynamics and aerodynamics, is a continuum description of the collective behavior of a large number of particles.

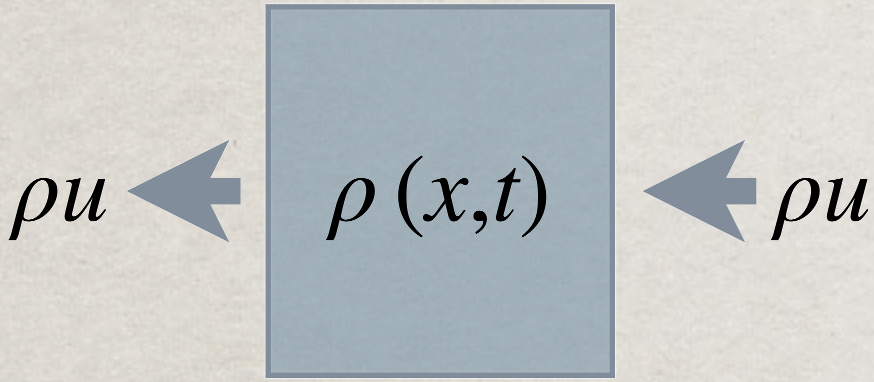
The equations of fluid dynamics can be derived from kinetic theory in the limit that the collisional mean free path, ℓ, is much smaller than the macroscopic scales of interest, L.

Thus we are concerned with the bulk velocity of the fluid, u, while the random velocity of individual fluid particles is only considered to the extent that they form an internal energy.

Key to the behavior of fluids is that they, like solids, deform under stress. However, unlike a solid, a fluid shows no tendency to return to the former state when the stress is removed.

Continuity

Quantity of matter can be described by the mass density, which changes in time and space.



The change of ρ with time in the box requires a “flux” of mass across the boundary at velocity u.

More formally,

$$\frac{d}{dt}\int_V \rho\, dV = -\int_S (\rho\vec{u})\cdot\vec{n}\, dS$$

Applying the time independence of V on the left and divergence theorem on the right yields

$$\int_V \frac{\partial\rho}{\partial t}\, dV = -\int_V \nabla\cdot(\rho\vec{u})\, dV$$

Since this is true for arbitrary V,

$$\frac{\partial\rho}{\partial t} + \nabla\cdot(\rho\vec{u}) = 0$$

Continuity Equation

Defining Derivatives

When considering a moving fluid, there are two natural frames of reference.

1) Eulerian Coordinates, which are fixed in space.

2) Lagrangian Coordinates, which move with the fluid.

To define a Lagrangian (or material) Derivative of a quantity f, we must consider both changes that are local in space and those that result from movement.

$$\frac{Df}{dt} = \frac{\partial f}{\partial t} + \vec{u} \cdot \nabla f$$

Since $\nabla \cdot (b\vec{a}) = \vec{a} \cdot \nabla b + b(\nabla \cdot \vec{a})$, the continuity equation can be transformed from

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{u}) = 0 \quad \text{into} \quad \frac{D\rho}{Dt} + \rho \nabla \cdot \vec{u} = 0$$

Causing Fluid to Move

We next need to understand what generates the velocity, u.

For a co-moving volume, the total momentum is $\int_V \rho u \, \mathrm{d}V$ and the time rate of change comes entirely from external forces.

These take the form of external body forces, f, e.g., gravity, and surface forces, e.g., pressure, P.

$$\frac{d}{dt}\int_V \rho\vec{u}\, dV = -\int_S P\vec{n}\, dS + \int_V \rho\vec{f} dV$$

Applying the Chain Rule to the left side yields,

$$\frac{d}{dt}\int_V \rho\vec{u}\, dV = \int_V \rho\frac{D\vec{u}}{Dt} dV + \int_V \vec{u}\frac{D\rho}{Dt} dV$$

0

since ρV is invariant for co-moving volumes.

Applying the divergence theorem to the right side yields

$$-\int_S P\vec{n}\, dS + \int_V \rho\vec{f} dV = \int_V (-\nabla P + \rho\vec{f}) dV$$

Euler Equation

Combining these yields

$$\int_V \rho \frac{D\vec{u}}{Dt} dV = \int_V (-\nabla P + \rho \vec{f}) dV$$

or, since this applies for arbitrary volumes,

$$\rho \frac{D\vec{u}}{Dt} = (-\nabla P + \rho \vec{f})$$

Written in terms of coordinates fixed in space, this becomes

$$\rho \left(\frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \nabla)\vec{u} \right) = (-\nabla P + \rho \vec{f})$$

Euler Equation

To make sure we see the physics of this equation, we can rewrite this as

$$\frac{\partial \rho \vec{u}}{\partial t} + \nabla \cdot \rho \vec{u}\vec{u} = -\nabla P + \rho \vec{f}$$

Viscosity

When we wrote the effect of the surface pressure as $\int_S P\,\vec{n}\,dS$, we implicitly assumed that viscosity was unimportant.

In the general case, $F_i = \int_S \Sigma_j P\,\sigma_{ij}\,n_j\,dS$, where σ_{ij} is the stress tensor, rather than $\int_S P\,n_i\,dS$.

For gases and simple liquids, we can define a dynamical viscosity, μ, in which case the stress tensor is

$$\sigma_{ij} = -P\delta_{ij} + \mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3}(\nabla\cdot\vec{u})\delta_{ij}\right)$$

In this case, the momentum equation becomes

Navier-Stokes Equation.

$$\rho\frac{D\vec{u}}{Dt} = -\nabla P + \rho\vec{f} + \mu\left(\nabla^2\vec{u} + \frac{1}{3}\nabla(\nabla\cdot\vec{u})\right)$$

μ is generally very small in astrophysics and the Reynolds number, the ratio of inertial forces to viscous forces, is large.

Mechanical Energy

The Euler equation includes a gradient of the kinetic energy, requiring an equation to evolve the kinetic energy.

Taking the dot product of u/ρ with the Euler equation

$$\frac{\vec{u}}{\rho} \cdot \rho \frac{D\vec{u}}{Dt} = \frac{\vec{u}}{\rho} \cdot (-\nabla P + \rho \vec{f})$$

provides such an equation

$$\frac{D}{Dt}\left(\frac{1}{2}\vec{u}^2\right) = -\frac{1}{\rho}\vec{u} \cdot \nabla P + \vec{u} \cdot \vec{f}$$

mechanical energy equation

Simply, the kinetic energy changes in response to work done by pressure and body forces.

This approach may seem arbitrary, but is equivalent to calculating the work done by a force as

$$uF = uma = um\frac{du}{dt} = \frac{d}{dt}\left(\frac{1}{2}u^2\right)$$

Energy Conservation

Of course, kinetic energy is not conserved, rather it is the total energy, kinetic + internal (thermal), $\frac{1}{2}u^2 + U$.

If we expand our energy equation to include the internal energy, we must add terms for the heat generated within the volume, ϵ, and the flux of heat across the boundary, F.

$$\frac{d}{dt}\int_V \left(\frac{1}{2}\vec{u}^2 + U\right)\rho\, dV = -\int_S \vec{u}\cdot P\vec{n}\, dS + \int_V \vec{u}\cdot\vec{f}\rho\, dV + \int_V \epsilon\rho\, dV - \int_S \vec{F}\cdot\vec{n}\, dS$$

Applying the divergence theorem to replace the surface integrals

$$\int_S \vec{F}\cdot\vec{n}\, dS = \int_V \nabla\cdot\vec{F}\, dV$$

$$\int_S \vec{u}\cdot P\vec{n}\, dS = \int_V \nabla\cdot P\vec{u}\, dV$$

Energy Equation

As in the prior derivation, the invariance of ρV for co-moving volumes simplifies the energy time derivative.

$$\frac{d}{dt}\int_V \left(\frac{1}{2}\vec{u}^2 + U\right)\rho\, dV = \int_V \rho \frac{D}{Dt}\left(\frac{1}{2}\vec{u}^2\right) + \int_V \rho \frac{DU}{Dt}\, dV$$

Once again, we can also remove the volume integral that appears in each term, yielding.

$$\rho \frac{D}{Dt}\left(\frac{1}{2}\vec{u}^2\right) + \rho \frac{DU}{Dt} = -\nabla \cdot P\vec{u} + \rho\vec{u}\cdot\vec{f} + \rho\epsilon - \nabla\cdot\vec{F}$$

Expanding the co-moving derivatives, and merging like terms, leaves

$$\frac{\partial}{\partial t}\rho(\frac{1}{2}u^2 + U) + \nabla\cdot(\rho(\frac{1}{2}u^2 + U)\vec{u}$$

$$= -\nabla \cdot P\vec{u} + \rho\vec{u}\cdot\vec{f} + \rho\epsilon - \nabla\cdot\vec{F}$$

Equation of State

The pressure, P, appears in both the momentum and energy equation, yet we have no equation for its evolution.

For all matter, there exist thermodynamic relations linking the pressure, density, temperature, internal energy, entropy…

These are the Equations of State (EoS).

The most widely known is the ideal (monatomic) gas EoS

$$P\,V = R\,T \text{ and } U = \tfrac{3}{2}\,R\,T, \text{ thus } P = \tfrac{2}{3}\,\rho\,U$$

The more generalized version is cast in terms of the adiabatic index $\gamma = C_P/C_V$, the ratio of specific heats.

$$P = (\gamma - 1)\,\rho\,U$$

where $\gamma = 5/3$ for a monatomic gas.

Polytropic Fluid

For the adiabatic case, the ideal gas EoS can be written as

$P = K\rho^{\gamma}$ in which case $U = \dfrac{P}{\gamma - 1} = \dfrac{K\rho^{\gamma}}{\gamma - 1}$

Such EoS are often written in the form

$P = K\rho^{1+\frac{1}{n}}$ where n is called the polytropic index.

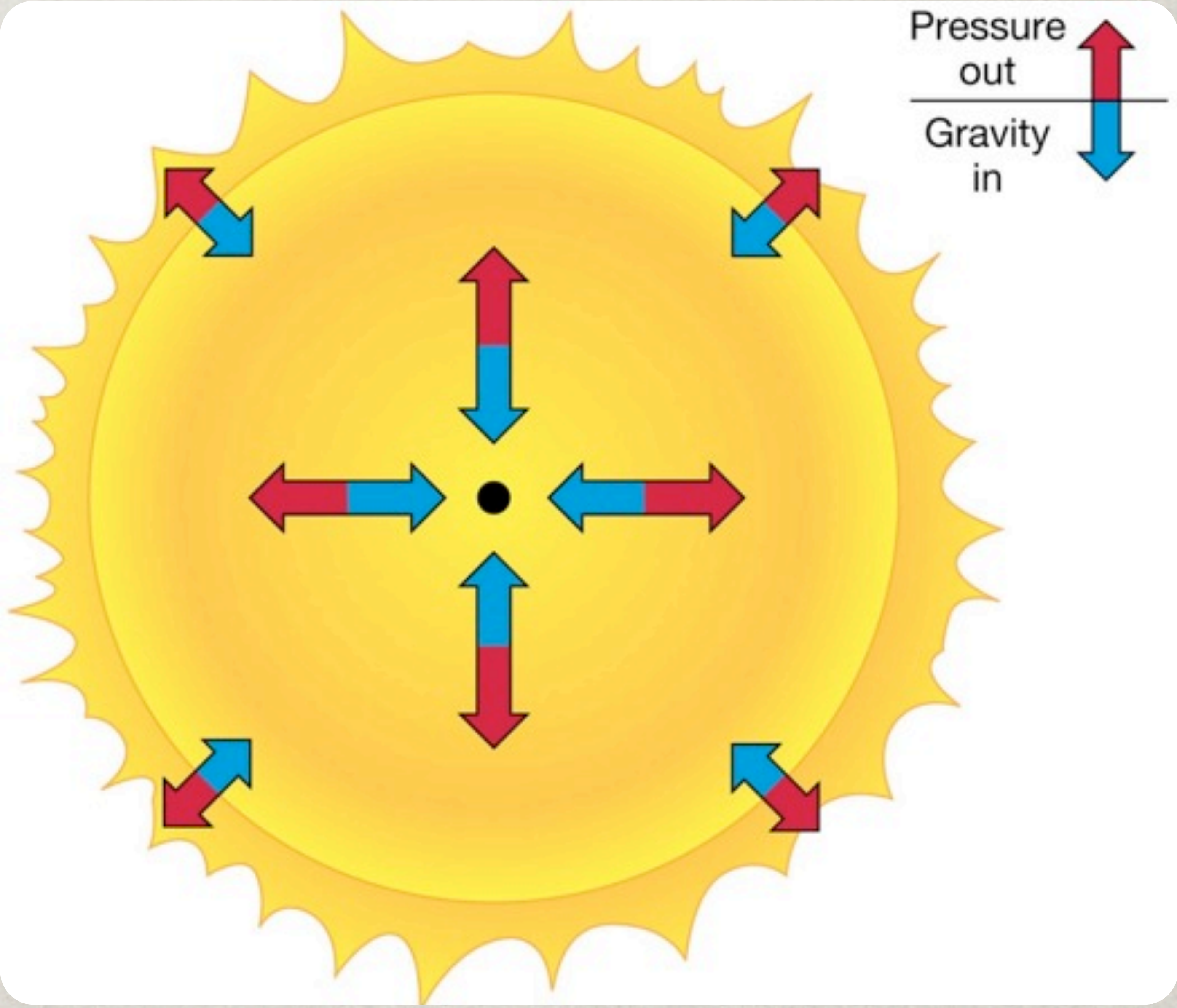
These polytopic EoS played a large role in early calculations of stellar structure and remain useful because a number of physical states behave approximately as polytropes.

For example, both the ideal monatomic gas and a non-relativistic degenerate gas obey $P = K\rho^{5/3}$.

For a relativistic degenerate gas $P = K\rho^{4/3}$, and stars in radiative equilibrium also follow this relation.

Hydrostatic Equilibrium

Simplest application of hydrodynamic is hydrostatics ($u = 0$). The hydrostatic limit of the Euler equation is *Hydrostatic Equilibrium*.



Numerically this takes the form

$$\frac{dP}{dr} = -\frac{GM(r)\rho(r)}{r^2}$$

Physically, this says the inward gravitational force must be balanced by the outward pressure.

This relationship is key to calculating the conditions in a star's interior.

Standard Solar Model

Hydrostatic versions of the continuity and energy equation give us

Mass Continuity

$$\frac{dM}{dr} = 4\pi r^2 \rho(r)$$

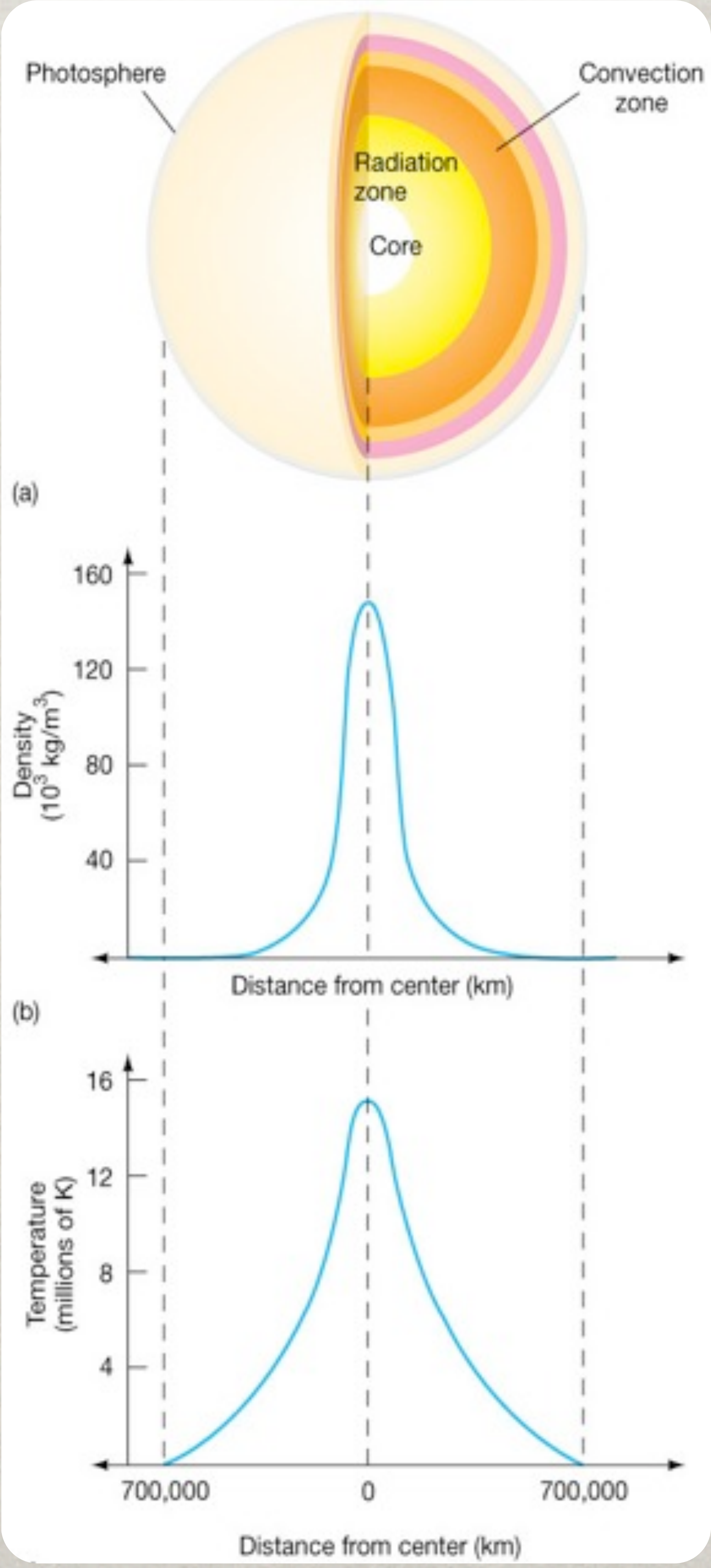
Energy Generation

$$\frac{dL}{dr} = 4\pi r^2 \rho(r)\epsilon(r)$$

Together with an equation for Energy Transport, which depends on the dominant energy transport process, these combined with boundary conditions like

$M(0) = 0$, $L(0) = 0$, &
$M(R_\odot) = M_\odot$, $L(R_\odot) = L_\odot$, etc.

allow us to calculated the stellar models.



The Equations We Solve

In VH-1, and many similar hydrodynamics codes, the 3D problem is *directionally-split* into separate 1D solutions along the representative directions. This simplifies the equations.

$$\frac{\partial \rho}{\partial t} + \frac{\partial A \rho u}{\partial V} = 0$$

To allow for different coordinate systems, we work in terms of a volume coordinate V with cell cross section A

$$\frac{\partial \rho u}{\partial t} + \frac{\partial A \rho u^2}{\partial V} = -\frac{\partial P}{\partial \chi} + \rho f$$

Gradients use a generalized spatial coordinate, χ

$$\frac{\partial \rho v}{\partial t} + \frac{\partial A \rho v u}{\partial V} = 0 = \frac{\partial \rho w}{\partial t} + \frac{\partial A \rho w u}{\partial V}$$

Momentum is also advected transversely.

$$\frac{\partial \rho E}{\partial t} + \frac{\partial A \rho E u}{\partial V} = -\frac{\partial A \rho P u}{\partial V} + \rho u f$$

Total energy $E = \frac{1}{2}(u^2+v^2+w^2) + U$

$$P = (\gamma - 1)\rho U$$

Equation of State

Spatial Differencing

Transforming continuous variables, $f(x)$, to variables represented on a discrete grid, f_j, we must approximate spatial derivates as differences. However, the choice is not unique.

For example, $\partial f/\partial x$ at $x = x_j$ can be written as

$$\left.\frac{\partial f}{\partial x}\right|_j \approx \frac{f_{j+1} - f_j}{\Delta x}$$ forward difference

$$\left.\frac{\partial f}{\partial x}\right|_j \approx \frac{f_j - f_{j-1}}{\Delta x}$$ backward difference

$$\left.\frac{\partial f}{\partial x}\right|_j \approx \frac{f_{j+1} - f_{j-1}}{2\Delta x}$$ centered difference

Higher order derivatives touch more points on the grid, e.g.,

$$\left.\frac{\partial^2 f}{\partial x^2}\right|_j \approx \frac{f_{j+1} - 2f_j + f_{j-1}}{\Delta x^2}$$

Order of Error

Difference choices of derivatives affect the error the comes from mapping to a discrete grid. One can estimate this error by calculating $f_{j+1} = f(x_{j+1}) = f(x_j+\Delta x)$ and $f_{j-1} = f(x_{j-1}) = f(x_j-\Delta x)$ using the Taylor series

$$f(x+h) = f(x) + h\frac{\partial f}{\partial x} + \frac{h^2}{2}\frac{\partial^2 f}{\partial x^2} + \cdots$$

to calculate the error as a function of Δx.

For both forward and backward differencing the leading error in the approximation of $\partial f/\partial x$ is $\propto (\Delta x)\ \partial^2 f/\partial x^2$, thus these approximations are $O(\Delta x)$. For centered differencing, the error is $O(\Delta x^2)$ because the $\partial^2 f/\partial x^2$ terms cancel.

While having a smaller truncation error, centered differencing has a tendency to spread sharp features which is determental in some circumstances.

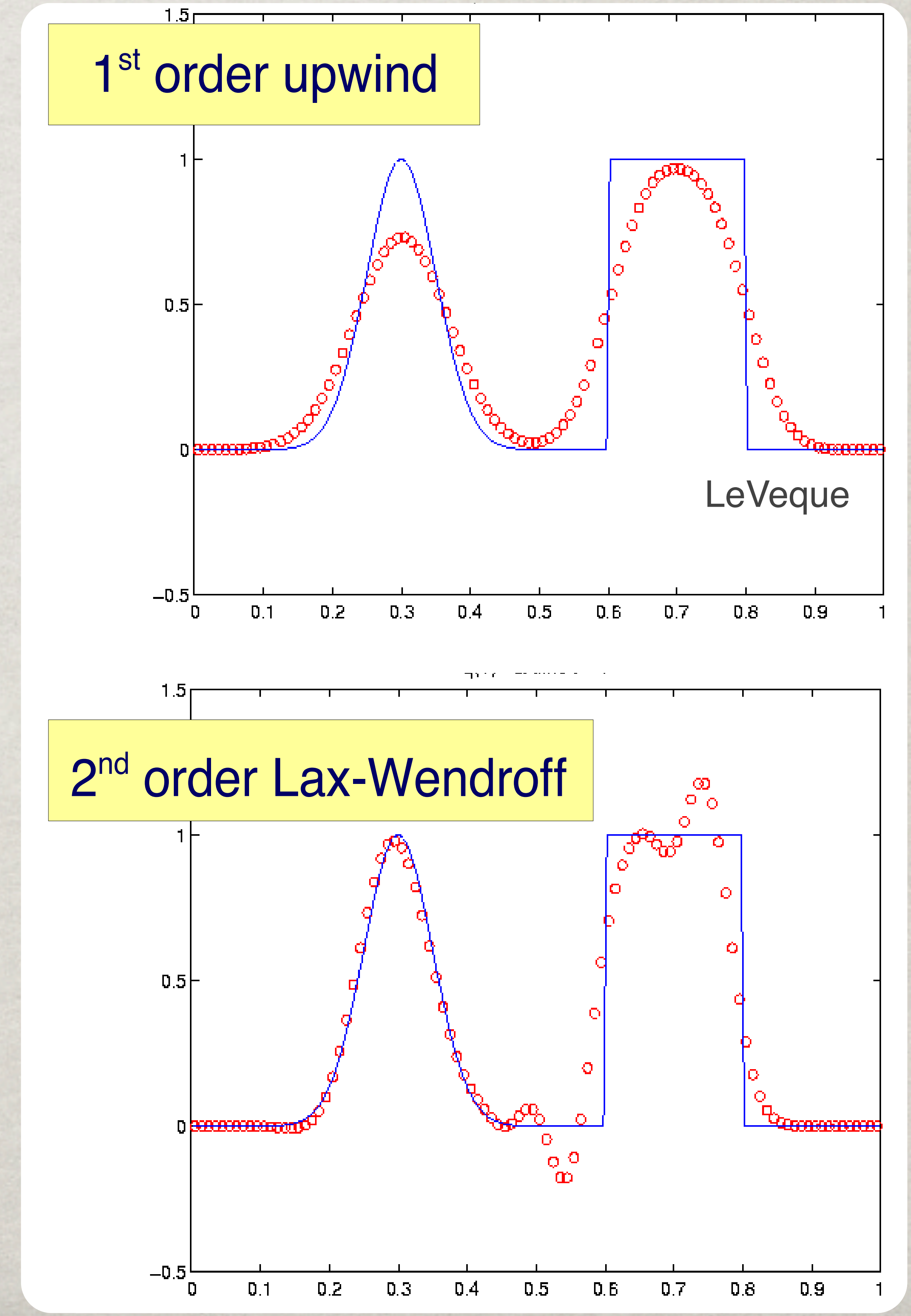
Capturing Shocks

Many problems in nuclear astrophysics include shocks and compositional (contact) discontinuities.

Simple differencing schemes are challenged by sharp flow features like these.

Low order methods tend to diffuse these features over many zones.

Higher order methods are less diffusive, but can add considerable dispersion (noise).



Riemann Problem

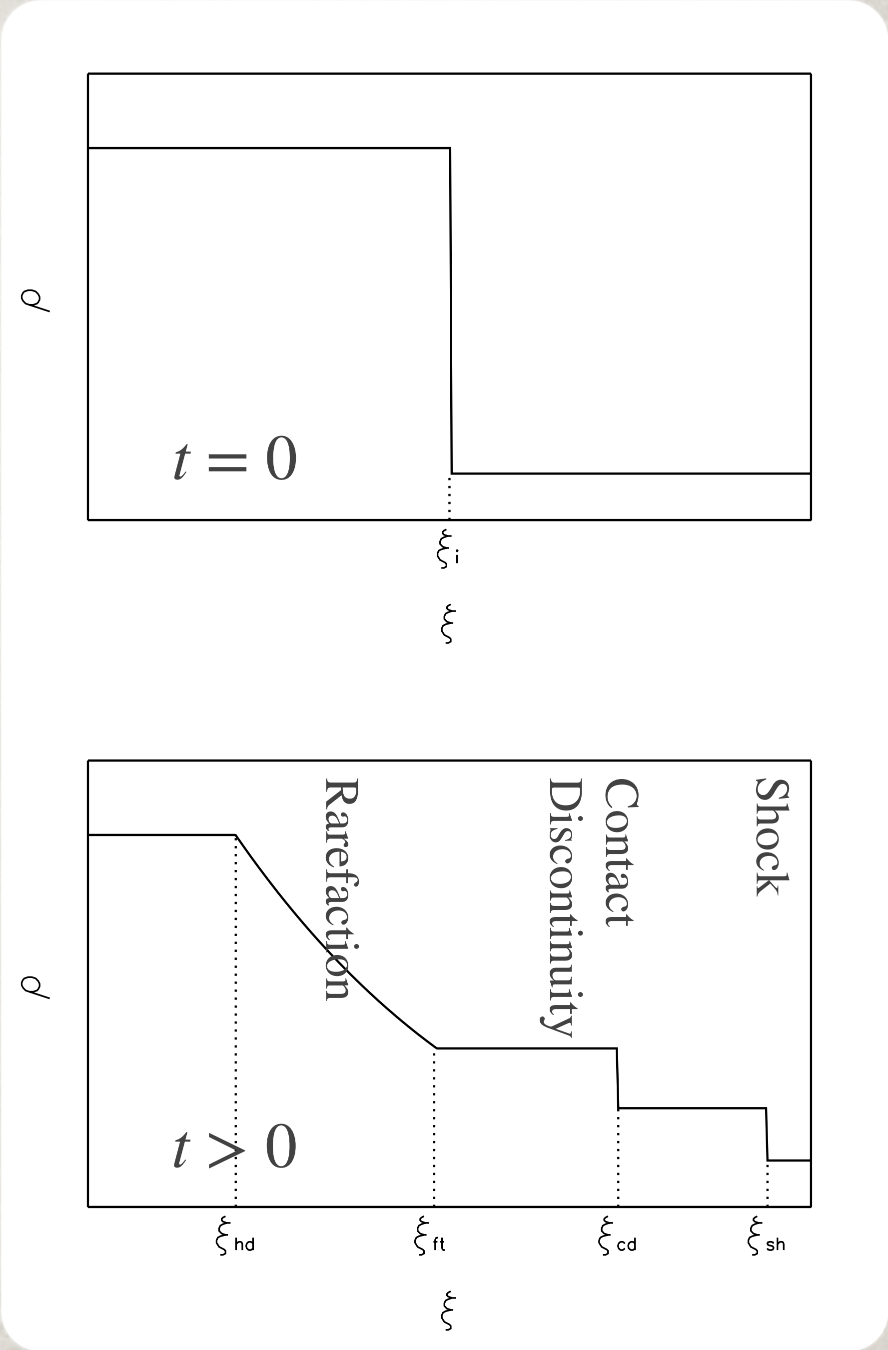
An alternative, from Godunov, is to calculate fluxes by assuming a Riemann problem at each interface.

For left wave, $P^{*} - P_l + W_l(u^{*} - u_l) = 0$

For right wave, $P^{*} - P_r - W_r(u^{*} - u_r) = 0$

where (P_r, u_r) & (P_l, u_l) characterize the unshocked right and left states, (P^{*}, u^{*}) are the unknown shocked state and

$$W_s = \rho_s c_s \left[1 + \frac{\gamma + 1}{2\gamma}\left(\frac{P^{*} - P_s}{P_s}\right)\right]^{1/2}$$



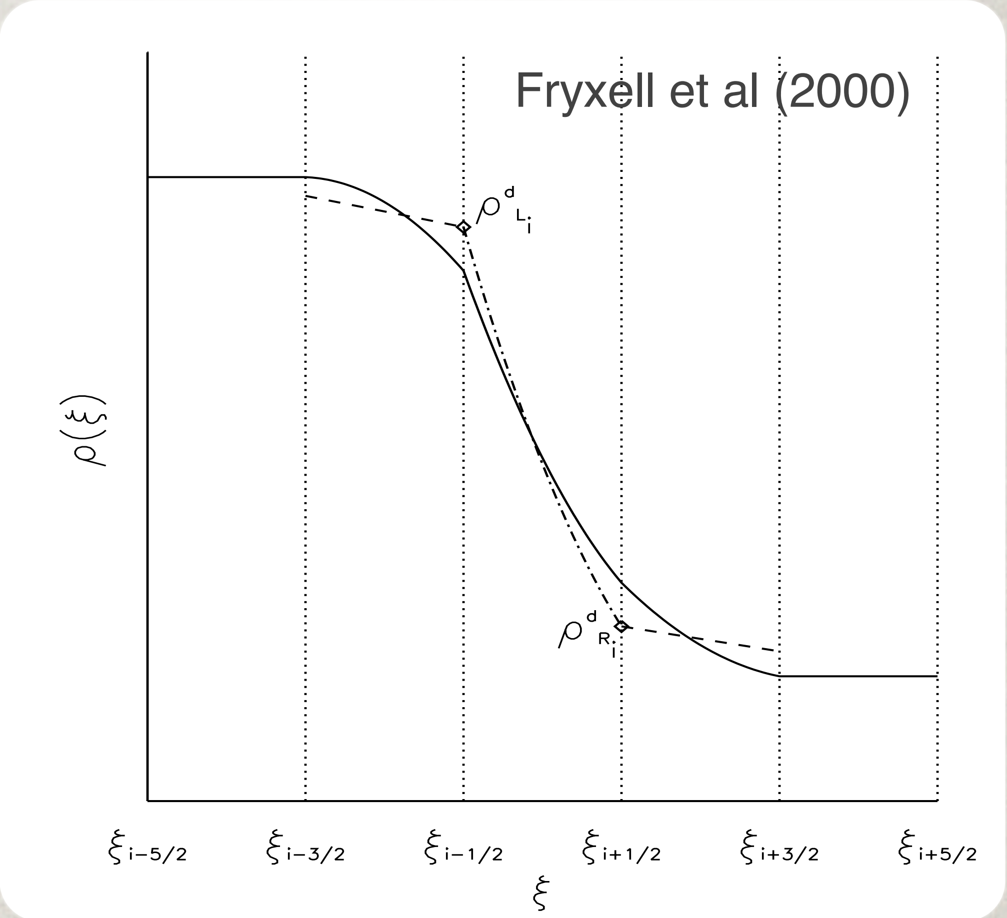
(P^{*}, u^{*}) can be calculated (iteratively) from the right and left wave equations and from these fluxes at the interfaces.

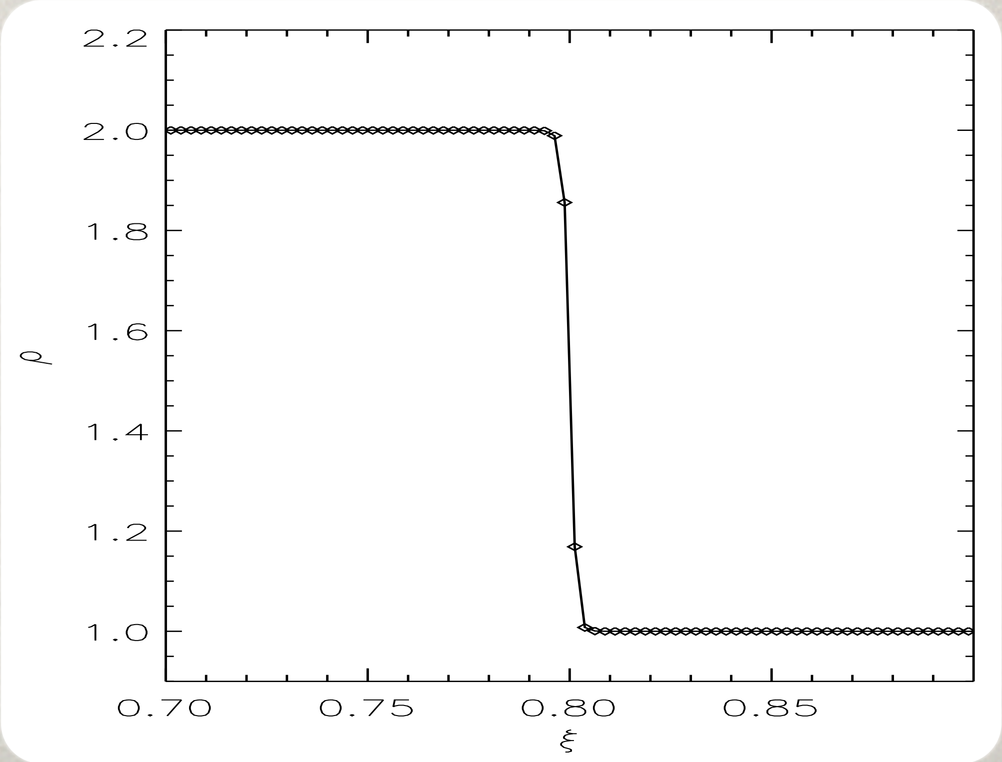
PPM

The Piecewise Parabolic Method, introduced by Colella & Woodward, improves on Godunov's method by using a piecewise parabolic reconstruction of flow variables (akin to Simpson's rule for integration) in place of piecewise constant.

It adds explicit steeping of contact discontinuities and flattening of overly narrow shocks.

FLASH and VH-1 are both implementation of PPM.





Heat Transport

To quantify how the tremendous heat of stellar interior is transferred to the surface, we need an equation for energy transport, $dT/dr = ?$

The form of the equation depends on the means by which the energy moves.

In general, there are **three** modes of heat transfer

Conduction: The transfer of energy through motions on the microscopic scale (atoms or molecules).

Convection: The transfer of energy through macroscopic fluid motions.

Radiation: The transfer of energy via electromagnetic radiation (other forms of radiation are possible).

Radiation Force

Blackbody radiation exerts a pressure equal to ⅓ of the radiation density.

$$P_{rad} = \frac{a}{3}T^4 \quad \text{where } a = \frac{4\sigma_{SB}}{c}$$

The decline in temperature from the center of the star toward the surface causes a spherical shell in the star, thickness *dr*, to experience a temperature gradient, *dT*.

Inner Surface:

$$P_{rad}(r) = \frac{a}{3}T^4$$

Outer Surface:

$$P_{rad}(r + dr) = \frac{a}{3}(T + dT)^4$$

$$= \frac{a}{3}T^4\left(1 + \frac{dT}{T}\right)^4 \approx \frac{a}{3}T^4\left(1 + 4\frac{dT}{T}\right)$$

This produces a net force,

$$F_{rad} = 4\pi r^2\left[P_{rad}(r) - P_{rad}(r + dr)\right] \approx -4\pi r^2\frac{a}{3}T^4\frac{4dT}{T}$$

$$\approx -\frac{16\pi}{3}r^2T^3dT$$

Radiation Momentum

An alternative way to approach the same problem is in terms of the momentum of photons that are absorbed.

The momentum of a photon is $p = E/c$.

The total rate of photon energy passing through the shell is $L(r)$, thus the total photon momentum is $L(r)/c$.

The fraction of the photons absorbed passing through a shell of thickness dr is

$$dI/I = -\, n(\mathrm{r})\, \sigma(r)\, dr \;= d\tau = -\, \rho(r)\, \kappa(r)\, dr \text{ if } d\tau \ll 1$$

The rate at which momentum is transferred to the matter by absorbed photons is a force,

$$F_{rad} = \frac{L(r)}{c}\frac{dI}{I} \quad = -\frac{L(r)}{c}\rho(r)\kappa(r)dr$$

Radiative Heat Transport

With 2 equations for F_{rad}, one in terms of a temperature change and the other in terms of a distance and opacity, a temperature gradient can be constructed.

$$F_{rad} \approx -\frac{16\pi}{3} r^2 T^3 dT = -\frac{L(r)}{c}\rho(r)\kappa(r)dr$$

The resulting gradient,

$$\frac{dT}{dr} = -\frac{3\rho(r)\kappa(r)L(r)}{16\pi ac\, T(r)^3 r^2} = -\frac{3\rho(r)\kappa(r)L(r)}{64\pi\sigma_{SB} T(r)^3 r^2}$$

is called the *equation of radiative energy transport*.

In the Sun, a typical value of this gradient is

$$\frac{\Delta T}{\Delta r} \approx \frac{T_{surface} - T_c}{R_\odot - 0} \approx \frac{5800\text{K} - 1.5\times 10^7\text{K}}{7.0\times 10^5\text{km}} \approx -20\text{ K km}^{-1}$$

Earth's Troposphere ≈ -7 K km^{-1}

Convective Transport

Convective energy transport is a turbulent process by which hotter, deeper parcels of fluid rise, forcing cooler fluid to sink, and carrying energy upward.

If you watch a pot of water on the stove, convection does not begin the moment the heat is applied to the bottom.

Instead, a temperature gradient between the heating element and the surface must build.

Convection begins only when it reaches a critical value.

This critical gradient is called the adiabatic temperature gradient.

Convective Stability

Consider a small blob of fluid, in a star or a cooking pot.

It has pressure P_b and density ρ_b, compared to the ambient pressure P and density ρ.

If the blob is perturbed upward, to a lower pressure region, $P + dP$ ($dP<0$), it will expand until $P_b + dP_b = P + dP$.

What happens next depends on the density. If it is denser than the new surroundings ($\rho_b+d\rho_b > \rho + d\rho$), it will sink back down and the fluid is stable.

However, if $\rho_b + d\rho_b < \rho + d\rho$, the blob is buoyant and will keep rising, marking the onset of convection.

Since initially $\rho_b = \rho$, the stability condition is $d\rho_b > d\rho$.

Adiabatic Expansion

If this blob moves upward rapidly, there is insufficient time for it to exchange heat with the ambient medium.

A process in which heat is neither gained or lost is called *adiabatic*. Adiabatic processes are also isentropic.

For an adiabatic process PV^{γ} is conserved.
γ is the adiabatic index (e.g., $\gamma = 5/3$ for monatomic gas).

Writing this in terms of density $P\rho^{-\gamma}$ and taking the derivative

$$\frac{d\rho_b}{\rho_b} = \frac{1}{\gamma}\frac{dP_b}{P_b}$$

Applying the initial condition, $\rho_b = \rho$ & $P_b = P$, and the requirement of hydrostatic equilibrium $dP_b = dP$,

$$d\rho_b = \frac{\rho_b}{\gamma}\frac{dP_b}{P_b} = \frac{\rho}{\gamma}\frac{dP}{P} = \frac{\rho}{\gamma P}\frac{dP}{dr}dr$$

Stability Condition

Applying this expression for $d\rho_b$ to the stability condition

$$d\rho_b = \frac{\rho}{\gamma P}\frac{dP}{dr}dr > d\rho = \frac{d\rho}{dr}dr$$

Simplifying yields
$$\frac{1}{\gamma P}\frac{dP}{dr} > \frac{1}{\rho}\frac{d\rho}{dr}$$

To compare the convective stability to the radiative energy transport requires conversion to dT/dr. For an ideal gas,

$$\frac{dP}{dr} = \frac{\rho k}{\mu m_p}\frac{dT}{dr} + \frac{kT}{\mu m_p}\frac{d\rho}{dr} = \frac{P}{T}\frac{dT}{dr} + \frac{P}{\rho}\frac{d\rho}{dr}$$

Rearrangement reveals,
$$\frac{1}{\rho}\frac{d\rho}{dr} = \frac{1}{P}\frac{dP}{dr} - \frac{1}{T}\frac{dT}{dr}$$

Thus the stability condition can be written,
$$\frac{1}{\gamma P}\frac{dP}{dr} > \frac{1}{P}\frac{dP}{dr} - \frac{1}{T}\frac{dT}{dr}$$

Adiabatic Gradient

Grouping the *dP/dr* terms, and multiplying by *T* yields

$$-\left(1-\frac{1}{\gamma}\right)\frac{T}{P}\frac{dP}{dr} > -\frac{dT}{dr}$$

The left-hand side of this equation is called the *adiabatic temperature gradient*. At any point that the actual temperature gradient (the right-hand side) obeys this relation, convection is suppressed and radiative transport dominates.

Where this relation is not met, convection results, forcing the actual temperature gradient toward the adiabatic temperature gradient.

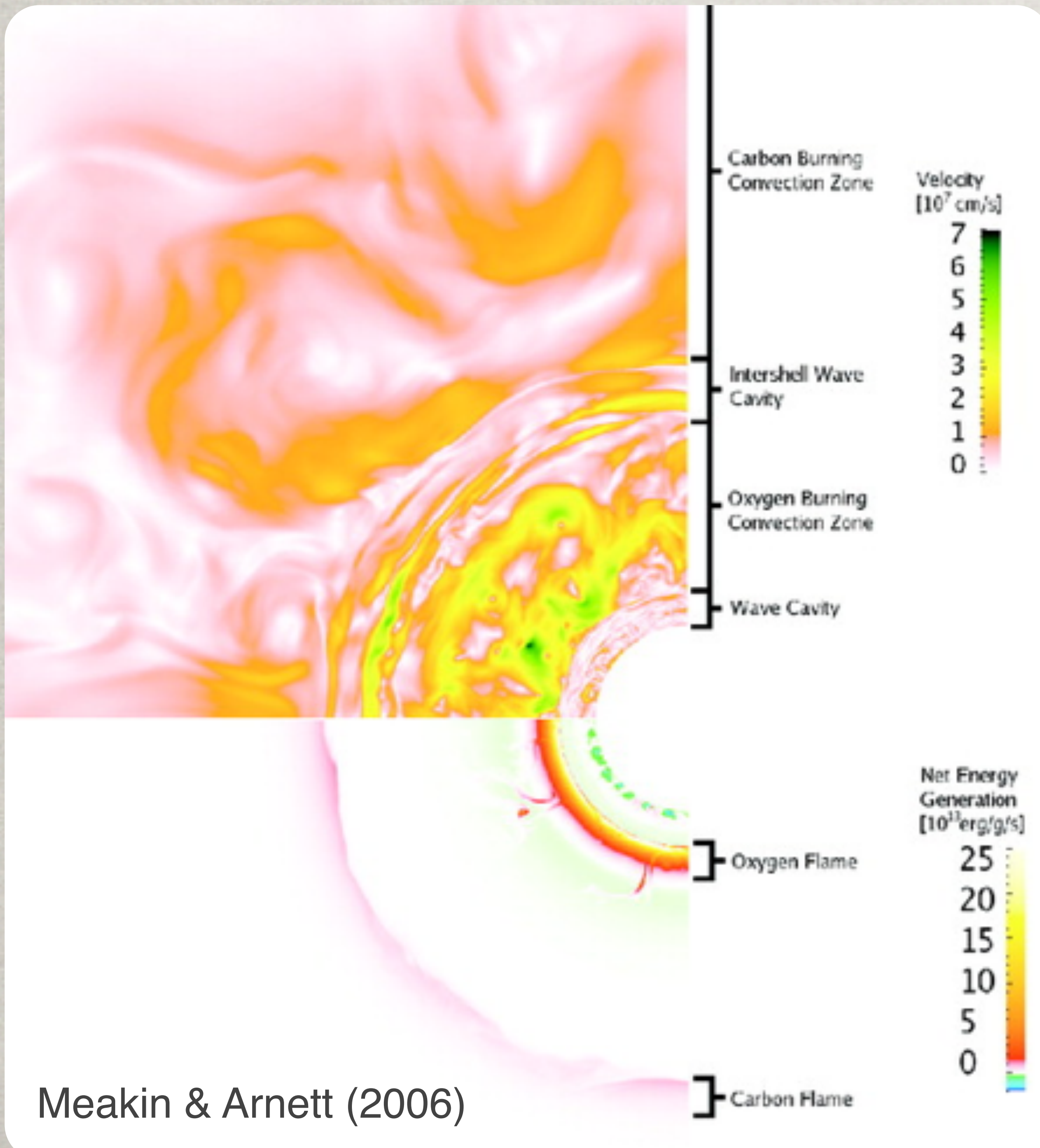
The equation of convective energy transport is therefore

$$\frac{dT}{dr} = \left(1-\frac{1}{\gamma}\right)\frac{T(r)}{P(r)}\frac{dP}{dr}$$

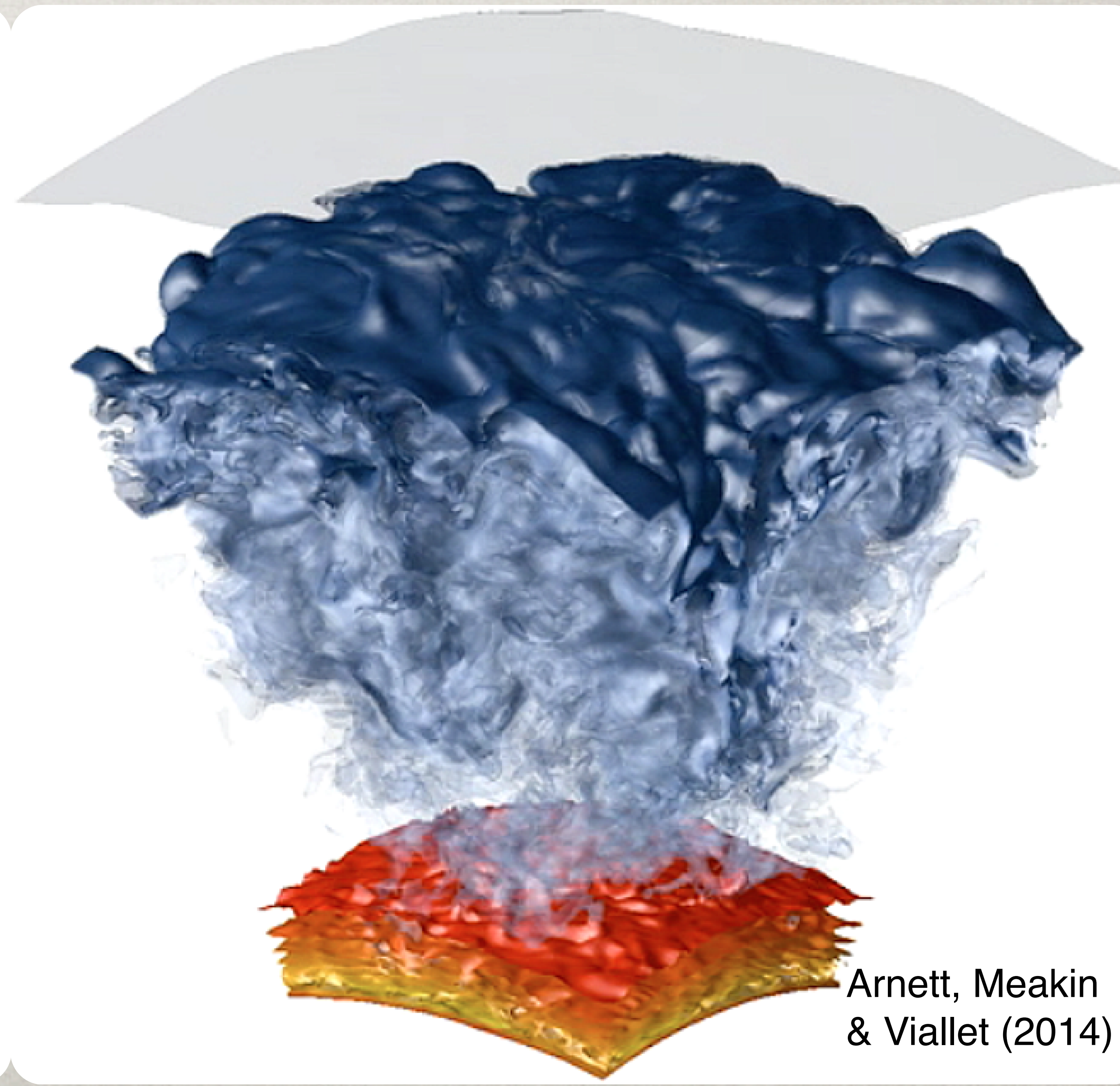
Stellar Convection

Convection is very important in stellar evolution.

Its proper treatment is a topic of much research (& debate).



Meakin & Arnett (2006)



Arnett, Meakin & Viallet (2014)

Rayleigh-Taylor

Another instability of interest to nuclear astrophysics is the Rayleigh-Taylor instability.

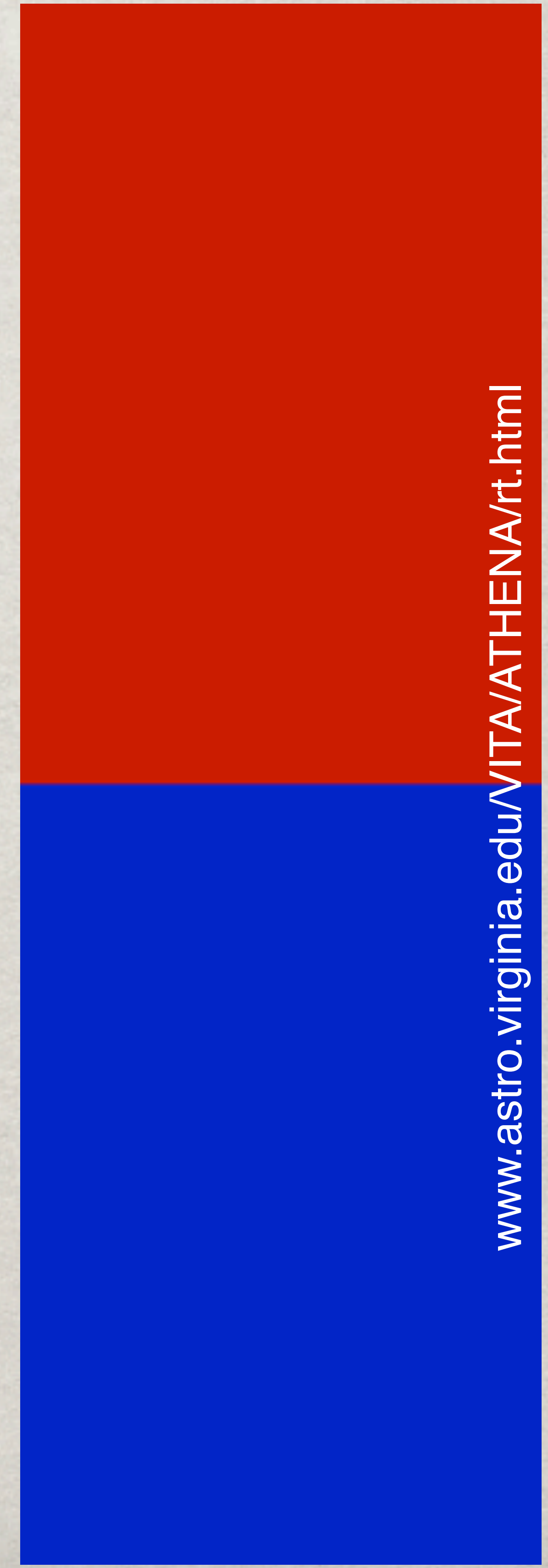
When a denser fluid lies "over" a lighter fluid, the amplitude, η, of a perturbation of the interface of wavelength $2\pi/k$ will grow exponentially

$$\eta(t)=\eta_0 \exp[(Agk)^{1/2} t]$$

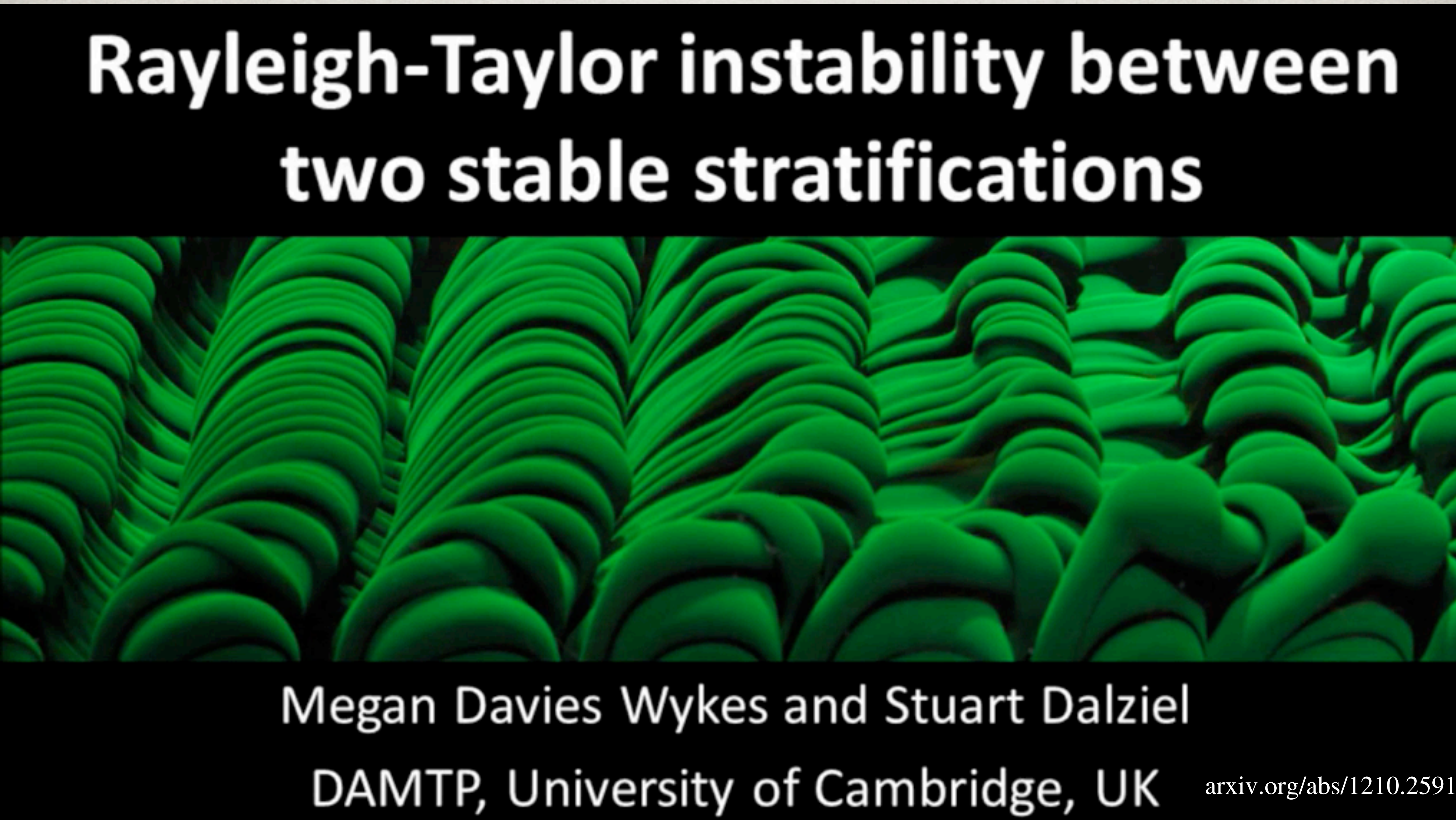
where A is the Atwood number $A \equiv \frac{\rho_u - \rho_l}{\rho_u + \rho_l}$

This ideal fluid solution is modified by viscosity and diffusivity, which inhibit high-wavenumber (short wavelength) growth.

This also occurs when acceleration, as by a shock, takes the place of an effective gravity.



3D RT



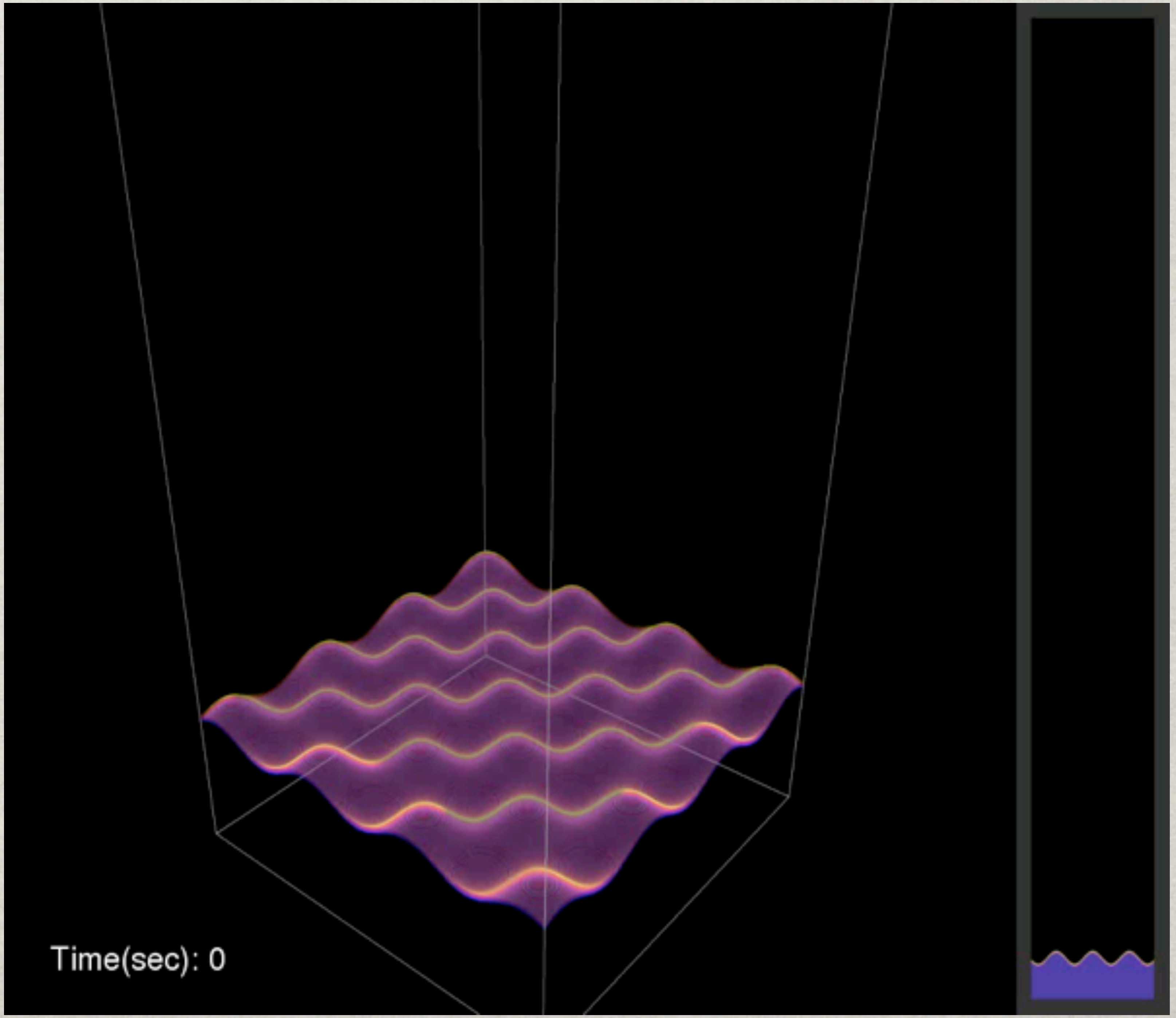
arxiv.org/abs/1210.2591

Thermonuclear RT

With ongoing energy production from nuclear reactions, the hot, low density matter remains lower in density as it rises.

This allows successive generations of Rayleigh-Taylor instability to build on each other.

Here we see a narrow region from a thermonuclear supernovae.



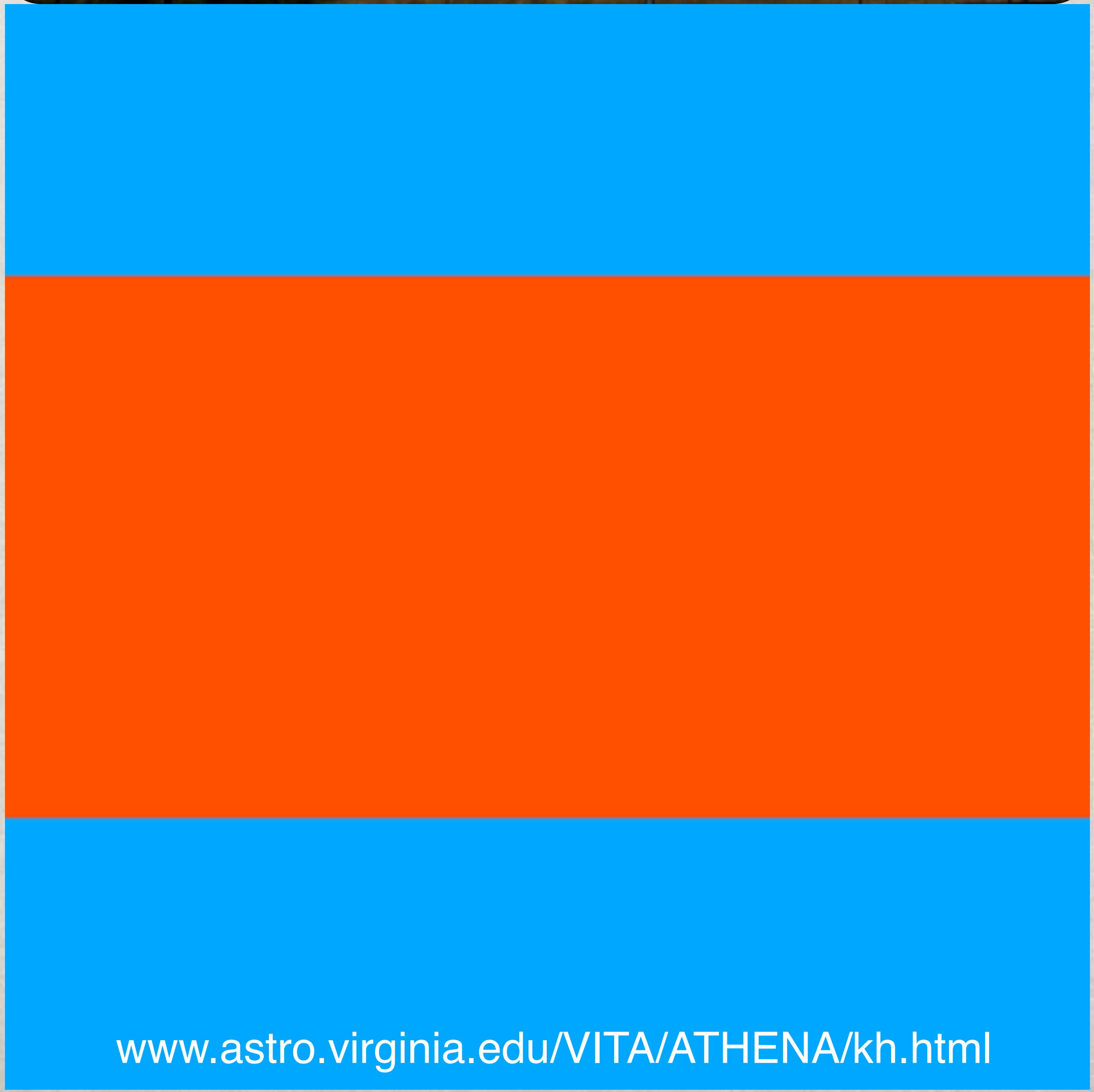
Kelvin-Helmholtz

Another commonly encountered hydrodynamic instability is the Kelvin-Helmholtz instability.

It occurs when a velocity shear exists between two layers in a fluid.

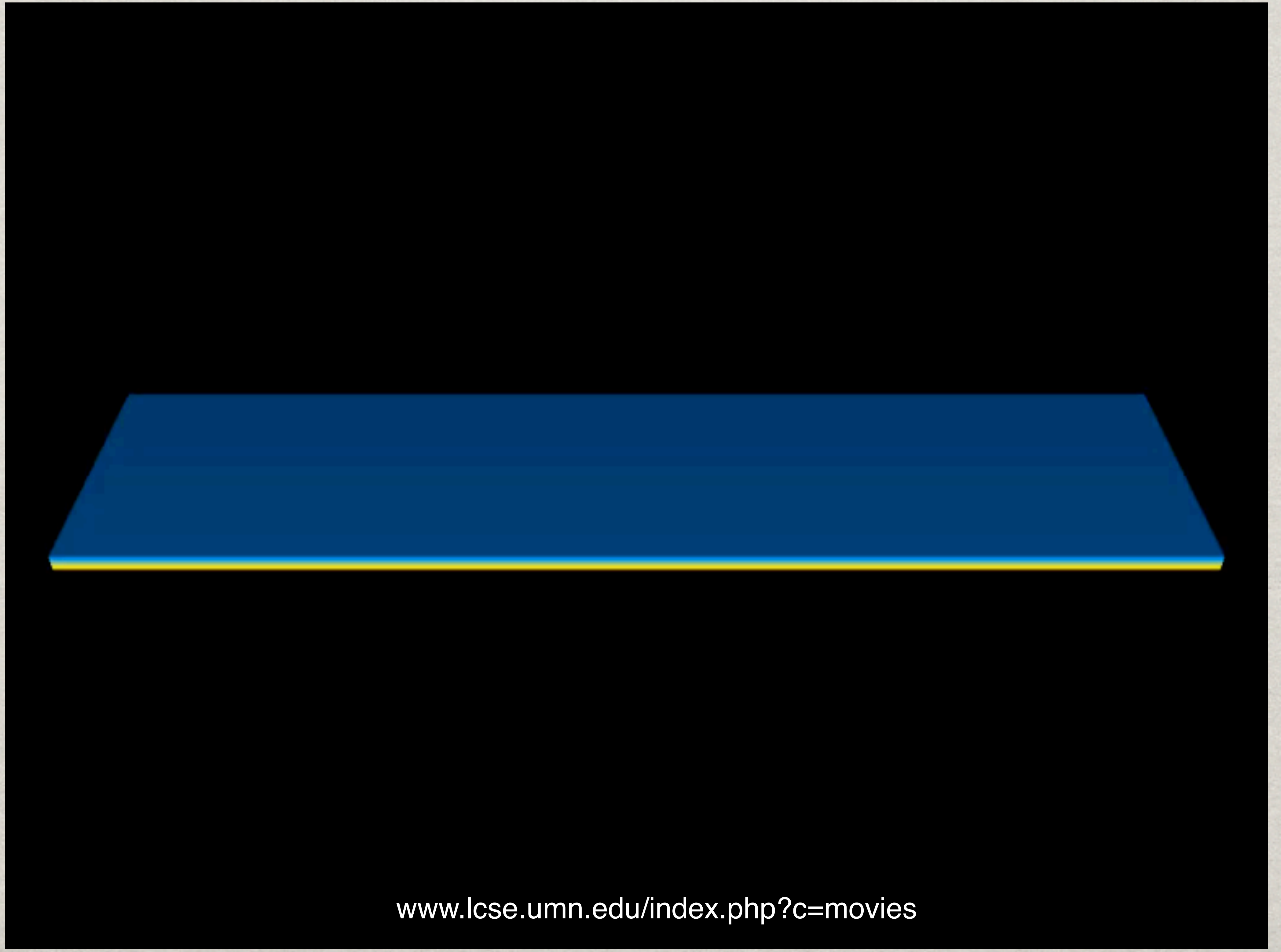
The motion of the higher velocity fluid introduces vorticity at the interface.

The unstable interface can grow to include the entire volume.



Kelvin-Helmholtz in 3D

In 3D, KH starts as in 2D, but soon develops lateral motions.

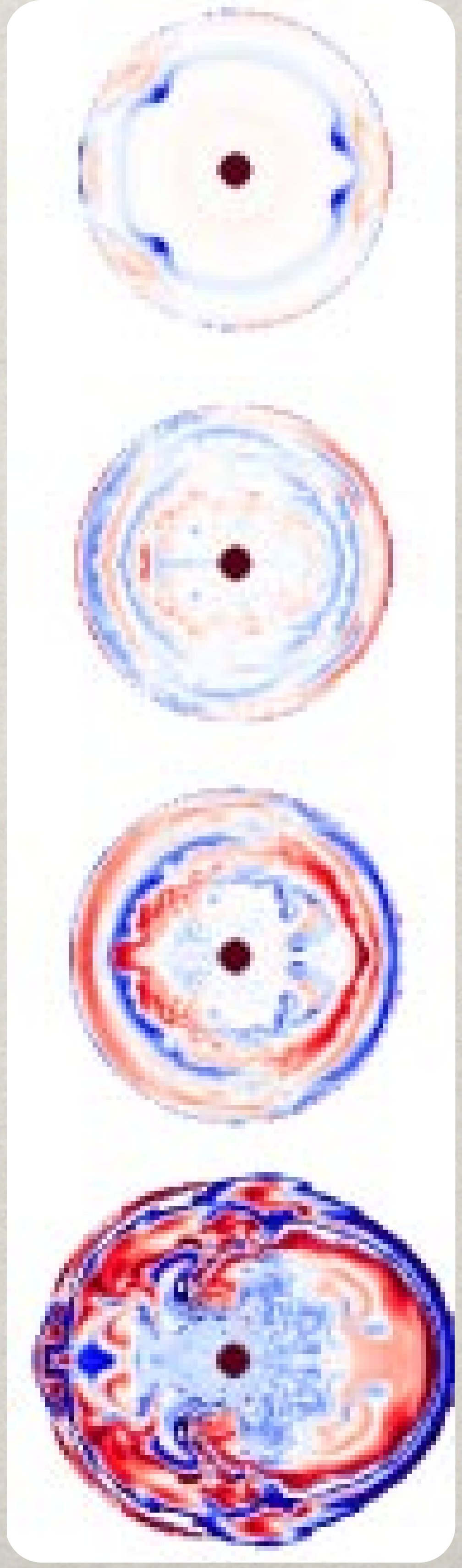


Standing Accretion Shock

Numerical simulations of a standing accretion shock, a phenomenon thought to occur during core-collapse supernovae, led to the discovery of a new instability by Blondin & Mezzacappa (2003).

Studied by many groups using simplified hydrodynamic models and seen by most groups doing realistic supernova models, in cases where the shock stalls for a sufficient time.

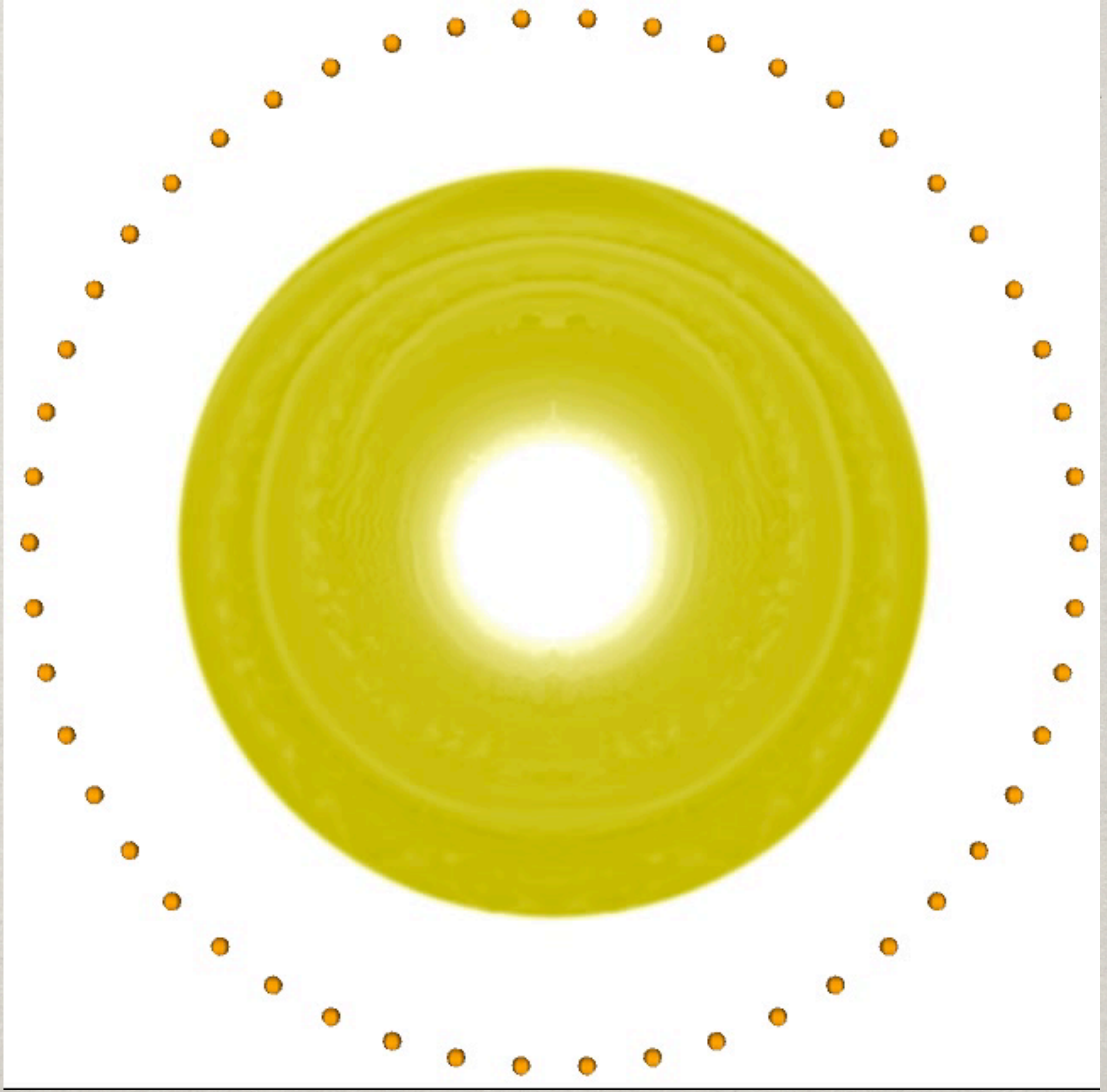
However, the mechanism is still a subject of debate, with some arguing it is an acoustic instability and others arguing advective-acoustic.



SASI in 2D

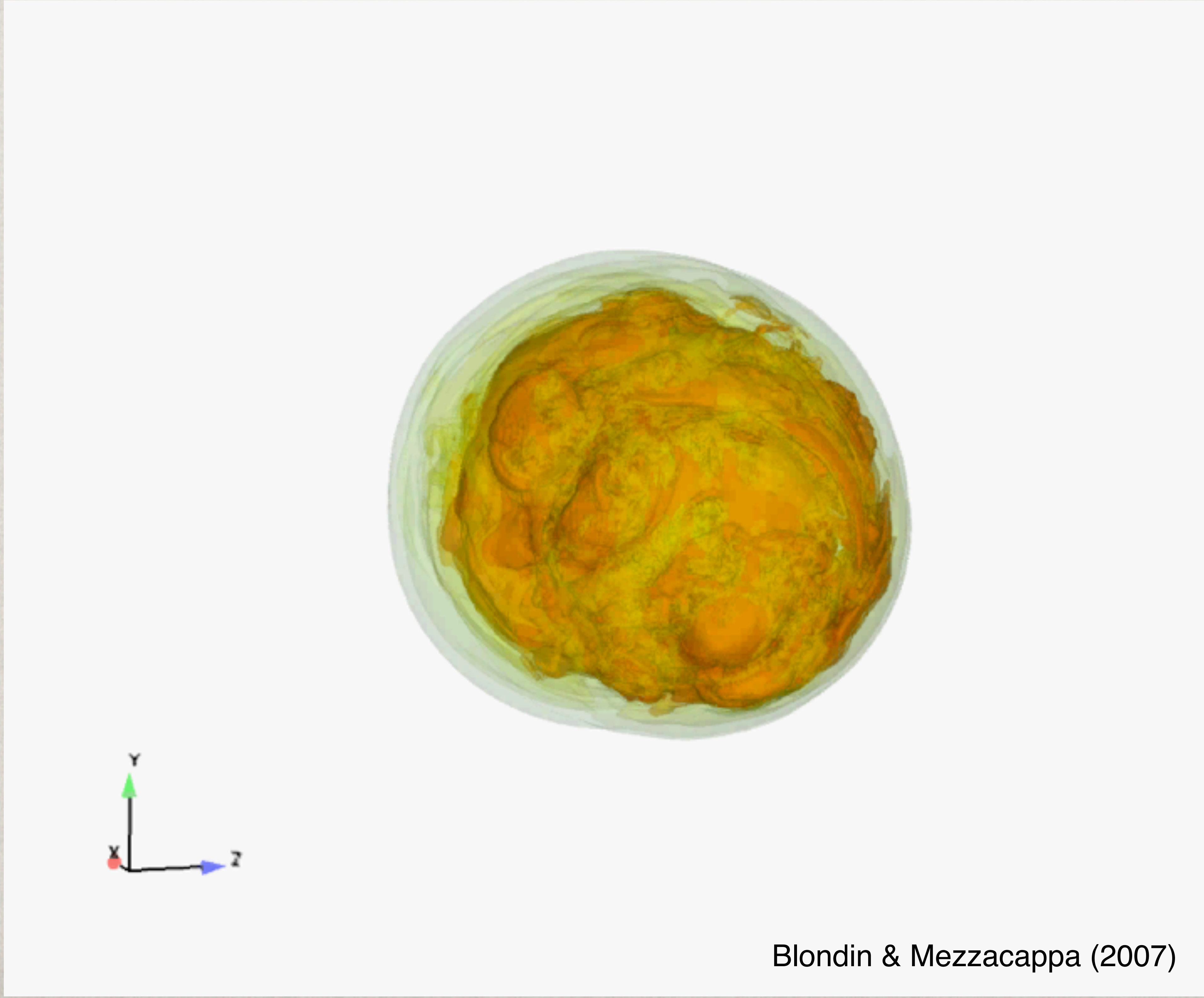
In 2D, the SASI is dominated by a sloshing mode dominated by the $l = 1$ component.

The net effect is to push the accretion shock boundary outward.



SASI in 3D

In 3D, the $l = 1$ sloshing mode transforms to an $m = 1$ spiral mode.

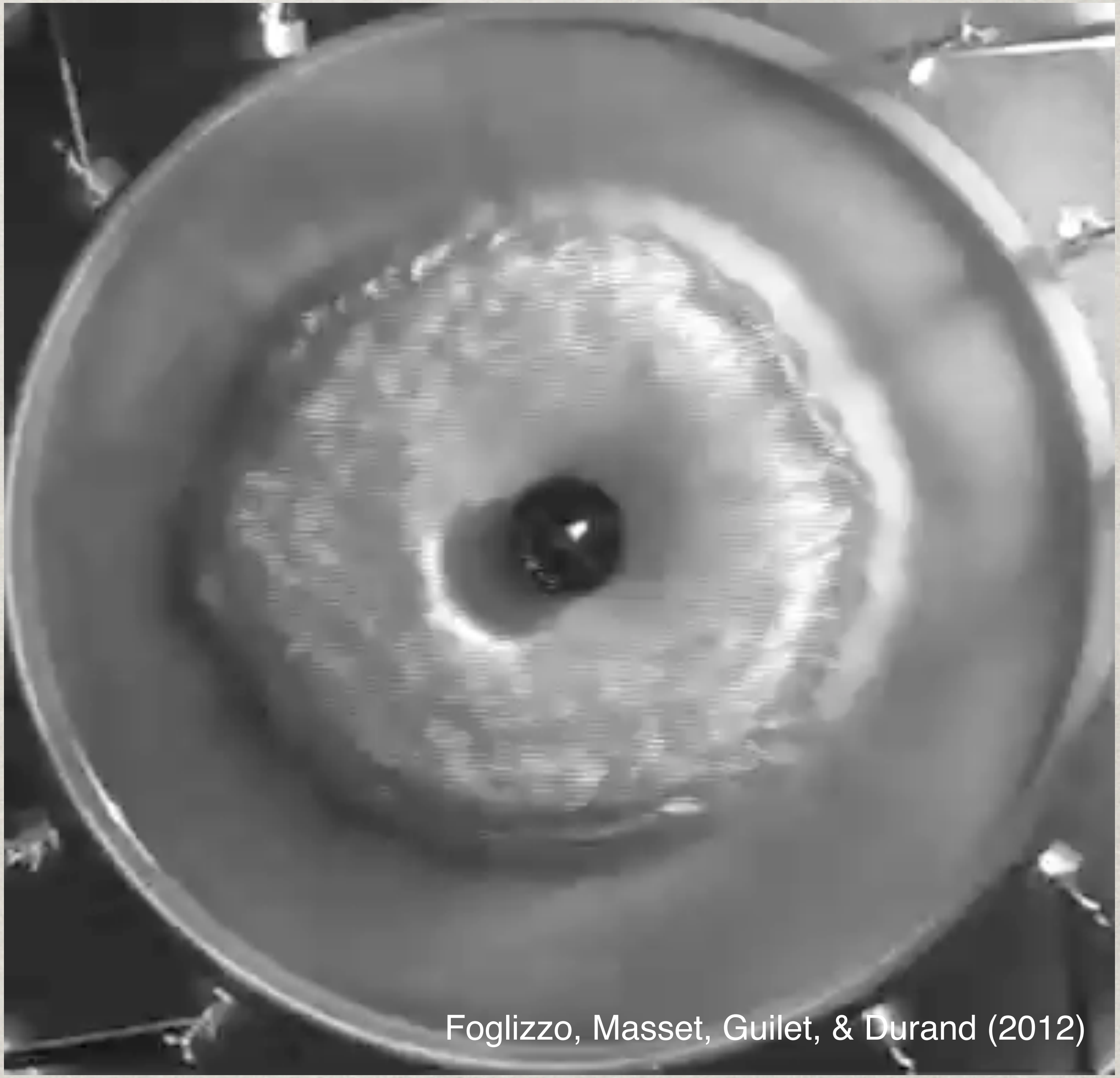


Blondin & Mezzacappa (2007)

Shallow Water Supernova?

Recently, Foglizzo and collaborators discovered a similar instability in a shallow water system, SWASI, a Shallow Water Analogue of a Shock Instability.

These also show initial sloshing modes that sometimes transition to spiral modes.


Foglizzo, Masset, Guilet, & Durand (2012)

Conclusions

Equations of Hydrodynamics are conservation equations for mass, momentum and total energy, as modified by external surface and body forces, internal energy generation and surface energy flow.

The Equation of State closes the system of equations.

Godunov methods, including PPM, use the solution of Riemann problems to calculate fluxes across cell boundaries, allowing better capture of shocks and other sharp features.

Convection and a number of other instabilities (Rayleigh-Taylor, Kelvin-Helmholtz, SASI) are important for nuclear astrophysics by altering the fluid flow in which nuclear reactions occur and the distribution of the newly formed elements.