Effect of Neutrino Magnetic Moment on Big Bang Nucleosynthesis

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Here the effect of the neutrino magnetic moment on the primordial abundances is preliminarily examined. Considering that additional couplings to electrons and photons can keep neutrinos in thermal equilibrium past 1 MeV, we look at consequences of including the magnetic pair-production process $e^+ + e^- \leftrightarrow \nu + \bar{\nu}$ during big bang nucleosynthesis. Although a limit on the neutrino magnetic moment using observed abundances was not able to be conclusively obtained here, the following illustrates how such a limit could be obtained if more than pair-production is considered.

PACS numbers:
Keywords:

I. LIMITS ON THE NEUTRINO MAGNETIC MOMENT AND APPLICATION TO BBN

Using astrophysical considerations to place upper bounds on the neutrino magnetic moment is not a new endevor. For example, examining the energy loss rate due to photon decay into a neutrino and antineutrino pair in the Sun [1] and low-mass red giant stars [2], authors have found limits $\mu_\nu < 10^{-10}\mu_B$ and $\mu_\nu < 3 \times 10^{-12}\mu_B$, respectively. However, the most reliable upper bounds come from reactor experiments, such as TEXONO and GEMMA, which use neutrino-electron scattering of low recoil electrons to obtain limits $\mu_\nu < 2.2 \times 10^{-10}\mu_B$ [4] and $\mu_\nu < 2.9 \times 10^{-11}\mu_B$ [3], respectively. The standard model, minimally extended to include massive neutrinos, predicts an even smaller value for the magnetic moment of Dirac neutrinos, on the order of $10^{-19} - 10^{-20}\mu_B$. However, since no astrophysical or experimental limits have been able to exclude down to the standard model prediction, there is still room for the inclusion of beyond the standard model physics to enhance the neutrino magnetic moment.

The effect of a higher magnetic moment on big bang nucleosynthesis is that the magnetic coupling, usually taken to be negligibly small, could keep the neutrinos in thermal equilibrium with the relativistic electrons and photons past the 1 MeV regime and thus further into the BBN epoch. Since the neutrino-electron interactions are the dominate neutrino equilibrium processes, here as a preliminary examination, we consider the enhancement of the equilibrium duration due to the magnetic pair-production process $e^+ + e^- \leftrightarrow \nu + \bar{\nu}$. The cross section for this process in the center of mass frame as a function of neutrino magnetic moment is given by [5]

$$\sigma = \frac{e^2\kappa_\nu^2}{24\pi}\left(1 - \frac{4m_e^2}{4E^2}\right)^{1/2} \sim \frac{\alpha^2\pi}{6m_e^2}\left[\frac{\mu_\nu}{\mu_B}\right]^2 \tag{I.1}$$

where E is the electron energy and μ_ν is the effective neutrino magnetic moment is units of Bohr magnetons. Inclusion of this process can keep the neutrinos coupled to the electrons past the traditional 1 MeV point, but of course, can only effect BBN up to 0.511 MeV where the cross section above is no longer applicable since the thermal neutrinos do not have enough energy to produce electron-positron pairs. We next find an expression for the neutrino decoupling temperature when this process is included.

II. NEUTRINO DECOUPLING TEMPERATURE

In order to find the neutrino decoupling temperature, consider that once the expansion rate is greater than the reaction rate, the neutrinos will no longer interact significantly. Thus the decoupling temperature is obtained by finding the temperature at which the reaction rate is comparable to the expansion rate, that is when

$$\Gamma \sim H \tag{II.1}$$

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where H is the expansion rate given by $H \sim \frac{T^2}{M_{pl}}$ and the reaction rate with the inclusion of magnetic pair-production is

$$\Gamma = n \langle \sigma v \rangle \sim n G_F^2 T^2 + n \frac{\alpha^2 \pi}{6 m_e^2} \left[\frac{\mu_\nu}{\mu_B} \right]^2 \tag{II.2}$$

Since the neutrinos are relativistic $n \sim T^3$ which gives the following cubic equation for the neutrino decoupling temperature

$$T^3 + \frac{1}{G_F^2} \frac{\alpha^2 \pi}{6 m_e^2} \left[\frac{\mu_\nu}{\mu_B} \right]^2 T - \frac{1}{G_F^2 M_{pl}} \sim 0 \tag{II.3}$$

then defining $r = \frac{1}{2 G_F^2 M_{pl}}$ the solution to the above cubic equation can be found to be

$$T_\nu^{decoup} \sim \left(r + \sqrt{\left(\frac{\alpha^2 \pi}{18 G_F^2 m_e^2} \left[\frac{\mu_\nu}{\mu_B} \right]^2 \right)^3 + r^2} \right)^{1/3} - \left(\left| r - \sqrt{\left(\frac{\alpha^2 \pi}{18 G_F^2 m_e^2} \left[\frac{\mu_\nu}{\mu_B} \right]^2 \right)^3 + r^2} \right| \right)^{1/3} \tag{II.4}$$

which is of course only applicable if the temperature is above 0.511 MeV. In Fig.1, the above neutrino decoupling tempertaure as a function of neutrino magnetic moment is shown. It is evident that when include the magnetic pair-production process alone, the decoupling temperature does not change from that given by the weak interaction alone until $\mu_\nu \sim 10^{-10} \mu_B$. This a region ruled out by the GEMMA experiment. However, since here we wish to explore the general procedure for examining the effect on abundances with additional neutrino couplings, we proceed even though the magnetic moment range we are to explore is not physically realizible.

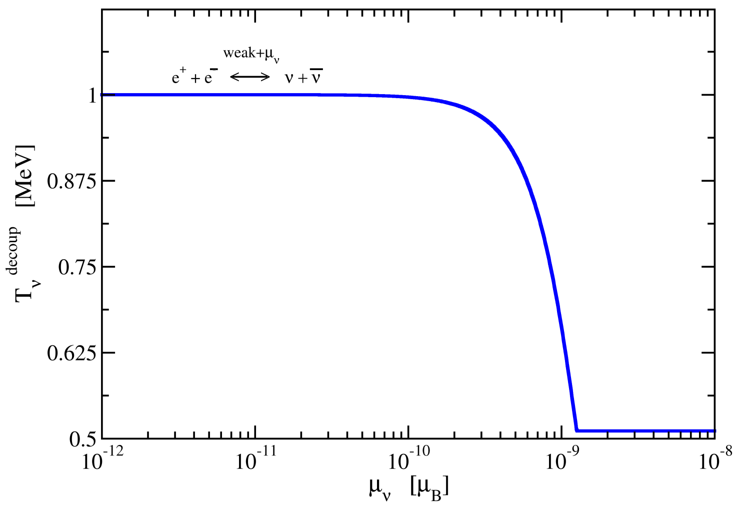

FIG. 1: (Color online) The neutrino decoupling temperature as a function of magnetic moment when only the magnetic pair-production process is considered.

III. EFFECT ON PRIMORDIAL ABUNDANCES

Keeping neutrinos coupled to the system longer can change the evolution of primordial abundances since it affects processes such as

$$e^- + p \leftrightarrow n + \nu_e \tag{III.1}$$
$$e^+ + n \leftrightarrow p + \bar{\nu}_e \tag{III.2}$$

which in turn affects processes such as $n + p \leftrightarrow d + \gamma$ and so on. Here we outline the procedure for implementing further coupling in a code which will then calculate abundances and reveal how they change (or don't change).

Using the big bang nucleosynthesis code developed by F.Timmes, we can explore the effect of changing the neutrino decoupling temperature. Although this code has issues with outputting the correct order of magnitude for the abundance for ^7Li, it is useful here for us to analyze the general trend relative to the output prior to any adjustments to include neutrino magnetic moment. The relevant quantity to adjust in the code is the neutrino temperature which is used to calculate the energy density

$$\rho \sim T_\gamma^4 \left[1 + \frac{7}{8} N_\nu \left(\frac{4}{11}\right)^{4/3}\right] \tag{III.3}$$

where

$$\left(\frac{4}{11}\right)^{1/3} = \frac{T_\nu}{T_\gamma} \tag{III.4}$$

assumes that the neutrinos are already decoupled. To keep neutrinos coupled to the system we must adjust this. However we must take care that we treat the point where the temperature reaches 0.511 MeV correctly. Thus the condition is the following

$$T_\nu^{decoup} > m_e \Longrightarrow \begin{cases} \rho \sim T^4 \left[1 + \frac{7}{8} N_\nu\right] & \text{if } T > T_\nu^{decoup} \\ \rho \sim T^4 \left[1 + \frac{7}{8} N_\nu \left(\frac{4}{11}\right)^{4/3} \xi\left(\frac{m_e}{T}\right)^{4/3}\right] & \text{if } T < T_\nu^{decoup} \end{cases} \tag{III.5}$$

$$T_\nu^{decoup} < m_e \Longrightarrow \begin{cases} \rho \sim T^4 \left[1 + \frac{7}{8} N_\nu\right] & \text{if } T > m_e \\ \rho \sim T^4 \left[1 + \frac{7}{8} N_\nu \left(\frac{4}{11}\right)^{4/3} \xi\left(\frac{m_e}{T}\right)^{4/3}\right] & \text{if } T < m_e \end{cases} \tag{III.6}$$

where $\xi(x)$ is the integral which takes into account the contribution from electrons which have become non-relativistic given by [6]

$$\xi(x) = 1 + \frac{45}{2\pi^4} \int_0^\infty y^2 dy \left[\sqrt{x^2 + y^2} + \frac{y^2}{3\sqrt{x^2 + y^2}}\right] \left[\exp\left(\sqrt{x^2 + y^2}\right) + 1\right]^{-1} \tag{III.7}$$

In practice the integral is not taken to infinity and the upper bound and lower bounds implemented by default in the code are $y_{max} = 50$ and $y_{min} = 10^{-6}$. Keeping these bounds is consistent with our conditions given in Eqns. III.5 and III.6 even though we require $T \leq T_\nu^{decoup}$ when we start the integral. This can be seen by considering that for the lower bound

$$y_{min} = \frac{p_{min}}{T_{max}} = \frac{p_{min}}{T_\nu^{decoup}} \to 0 \tag{III.8}$$

since the electron can in principle go to having only a rest energy. Similarly for the upper bound, if cut off the integral at a sufficiently low temperature ($T_{min} \sim 0.01$ MeV), having $y_{max} \sim 50$ implies taking $p_{max} = m_e$ which is indeed where the author of [6] suggests this integral to become important. Thus no additional constraints on the bounds of the integral need to be enforced when one changes the neutrino decoupling temperature.

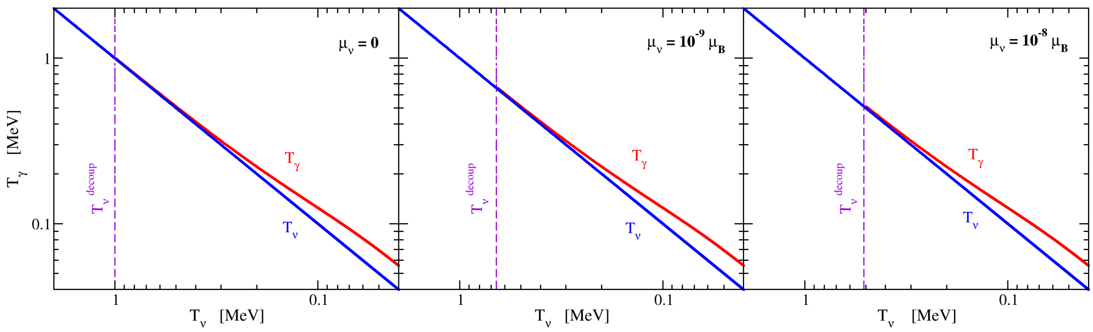

FIG. 2: (Color online) Left: Neutrino (blue) and photon(red) temperature evolution for $\mu_\nu = 0$; Center: Neutrino and photon temperature evolution for $\mu_\nu = 10^{-9}\mu_B$; Right: Neutrino and photon temperature evolution for $\mu_\nu = 10^{-8}\mu_B$.

A first check that the neutrino temperature was indeed following the desired behavior as in Eqns. III.5 and III.6 was to produce a plot of the photon temperature as a function of the neutrino temperature for different values of the neutrino magnetic moment. As seen in Fig.2 for a magnetic moment of zero, we are decoupling at 1 MeV as should be the case, while for the intermediate value of $10^{-9}\mu_B$ we decouple at about 0.65 MeV, and for a value of the magnetic moment which is forced to decouple at 0.511 MeV, such as $10^{-8}\mu_B$, we are indeed seeing the proper behavior.

Now that the proper decoupling temperature behavior has been confirmed, we first produce the Schramm plot (which illustrates how abundances depend on the baryon-to-photon ratio) for different values of the neutrino magnetic moment. To examine the maximal effect of the magnetic moment, only the case for $\mu_\nu = 0$ (black line) and $\mu_\nu = 10^{-8}\mu_B$ (blue line) are plotted in Fig.3.

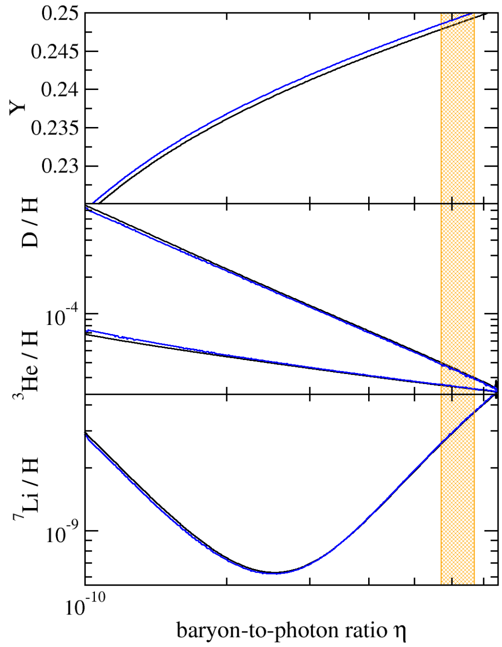

FIG. 3: (Color online) Schramm plot with (blue) and without (black) neutrino magnetic moment coupling of $10^{-8}\mu_B$; the vertical region (orange) is the obervational baryon-to-photon ratio range of $(5.7 - 6.7) \times 10^{-10}$.

Clearly, in the observed range of η, only the ^4He abundance displays a discernible effect. However, it should be stressed that the Schramm plot shown here only illustrates a relative change since the output abundances of the code prior to any alterations are all shifted relative to those given by the Particle Data Group [7], with ^7Li being an order of magnitude off.

The last goal in examing the effect of a non-negligible neutrino magnetic moment was to use the observed abundance ranges in order to find an upper limit on the magnetic moment. For the magnetic pair-production process which we are considering the most interesting range of neutrino magnetic moment is between 2×10^{-10} and $2 \times 10^{-9}\,\mu_B$ (as is evident from Fig.1). Thus to explore this region in more detail, and possibly obtain the desired upper limit, a plot of abundance versus magnetic moment was produced and is shown in Fig.4.

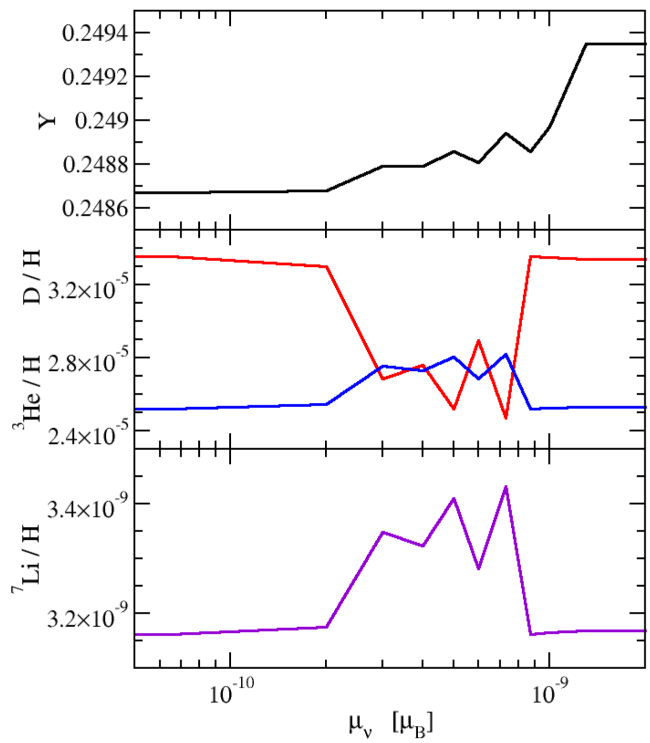

FIG. 4: (Color online) Schramm-like plot but with abundances as a function of the neutrino magnetic moment.

This plot may be revealing some interesting behavior in the intermediate 2×10^{-10} to 9×10^{-10} region, or it could just be noise. There are some general trends which match up, for example peaks in Y result in drops in D/H along with a rise in $^3He/H$. However, it is unclear why the abundances in this region should be so different from the abundance values at $1 \times 10^{-9}\mu_B$ since at this value of magnetic moment the neutrinos have been coupled for approximately as long as when $\mu_\nu \sim 8 \times 10^{-10}$. Since it seems ^4He has the most consistent trend (and the most overall variation) for the magnetic pair-production case considered here, in order to examine how this curve fits within the observationally allowed abundance regions, the $\mu_\nu = 0$ point was adjusted to be at the mean value given in [7] and all other data points were shifted accordingly. The resultant plot is shown in Fig.5.

From Fig. 5 see that we are unable to yield a limit of any kind as all points fall in the allowed region. However this plot nicely illustrates how a more thorough treatment of the problem (for example with the inclusion of electron-neutrino magnetic scattering) could allow limits on the neutrino magnetic moment to be placed via their agreement (or disagreement) with observed primordial abundances.

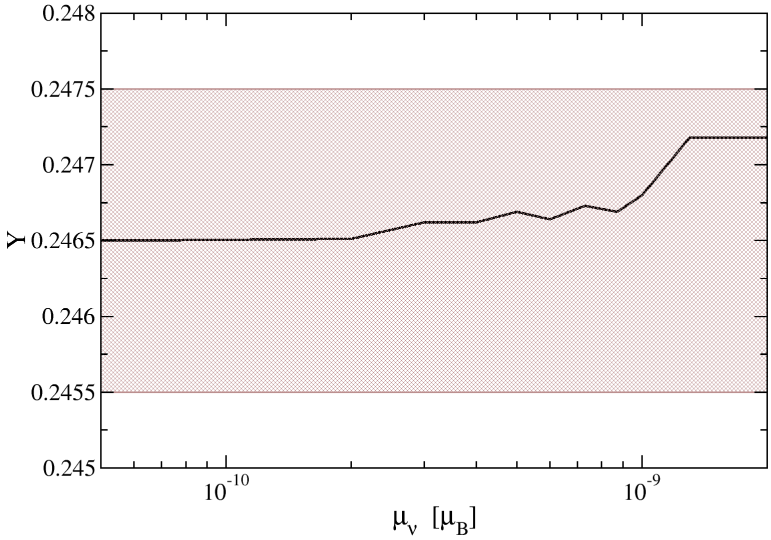

FIG. 5: (Color online) Adundance of ^4He as function of magnetic moment with the $\mu_\nu = 0$ point adjusted to be at the mean value given in [7]. The observationally allowed region for the abundance is shown.

IV. CONCLUSIONS AND OUTLOOK

Although some interesting results were generated, there are of course several problems. The first, and most important, is that since here we examined the effect of the subdominate magnetic pair-production process $e^+ + e^- \leftrightarrow \nu + \bar{\nu}$, we will not see any effects in abundances until $\mu_\nu \sim 10^{-10}\mu_B$ which is an unphysical region as it has been ruled out by the GEMMA experiment. A more thorough treatment of this problem would also include the $\nu + e^- \leftrightarrow \nu + e^-$ magnetic scattering channel whose cross section is potentially enhanced relative to the pair-production cross section when the electron recoil is low [5]. This process was not included here since we wished to observe overall trends and examine how much of an effect the subdominate pair-production process could have on abundances.

The other issue with this analysis is that the abundances given by the code, particularly for ^7Li, are off which makes the goal of using allowed abundance regions to find limits on the magnetic moment difficult. And so to do this analysis properly, either the rates the code uses must be cross checked, or a different code must be used. Additionally, it is unclear whether the output abundances of the code in the most interesting region of 2×10^{-10} - $9 \times 10^{-10}\mu_B$ can be interpreted as proper nucleosynthesis behavior, or simply numerical noise. Thus cross checking the results of the code used here with a different BBN code would not only be enlightening, but necessary.

V. APPENDIX: IMPLEMENTING ADDITIONAL COUPLING IN BBN CODE

A. Defining the Decoupling Temperature

In order to vary the magnetic moment and feed the code the new decoupling temperature, the eta1 main loop at the beginning was hijacked and a file records the variable $0.511/T_\nu^{decoup}$ to be referenced later.

```
!for bigbang munu
            open(unit=2,file='bbang_NMM.dat',status='unknown')
            write(6,77) 'nu magmom','den','omegab','d','he3','he4','li6','li7','heavy'
            write(2,77) 'nu magmom','den','omegab','d','he3','he4','li6','li7','heavy'
    77    format(1x,t4,a,t16,a,t28,a,t40,a,t52,a,t64,a,t76,a,t88,a,t100,a)
```

```
      nmmlo = -10.0
      nmmhi = -8.0

      eta1 = 6.2d-10

      nstep = 500
      step = 0.0d0
      if (nstep .ne. 1) step = (nmmhi - nmmlo)/float(nstep - 1)
      do kkk=1,nstep

       magmomplease = nmmlo + step * float(kkk-1)
       magmomplease = 10.0d0**magmomplease

       planckiemass = 1.0d22
       gfermico = 1.0d-11

       sigmasofine = ((finestruct**2)*pi*(magmomplease**2))/(6.0d0*(0.511**2))

       que = (third*sigmasofine/(gfermico**2))
       arrr = 0.5d0*(1/((gfermico**2)*planckiemass))

       comboqarr = ((que**3) + (arrr**2))

       termie1 = (arrr+sqrt(comboqarr))**(third)
       termie2 = -(abs(arrr-sqrt(comboqarr)))**(third)

       Tnewdecoup = termie1 + termie2

       bazinga = 0.511d0/Tnewdecoup       !form needed to compare with x=me/T

       open(unit=8,file='bazinga.dat')
       write(8,*) bazinga
       close(8)

 !set the initial density from the temperature and eta1
       din = f1 * eta1 * tin**3

 !burn it

       call burner(time_start,time_end,dtsav,&
                   tin,din,vin,zin,ein,xin, &
                   tout,dout,eout,xout, &
                   conserv,nok,nbad)

 !output a summary of the integration

      call net_summary(time_end,tin,din,ein, &
                       tout,dout,eout,conserv, &
                       nbad,nok,xout)

! bigbang only, write the final composition
      if (bbang) then

!the present day density
       zden  = f1 * eta1 * cmbtemp**3

! critical density, convert hubble constant to cm
       rhocrit = 3.0d0 * (hubble/(pc*10.0d0))**2 / (8.0d0*pi*g)
```

```
    ! ratio of baryon to critical

            omegab  = zden/rhocrit

            write(2,01) magmomplease,zden,omegab, &
                        xout(ih2),xout(ihe3),xout(ihe4), &
                        xout(ili6),xout(ili7)+xout(ibe7), &
                        (xout(ib11)+xout(ic11))

            write(6,01) magmomplease,zden,omegab, &
                        xout(ih2)/xout(iprot),xout(ihe3)/xout(iprot), &
                        xout(ihe4),(xout(ili7)+xout(ibe7))/xout(iprot)

     01     format(1x,1p10e12.4)
              end if

           end do

         !end of bigbang nmm loop

         end do
```

B. Modifying Functions which use T_ν^{decoup}

The place in the code of F. Timmes which makes use of the neutrino temperature is about 10,000 lines into the code inside the functions **wien2(x)** and **dwien2dx(x)**. The conditions given in Eqns.III.5 and III.6 were used to adjust the code in the following way:

Modified wien2(x)

```
      double precision function wien2(x)
      include 'implno.dek'
      include 'const.dek'
      include 'network.dek'

! this is the function given in
! weinberg's "gravitation and cosmology" page 537, equation 15.6.40

! declare the pass
      double precision x              ! note from Weinberg x = me/T


! communicate xcom via common block
      double precision xcom
      common /tes1/    xcom


! communicate the number of neutrino families
! using 2 families of neutrinos duplicates the time-temperature
! table in wienberg's "gravitation and cosmology", page 540, table 15.4

! brought in through network.dek
!      double precision xnnu
!      common /nufam/   xnnu
```

```
! local variables
      external         func2
      double precision func2,f2,wien1


! the integration limits ylo and yhi, along with the integration
! tolerance tol, will give at least 9 significant figures of precision

      double precision ylo,yhi,tol,con1,con2,con3,fthirds,bbb
      double precision con2flip,flipped,third,bazinga,conwein

      parameter        (ylo     = 1.0d-6, &
             yhi     = 50.0d0, &
                        tol     = 1.0d-10, &
                        con3    = 30.0d0/(pi*pi*pi*pi), &
                        fthirds = 4.0d0/3.0d0, &
                        con2 = 4.0d0/11.0d0, &
                        third = 1.0d0/3.0d0)

             ! for quadrature
               integer          nquad,ifirst
               parameter        (nquad = 100)
               double precision xquad(nquad),wquad(nquad)
               data             ifirst/0/


             open(unit=8,file='bazinga.dat')

             read(8,*) bazinga

             close(8)

             bbb = bazinga

             ! initialization of the quadrature abcissas and weights
                   if (ifirst .eq. 0) then
                    ifirst = 1
                    call bb_gauleg(ylo,yhi,xquad,wquad,nquad)
             ! a constant that depends on the number of neutrino families
                    con1 = xnnu * 7.0d0/8.0d0
                  end if



             ! don't do any integration if x is large enough
               if (x .gt. 50) then
                 wien2 = 1.0d0 + con1*con2**fthirds

            ! do the integration
               else

                 xcom = x

            ! call bb_qromb(func2,ylo,yhi,tol,f2)
              call bb_qgaus(func2,xquad,wquad,nquad,f2)
```

```
          ! here is the loop modification to keep neutrinos coupled below the decoupling temperature

               if (bbb .lt. 1.0) then

                  if (x .le. bbb) then

                  wien2 = 1.0d0 + con1*(1.0d0) + con3 * f2

                else

                  wien2 = 1.0d0 + con1 * (con2 * wien1(x))**fthirds + con3 * f2

                end if

               else

                 if (x .lt. 1.0) then

                    wien2 = 1.0d0 + con1*(1.0d0) + con3 * f2

                 else

                    wien2 = 1.0d0 + con1 * (con2 * wien1(x))**fthirds + con3 * f2

                 end if

              end if

             end if

      return
      end
```

Modified dwien2dx(x)

```
      double precision function dwien2dx(x)
      include 'implno.dek'
      include 'const.dek'
      include 'network.dek'

! this is the derivative with respect to x of the function given in
! weinberg's "gravitation and cosmology" page 537, equation 15.6.40

! declare the pass
      double precision x



! communicate xcom via common block
      double precision xcom
      common /tes1/    xcom


! communicate the number of neutrino families
! using 2 families of neutrinos duplicates the time-temperature
! table in wienberg's "gravitation and cosmology", page 540, table 15.4
```

```
! brought in through network.dek
!      double precision xnnu
!      common /nufam/   xnnu


! local variables
      external         dfunc2dx
      double precision dfunc2dx,df2,wien1,w1,dwien1dx,dw1,wbaz


! the integration limits ylo and yhi, along with the integration
! tolerance tol, will give at least 9 significant figures of precision

      double precision ylo,yhi,tol,con1,con2,con3,fthirds,bbb
      double precision con2flip,flipped,third,bazinga,conwein

      parameter        (ylo     = 1.0d-6, &
               yhi     = 50.0d0, &
                        tol     = 1.0d-10, &
                        con3    = 30.0d0/(pi*pi*pi*pi), &
                        fthirds = 4.0d0/3.0d0, &
                        con2 = 4.0d0/11.0d0, &
                        third = 1.0d0/3.0d0)

             ! for quadrature
             integer          nquad,ifirst
             parameter        (nquad = 100)
             double precision xquad(nquad),wquad(nquad)
             data             ifirst/0/

             open(unit=8,file='bazinga.dat')

             read(8,*) bazinga

             close(8)

             bbb = bazinga


      ! initialization of the quadrature abcissas and weights
            if (ifirst .eq. 0) then
             ifirst = 1
             call bb_gauleg(ylo,yhi,xquad,wquad,nquad)

      ! a constant that depends on the number of neutrino families
             con1 = xnnu * 7.0d0/8.0d0
            end if

      ! don't do any integration if x is large enough
            if (x .gt. 50.0) then
             dwien2dx = 0.0d0

      ! do the integration
            else

             xcom = x
```

```
!     call bb_qromb(dfunc2dx,ylo,yhi,tol,df2)
      call bb_qgaus(dfunc2dx,xquad,wquad,nquad,df2)

! here is the loop modification to keep neutrinos coupled below the decoupling temperature

          if (bbb .lt. 1.0) then

      if (x .le. bbb) then

        dwien2dx = con3*df2

      else

        w1 = wien1(x)
dw1  = dwien1dx(x)
                dwien2dx = fthirds*con1*(con2*w1)**third * con2*dw1 + con3*df2

      end if

  else

      if (x .lt. 1.0) then

        dwien2dx = con3*df2

      else

        w1 = wien1(x)
dw1  = dwien1dx(x)
                dwien2dx = fthirds*con1*(con2*w1)**third * con2*dw1 + con3*df2

      end if

          end if

          !w1 = wien1(x)
          !dw1  = dwien1dx(x)
          !w2   = 1.0d0 + con1*(con2*w1)**fthirds + con3*f2
          !dwien2dx = fthirds*con1*(con2*w1)**third * con2*dw1 + con3*df2

       end if

   return
   end
```


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