

Covariance Analysis of Symmetry Energy Observables from Heavy Ion Collision

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Abstract

Using covariance analysis, we quantify the correlations between the interaction parameters in a transport model and the experimental observables commonly used to extract information of the Equation of State of Asymmetric Nuclear Matter. Using the simulations of $^{124}\text{Sn}+^{124}\text{Sn}$, $^{124}\text{Sn}+^{112}\text{Sn}$ and $^{112}\text{Sn}+^{112}\text{Sn}$ reactions at beam energies of 50 and 120 MeV per nucleon, we have identified that the nucleon effective masses splitting are most strongly correlated to the neutrons and protons yield ratios from central collisions especially at high incident energy. The best observable to determine the slope of the symmetry energy, L , at saturation density is the isospin diffusion observable at low incident energy even though the correlation is not very strong (~ 0.7). Similar magnitude of correlation but opposite in sign exists for isospin diffusion and nucleon isoscalar effective masses. At 120 MeV/u, the effective mass splitting and the isoscalar effective mass also have opposite correlation for the double n/p and isoscaling p/p yield ratios. By combining the data and simulations at high and low energy, it should be possible to disentangle the effects the slope of symmetry energy (L), isoscalar effective mass (m_s^*/m) and effective mass splitting and place the constraints on L , m_s^*/m and effective mass splitting with reasonable uncertainties.

Keywords: nucleon effective mass and effective mass splitting, symmetry energy, heavy ion collisions, covariance analysis

Knowledge about the isospin asymmetric nuclear equation of state is of

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fundamental importance to our understanding of nature's most asymmetric objects including neutron stars and heavy nuclei composed of very different numbers of neutrons and protons. In theory, there are two approaches to predict its properties. One is the microscopic approach which starts from a realistic two-body free NN and three-body NNN interaction[1, 2]. Based on many body approaches, different approximations are used, such as the relativistic Dirac-Bruckner-Hartree-Fock (DBHF) and its nonrelativistic counterpart Bruckner-Hartree-Fock (BHF)[3, 4, 5, 6, 7], Relativistic-Hartree-Fock (RHF)[8, 9] and chiral effective field[10] theories. Another approach is to use effective density dependent many body interactions such as the zero range Skyrme interaction [11, 12] and finite range Gogny interaction [13]. Among them, the effective Skyrme interactions are more commonly used in nuclear structure, reactions and astrophysics studies because they include translational invariance, Galilei invariance, rotational invariance, isospin invariance, parity invariance and time reversal invariance. The effective Skyrme force parameters allow a quantitative description of heavy nuclei properties while being sufficiently simple mathematically to make it computationally feasible[14]. They are usually obtained by fitting the properties of symmetric nuclear matter (such as the saturation density and the corresponding energy per nucleon and its incompressibility), asymmetric properties of nuclear matter (such as symmetry energy and isovector effective mass at saturation density), finite nuclei properties (such as binding energies and r.m.s radius of selected set of doubly magic nuclei) etc.[15]. In Ref [16], 240 Skyrme parameter sets [16] that fit the ground-state properties of stable nuclei and of symmetric and asymmetric nuclear matter are compiled. The large number of parameterization arises in part because there are strong correlations between some individual parameters or groups of parameters, with particular physical properties of the many-body nuclear system. These sets lead to very different neutron Equation of states[17, 18, 19] which may have different incompressibility $K_0 = 9\rho^2 \frac{\partial^2 \epsilon / \rho}{\partial \rho^2}$, symmetry energy coefficient $S_0 = S(\rho_0)$, slope of symmetry energy $L = 3\rho_0 \frac{\partial S(\rho)}{\partial \rho} |_{\rho_0}$, isoscalar effective mass $m_s^* = (1 + \frac{m}{k} [\frac{\partial U(k, E)}{\partial k}])^{-1}$, and isovector effective mass $m_v^* = \frac{1}{1+\kappa}$, where κ is the enhancement factor of the Thomas-Reich-Kuhn sum rule [20]. In this work, the magnitude of effective mass splitting, $(m_n^* - m_p^*)/m$ was not used as input variables since its form is much more complicated to be incorporated into the code. Instead we use $f_I = \frac{1}{2\delta} \frac{\partial(U_n - U_p)}{\partial E_k} = \frac{1}{2\delta} (\frac{m}{m_n^*} - \frac{m}{m_p^*}) = \frac{m}{m_s^*} - \frac{m}{m_v^*}$, where $\delta = (\rho_n - \rho_p)/(\rho_n + \rho_p)$, ρ_n and ρ_p are the neutron and proton den-

sity, m_n^* , m_p^* and m are the neutron, proton effective mass and free nucleon mass. In Skyrme Hartree-Fock approximation[12, 15, 16], f_I increases with increasing density, but is independent of the momentum and kinetic energy which is similar to the behaviors in ImQMD-Sky. In the DBHF and RHF approximations [9, 21, 22], f_I not only depends on the density but also on the kinetic energy of the in-medium nucleon.

To obtain the information of symmetry energy with heavy ion collision data, the symmetry potential used in transport models is changed by varying its input parameters, corresponding to the different values of S_0 and/or L in the expression of density dependence of symmetry energy. The results of the calculations are then compared with data to find the best parameter sets. Recently, a consistent set of constraints on the symmetry energy near saturation density between S_0 , and its slope, L , has been obtained from observables measured in both nuclear structure and nuclear reaction experiments[19, 23, 24, 25].

In Skyrme parametrizations, the symmetry energy parameters are expected to be correlated to other parameters such as those associated with the nucleon effective mass. Such correlations would affect the eventual symmetry energy constraints obtained from heavy ion collision data. In this work, we use the covariance analysis to quantitatively examine the correlations between model parameters and experimental observables. The correlation coefficient C_{AB} between variable A and observable B is calculated as follows[26]:

$$C_{AB} = \frac{cov(A, B)}{\sigma(A)\sigma(B)} \quad (1)$$

$$cov(A, B) = \frac{1}{N-1} \sum_i (A_i - \langle A \rangle)(B_i - \langle B \rangle) \quad (2)$$

$$\sigma(X) = \sqrt{\frac{1}{N-1} \sum_i (X_i - \langle X \rangle)^2}, X = A, B \quad (3)$$

$$\langle X \rangle = \frac{1}{N} \sum_i X_i, i = 1, N. \quad (4)$$

$C_{AB} = 1$ means there is a linear dependence between A and B , and $C_{AB} = 0$ means no correlations. In this work, we use the Improved Quantum Molecular Dynamic Model which incorporates the effective Skyrme interactions (ImQMD-Sky)[27] to simulate the collisions of heavy ions with parameter

set i as in Table I. A_i represents the i th parameter set of the transport variable $A = K_0, S_0, L, m_s^*, m_v^*$ or f_I used as input to the ImQMD-Sky.

In ImQMD-Sky, the nucleonic potential energy density is $u_{loc} + u_{md}$, where

$$\begin{aligned}
u_\rho &= \frac{\alpha \rho^2}{2 \rho_0} + \frac{\beta}{\eta + 1} \frac{\rho^{\eta+1}}{\rho_0^\eta} + \frac{g_{sur}}{2\rho_0} (\nabla \rho)^2 \\
&+ \frac{g_{sur,iso}}{\rho_0} [\nabla(\rho_n - \rho_p)]^2 \\
&+ A_{sym} \rho^2 \delta^2 + B_{sym} \rho^{\eta+1} \delta^2
\end{aligned} \tag{5}$$

and the energy density of Skyrme-type momentum dependent interaction are written based on its interaction form $\delta(r_1 - r_2)(p_1 - p_2)^2$ [11, 12],

$$\begin{aligned}
u_{md} &= u_{md}(\rho\tau) + u_{md}(\rho_n\tau_n) + u_{md}(\rho_p\tau_p) \\
&= C_0 \int d^3p d^3p' f(\vec{r}, \vec{p}) f(\vec{r}, \vec{p}') (\vec{p} - \vec{p}')^2 + \\
&D_0 \int d^3p d^3p' [f_n(\vec{r}, \vec{p}) f_n(\vec{r}, \vec{p}') (\vec{p} - \vec{p}')^2 \\
&+ f_p(\vec{r}, \vec{p}) f_p(\vec{r}, \vec{p}') (\vec{p} - \vec{p}')^2].
\end{aligned} \tag{6}$$

The 9 parameters $\alpha, \beta, \eta, A_{sym}, B_{sym}, C_0, D_0, g_{sur}, g_{sur,iso}$ used in ImQMD-Sky can be derived from standard Skyrme parameter sets with 9 parameters $\{t_0, t_1, t_2, t_3, x_0, x_1, x_2, x_3, \sigma\}$ [27, 28]. The coefficients of surface terms are set as $g_{sur} = 24.5 MeV fm^2$ and $g_{sur,iso} = -4.99 MeV fm^2$ which are the same values derived from SLy4 parameter set [15], and varying of g_{sur} and $g_{sur,iso}$ for different Skyrme interactions have negligible effects on the experimental observables at intermediate energy. By using the relationship which was derived in reference [29, 30], the reduced 7 Skyrme parameter sets $\{\alpha, \beta, \eta, A_{sym}, B_{sym}, C_0, D_0\}$ can be replaced by the parameter sets $\{\rho_0, E_0, K_0, S_0, L, m_s^*, m_v^*\}$ which is directly related to the properties of nuclear matter at saturation density ρ_0 . Choosing the experimental values of $\rho_0 = 0.16 fm^{-3}$, $E_0 = -16 MeV$, the parameter sets are further reduced to 5 variables, $A = K_0, S_0, L, m_s^*, m_v^*$. As explained above, to simplify the coding, the input variables used in ImQMD-Sky are $A = K_0, S_0, L, m_s^*, f_I(m_s^*, m_v^*)$.

For experimental observables, we adopt the ratios constructed from nucleon spectra. Most of the transport models over-predict the absolute yield of free nucleons due to inadequacies of the model in describing cluster production [31, 32, 33]. To minimize these effects, we construct the coalescence

invariant (CI) nucleon yield spectra and their ratios the same way as in Ref[34, 37, 38] by combining the nucleons in the light particles and free nucleons at given kinetic energy per nucleon as follows,

$$\frac{dM_{n,CI}}{dE_{c.m.}/A} = \sum N \frac{dM(N, Z)}{dE_{c.m.}/A} \quad (7)$$

$$\frac{dM_{p,CI}}{dE_{c.m.}/A} = \sum Z \frac{dM(N, Z)}{dE_{c.m.}/A} \quad (8)$$

The summation is up to $Z \leq 6$ and $A \leq 16$ particles in the calculations. Such CI neutron and proton yields obtained with ImQMD calculations with selected Skyrme parameter sets reproduce the Sn+Sn data[39] reasonably well especially at high beam energy. For simplicity, the CI neutron and proton yields from reaction i are represented as $Y_i(n)$ and $Y_i(p)$ respectively in the following. By convention, the more neutron-rich reaction is represented by the subscript "2" and in this work, $i=1,2$ represent the reactions $^{112}\text{Sn}+^{112}\text{Sn}$ and $^{124}\text{Sn}+^{124}\text{Sn}$.

We simulate 10,000 events for each of the three systems, $^{124}\text{Sn}+^{124}\text{Sn}$, $^{124}\text{Sn}+^{112}\text{Sn}$ and $^{112}\text{Sn}+^{112}\text{Sn}$. We construct four ratios from the coalescence invariant nucleon spectra: 1.) single n/p ratio $\text{CI-R}_2(n/p)=Y_2(n)/Y_2(p)$, 2.) double n/p ratios $\text{CI-DR}(n/p)=\text{CI-R}_2(n/p)/\text{CI-R}_1(n/p)=\text{CI-R}_{21}(n/n)/\text{CI-R}_{21}(p/p)$, 3.) isoscaling ratios $\text{CI-R}_{21}(n/n)$ and 4.) $\text{CI-R}_{21}(p/p)$ ratios. The experimental isoscaling ratios $\text{CI-R}_{21}(n/n)=Y_2(n)/Y_1(n)$ and $\text{CI-R}_{21}(p/p)=Y_2(p)/Y_1(p)$ and the double ratios $\text{CI-DR}(n/p)$ have the advantage that they are minimally affected by detector systematic uncertainties and sequential decays[40, 41]. 5.) last isospin observable we analyze is the isospin transport ratios[42], R_{diff} , that quantify diffusion of the nucleons in the neck region during the nuclear collisions. R_{diff} has been used to constrain the symmetry energy at subsaturation density. These constraints are consistent with constraints obtained in nuclear structure experiments[19].

In experiments, R_{diff} is defined as [42], $R_{diff} = (2X - X_{aa} - X_{bb})/(X_{aa} - X_{bb})$ where $X = X_{ab}$ is an isospin observable. In this work, we use the subscripts a and b to denote the projectile (first index) and target (second index) combination. By convention, aa and bb represent the neutron-rich and neutron-poor reactions, respectively. In theoretical calculations, X is the isospin asymmetry of the emitting source, which is constructed from all the emitted nucleons and fragments with certain velocity cuts and it is linearly related to X_{exp} as discussed in [42, 43], and have been used to

compare with data. One should note that, sensitivities of nucleon ratios to effective mass splitting such as $R_i(n/p)$, $DR(n/p)$ rely mainly on high energy nucleons[27, 44, 45, 46, 47, 48], while R_{diff} are constructed with at least three reaction systems to cancel the drift and mainly retain the information of the diffusion, and carries the information of "nucleons" that have lower energy. Thus complimentary information is obtained from the nucleon ratios and the isospin transport ratios.

We first perform calculations by separately varying each variable of the parameter set K_0 , S_0 , L , m_s^* , f_I . In the following studies, 12 parameter sets are constructed and listed in Table I. The average values of each variable in the parameter space we used are, $\langle K_0 \rangle = 242.5 MeV$, $\langle S_0 \rangle = 32 MeV$, $\langle L \rangle = 54.5 MeV$, $\langle m_s^*/m \rangle = 0.7375$ and $\langle f_I \rangle = -0.1835$.

Table 1: List of twelve parameters used in the ImQMD calculations. $\rho_0 = 0.16 fm^{-3}$, $E_0 = -16 MeV$, and $g_{sur} = 24.5 MeV fm^2$, $g_{sur,iso} = -4.99 MeV fm^2$

Para.	K_0 (MeV)	S_0 (MeV)	L (MeV)	m_s^*/m	f_I
1	230	32	46	0.7	-0.238
2	280	32	46	0.7	-0.238
3	330	32	46	0.7	-0.238
4	230	30	46	0.7	-0.238
5	230	34	46	0.7	-0.238
6	230	32	60	0.7	-0.238
7	230	32	80	0.7	-0.238
8	230	32	100	0.7	-0.238
9	230	32	46	0.85	-0.238
10	230	32	46	1.00	-0.238
11	230	32	46	0.7	0.0
12(SLy4)	230	32	46	0.7	0.178

To study the sensitivity of different force parameters in Skyrme interactions on isospin observables, we calculate the covariance coefficients C_{AB} based on Eq.(1)-(4) using five force parameters: $A=K_0$, S_0 , L , m_s^* and f_I and five HIC observables: $B=CI-R_2(n/p)$, $CI-DR(n/p)$, $CI-R_{21}(n/n)$, $CI-R_{21}(p/p)$ and R_{diff} . All the nucleon yield observables in the simulations are obtained for $^{124}Sn+^{124}Sn$ and $^{112}Sn+^{112}Sn$ collisions at incident energies of 50 and 120 MeV per nucleon at impact parameter 2fm. The energy cut $E_{c.m.}/A > 40 MeV$ and angular cut with $70^\circ < \theta_{c.m.} < 110^\circ$ for $CI-R_2(n/p)$,

CI-DR(n/p), CI- $R_{21}(n/n)$ and CI- $R_{21}(p/p)$ are imposed since nucleons with high kinetic energy are less influenced by sequential decay and are more sensitive to the effective mass splitting[27, 44, 45, 46, 47, 48]. For R_{diff} , an additional mixed reaction $^{124}\text{Sn}+^{112}\text{Sn}$ is included and the calculations are performed with mid-peripheral impact parameter of 6 fm where a low density neck is formed and isospin transport between projectile and target occurs.

Figure 1 shows the covariance coefficient C_{AB} for Sn+Sn reactions at $E_{beam}=50$ MeV/u (upper panels) and at $E_{beam}=120$ MeV/u (lower panels). Red color bars represent positive correlations which mean observable B increases with parameter A, and blue shaded color bars show negative correlations which means observable B decreasing with increasing parameter A. To focus our search for the best experimental observables which are most sensitive to the model parameters, we will discuss only C_{AB} with values greater than 0.5. With this criterion, S_0 and K_0 are not very sensitive to any of the observables studied here.

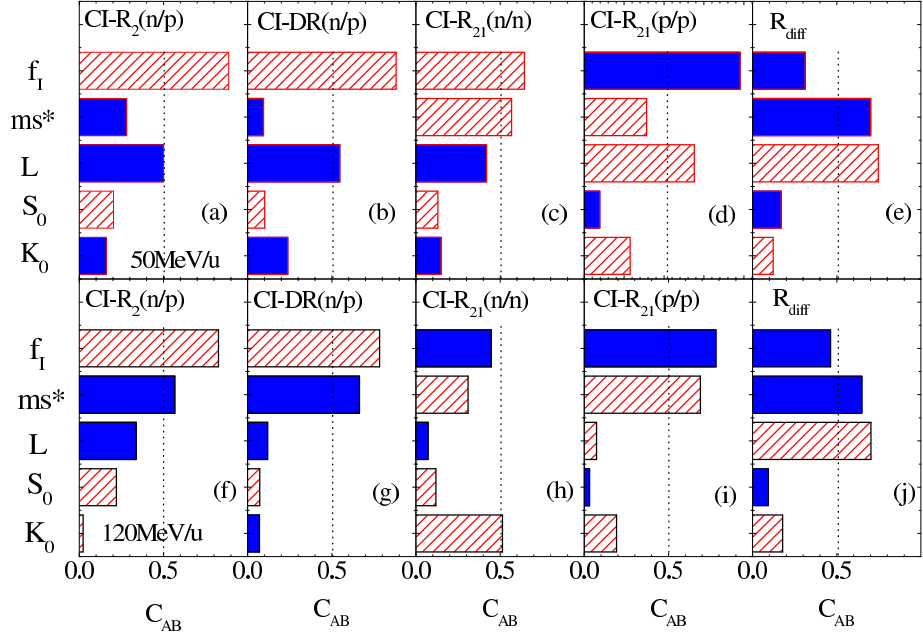


Figure 1: (Color online) Correlations of five observables, CI- $R_2(n/p)$ (a), CI-DR(n/p) (b), CI- $R_{21}(n/n)$ (c), CI- $R_{21}(p/p)$ (d), R_{diff} (e) with five force parameter, K_0 , S_0 , L, m_s^* and f_I . Up panels are the results for 50 MeV per nucleon, and bottom panels are for 120 MeV per nucleon.

In the case of L, slope of the symmetry energy at saturation density,

$C_{LRdiff} \sim 0.7$ meets this criterion. Our analysis also shows similar influence from m_s^* , $C_{m_s^*, Rdiff} \sim 0.64 - 0.70$. The observation that R_{diff} is also sensitive to m_s^* is consistent with the calculations from BUU approaches with and without momentum dependent interaction [35, 36, 38]. Interestingly, the sign of the correlation for $C_{L, Rdiff}$ and $C_{m_s^*, Rdiff}$ are opposite suggesting that the data could determine both L and m_s^* values with reasonable uncertainties. Since isospin diffusion decreases with increasing incident energy due to the shorter contact time between projectile and target, one expects R_{diff} values closer to ± 1 (no diffusion) at 120 MeV per nucleon beam energy and it becomes more difficult to measure R_{diff} experimentally. Thus it is not clear if the correlation value of $C_{L, Rdiff}$ at 120 MeV per nucleon is a robust correlation in practice.

f_I shows the largest correlations to the experimental coalescence invariant nucleon observables, the single and double n/p yield ratios at both incident energies. As shown in panels (a), (f) and (b), (g) of Figure 1, the correlation between the single and double n/p yield ratios and f_I reach above 0.8. However, strong correlation with m_s^* is observed mainly at high energy, because the more violent nucleon-nucleon collisions at 120MeV/u causes larger momentum transfer and leads to the momentum dependent interaction (which can be characterized by the effective mass) to play more important roles. It can also be understood from the expression of the density dependence of symmetry energy in Skyrme interaction, where the density dependence of symmetry energy not only depends on isovector effective mass m_v^* , but also on the isoscalar effective mass m_s^* , i.e. $S(\rho) = \frac{1}{3}\epsilon_F \rho^{2/3} + A_{sym}\rho + B_{sym}\rho^{\sigma+1} + C_{sym}(m_s^*, m_v^*)\rho^{5/3}$.

For isoscaling ratios, the sensitivity of CI- $R_{21}(n/n)$ to all variables K_0 , S_0 , L , m_s^* and f_I are borderline. On the other hand, CI- $R_{21}(p/p)$ are particularly sensitive to f_I and m_s^* at 120 MeV/u. Since the proton felt attractive from symmetry potential, it leads to negative correlation between CI- $R_{21}(p/p)$ and f_I . Due to the Coulomb repulsion, protons accelerate to higher kinetic energy than neutrons. At higher energy, the influence of effective mass splitting become more important, and it causes the very negative correlation between CI- $R_{21}(p/p)$ and f_I . The stronger positive correlation between CI- $R_{21}(p/p)$ and m_s^* appears due to the violent nucleon-nucleon collisions at 120MeV/u. The correlation signs are opposite for them and the single and double n/p ratios. This opposite correlation signs can be exploited to extract constraints for both f_I and m_s^* from the single and double n/p ratios as well as the p/p ratios at 120MeV/u incident energy.

In summary, by separately varying the interaction parameter sets in the ImQMD-Sky code, we study the influence of K_0, S_0, L, m_s^* , and f_I on the high energy coalescence invariant neutron and proton yield ratios, for $^{124}\text{Sn}+^{124}\text{Sn}$ and $^{112}\text{Sn}+^{112}\text{Sn}$ at 50MeV/u and 120MeV/u incident energy. Sensitivities to S_0 and K_0 are relatively small, $C_{AB} < 0.5$. Instead, at incident energy of 120MeV/u, strong correlation are observed between observables constructed from coalescence invariant nucleon spectra at $E_{c.m.}/A > 40$ MeV, such as CI- $R_2(n/p)$, CI-DR(n/p), and CI- $R_{21}(p/p)$, and the effective mass splitting and the isoscalar effective mass, important input parameters to the transport models. The calculations also confirm the sensitivity of L to the isospin diffusion observable at low beam energy. However, the same observable is also sensitive to isoscalar effective mass but has opposite correlations-which should allow one to extract the constraints of m_s^* and L with reasonable uncertainties. Similarly the opposite correlations of the nucleon yield ratios, such as CI-DR(n/p) and CI- $R_2(n/p)$ ratios to f_I and m_s^* , at 120 MeV/u reactions would allow one to disentangle the effects of the effective nucleon mass splitting, isovector effective mass.

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