The impact of energy conservation in transport models on the π^-/π^+ multiplicity ratio in heavy-ion collisions and the symmetry energy

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Abstract

The charged pion multiplicity ratio in intermediate energy central heavy-ion collisions has been proposed as a suitable observable to constrain the high density dependence of the isovector part of the equation of state, with contradicting results. Using an upgraded version of the Tübingen QMD transport model, which allows the conservation of energy at a local or global level by accounting for the potential energy of hadrons in two-body collisions and leading thus to particle production threshold shifts, we demonstrate that compatible constraints for the symmetry energy stiffness can be extracted from pion multiplicity and elliptic flow observables. Nevertheless, pion multiplicities are proven to be highly sensitive to the yet unknown isovector part of the in-medium $\Delta(1232)$ potential which hinders presently the extraction of meaningful information on the high density dependence of the symmetry energy. A solution to this problem together with the inclusion of contributions presently neglected, such as in-medium pion potentials and retardation effects, are needed for a final verdict on this topic.

Keywords: equation of state of nuclear matter, quantum molecular dynamics, heavy-ion collisions, symmetry energy

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1. Introduction

The isovector part of the equation of state (asy-EoS), commonly known as symmetry energy (SE), has an important impact on the structure of rare isotopes, dynamics and spectra of heavy-ion collisions and on certain astrophysical processes such as neutron star cooling, their chemical composition, and supernovae explosions [1, 2]. It is defined as the coefficient of the second term of the Taylor expansion of the nuclear matter equation of state in terms of the isospin asymmetry β,

$$E/N(\rho,\beta) = E/N(\rho,0) + S(\rho)\beta^2 + \mathcal{O}(\beta^4). \tag{1}$$

Information on its dependence on density is encoded in the coefficients of its Taylor expansion around saturation density, in particular the slope L and curvature K_{sym} parameters

$$S(\rho) = S_0 + L/3\,u + K_{sym}/18\,u^2 + \ldots, \tag{2}$$

where in the above $\beta=\frac{\rho_n-\rho_p}{\rho_n+\rho_p}$ and $u=\frac{\rho-\rho_0}{\rho_0}$; ρ_n, ρ_p, ρ and ρ_0 denote the neutron, proton, total and saturation densities respectively.

Intermediate energy nuclear reactions involving stable and radioactive beams have allowed, by studying the thickness of neutron skins, binding energies, isospin diffusion, pygmy and giant dipole excitations and dipole polarizabilities of nuclei to constrain the density dependence of SE at densities close or below saturation to a value for the slope parameter in the range L=40÷80 MeV [3]. While existing theoretical

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models generally agree with each other in this density regime, their predictions start to differ substantially well before regions of densities $\rho \geq 2\rho_0$ are reached [2]. It is thus of uttermost importance to study experimentally nuclear matter at suprasaturation densities, a regime reached in Earth based laboratories only in the process of heavy-ion collisions. To this end several promising observables have been identified: the ratio of neutron/proton yields of squeezed out nucleons [4], light cluster emission [5], π^-/π^+ multiplicity ratio (PMR) in central collisions [6, 7, 8], elliptic flow related observables [9] and others.

Constraints for L from the elliptic flow ratio of neutrons vs. protons and neutrons vs. hydrogen have been extracted using the UrQMD transport model and power-law [10] and respectively contact Skyrme interactions [11] parametrizations of the symmetry potential with the results L=83±52 MeV and respectively L=89±45 MeV. A similar analysis making use of the Tübingen QMD (TüQMD) transport model and the Gogny parametrization of the symmetry potential [12] and using the experimental data for either elliptic flow difference or ratio of neutrons and protons has resulted in a higher average value for the slope parameter of the asy-EoS: $L=123^{+33}_{-52}$ MeV. Half of the difference between the UrQMD and TüQMD extracted values for L can be attributed to the procedure used in the latter study to obtain impact parameter integrated experimental data [11] while the remaining half may be due to left over model dependence. Very similar constraints for the value of L can be extracted using the IBUU transport model too [13]. In all cases reanalyzed sets of the 90's experimental data for ^{197}Au+^{197}Au collisions at an incident projectile energy of 400 MeV/nucleon, obtained by the FOPI-LAND collaboration [14, 15], have been employed.

Attempts to constrain the stiffness of asy-EoS making use of the FOPI experimental [16] data for the π^-/π^+ multiplicity ratio (PMR) have resulted in a series of contradictory results: Xiao *et al.* [6] made use of the IBUU transport model supplemented by the isovector momentum dependent Gogny inspired parametrization of SE [17] to point toward a soft asy-EoS, the study of Feng and Jin [7], which employed IQMD and a power-law parametrization of the potential part of the SE, favors a stiff asy-EoS and lastly Xie *et al.* [8] addressed the same issue within the Boltzmann-Langevin approach and a power-law parametrization of asy-EoS presenting support for a super-soft scenario for the SE. Additionally, the constraints extracted for L from elliptic flow and pion multiplicity ratios data are often in contradiction to each other for a given transport model. Still, the recent study of Hong *et al.* [18] presents the claim that the π^-/π^+ multiplicity ratio is insensitive to the slope of SE at saturation, which is suggested to may be due to the inclusion of the S-wave pion potential, a feature that models previously employed in computing the PMR do not share.

The described state of affairs on the theoretical side is clearly unacceptable and, in view of the upcoming new experimental measurements (ASYEOS [19] for elliptic flow and SAMURAI [20] for PMR), the observed large discrepancy between the asy-EoS stiffness extracted from elliptic flow versus charged pion multiplicity ratio data has to be eliminated. Previous attempts in this direction have aimed at studying the impact of the change of in-medium properties of π mesons and Δ baryons on the π^-/π^+ multiplicity ratio within a thermal model [21, 22]. The impact of medium effects on the S and P wave contributions to the pion-nucleon interaction close to threshold proved to have opposite signs almost canceling each other and thus leading only to a small effect of at most 5% at a projectile energy of T=200 MeV/nucleon and towards lower PMR values. The neutron skin thickness has also been proven to influence the size of PMR value at a 10 % level at mid-rapidity in peripheral collisions [23]. Most recently, in-medium modifications of the kinetic part of the SE, which impact the magnitude of the symmetry potential, have also been shown to impact PMR reducing its value [24].

In alternative approaches, particle self-energies contributions to the collision term in transport models have been included, leading to modifications of the production thresholds of hadrons in dense and/or asymmetric nuclear matter with impact on particle yields. The studies of Ferini *et al.* in Refs. [25, 26] have concentrated on production multiplicities and ratios of π^-/π^+ and K^0/K^+ within the relativistic mean-field approximation of quantum hadrodynamics for ^{197}Au+^{197}Au collisions in the beam energy range 0.8-1.8 GeV/nucleon. In this energy regime the effect is, non-surprisingly, larger for the kaon ratio, however a comparison with experimental PMR FOPI data [16] at 800 MeV/nucleon still favors a rather soft asy-EoS. In the recent study of Song *et al.* [27] particle self-energies are implemented, in a covariant fashion, in a RVUU type transport model to conclude that the inclusion of medium modifications of the pion production threshold results in a higher value for the PMR for a stiffer asy-EoS compared to a softer one, opposite to the situation when such effects are neglected. The difference between the two cases examined proved

x	L_{sym}(MeV)	K_{sym}(MeV)
-2	152	418
-1	106	127
0	61	-163
1	15	-454
2	-31	-745

Table 1: Values for L and K_{sym} coefficients appearing in the Taylor expansion of the symmetry energy around saturation density, see Eq. (2), for given values of the stiffness parameter x.

however not to be sizable enough to allow an extraction of the slope parameter L.

The present Letter reports on an upgrade of the Tübingen QMD transport model, which includes the impact of the mean field potential energies on the threshold production of hadrons, and results obtained with it for the PMR supplemented with a brief description of the impact on neutron-proton elliptic flow ratio. Such effects are considered within two "scenarios": local and global conservation of the energy, the former being a non-relativistic counterpart of the approaches in Refs. [25, 26, 27] while the latter implements the conservation of total energy of the system, a feature that is at this moment, to our best knowledge, unique for heavy-ion transport models applicable to an energy regime of a few hundred MeV/nucleon. Additionally, the impact of poorly known ingredients, in particular the isovector part of the in-medium $\Delta(1232)$ potential, on the value of PMR is studied and conclusions upon the feasibility of using it, given the current knowledge, for constraining the high density dependence of the symmetry energy are drawn.

2. The model

Heavy-ion collisions have been simulated by using the QMD transport model developed in Tübingen [28, 29]. The model has been previously applied to the study of dilepton emission in heavy-ion collisions [30, 31, 32], stiffness of the equation of state of symmetric nuclear matter [33] and various in-medium effects relevant for the dynamics of heavy-ion collisions [29, 34]. It has been upgraded to accommodate for density dependent cross-sections and an isospin dependent EoS [35]. For the present study the Gogny inspired momentum dependent parametrization of the isovector part of the equation of state [17] has been selected. It contains a parameter denoted x which has been introduced to allow adjustments in the stiffness of asy-EoS, negative and positive values corresponding to a stiff and a soft density dependence of the symmetry energy, respectively. Corresponding values for the L and K_{sym} coefficients for the values of x encountered in this study can be read from Tab. (1). We do however caution that quoting the value of L alone (rather than the value of x) is misleading as the observables usually employed in the study of heavy-ion collisions to constrain the stiffness of asy-EoS probe also the suprasaturation density regime (reaching densities $\rho \geq 2\,\rho_0$) where higher order terms in the Taylor density expansion contribute or are even dominant.

The strength of baryon resonances potentials in nuclear matter is presently uncertain at best. Results for the isoscalar part of the $\Delta(1232)$-potential differ considerably, depending on the source. Phenomenological studies of inclusive electron-nucleus (He, C, Fe) scattering data favor an attractive potential, deeper than that of the nucleon-nucleus system [36]. Ab-initio calculations, using as input well established microscopical nucleon-nucleon potentials (Argonne v_{28}) within the framework of the Bethe-Brueckner-Goldstone method [37] or one-boson exchange nucleon-nucleon potentials in the relativistic Dirac-Brueckner model which allow also a good reproduction of the elastic pion-nucleon P_{33} phase-shift [38], do however arrive at a mildly repulsive isoscalar Δ-potential. The latter result is due to dominant repulsive contributions of total isospin I=2, a channel which cannot be sufficiently constrained by elastic nucleon-nucleon scattering data. In view of these results it is customary to choose, in transport models, the isoscalar component of baryon resonances potential equal to that of the nucleon, a choice adopted here too. To our best knowledge, there have been no attempts reported in the literature to extract information on the isovector part of the $\Delta(1232)$ potential. The choice most often employed for isospin 3/2 resonances is guided by the respective decay branching ratios of the isospin quadruplet components into the possible pion-nucleon pairs [25], leading to

$$\begin{array}{lcrcr} V_{\Delta^-} & = & V_n & & \\ V_{\Delta^0} & = & (2/3)\,V_n & + & (1/3)\,V_p \\ V_{\Delta^+} & = & (1/3)\,V_n & + & (2/3)\,V_p \\ V_{\Delta^{++}} & = & & & V_p \end{array} \tag{3}$$

while for the isospin 1/2 resonances the potentials of the isospin partners are taken the same as those of the corresponding component of the nucleon isospin dublet.

The enforcement of the principle *action* = *reaction* allows energy conservation at an event by event basis if the transport model is employed in the Vlasov mode. For that to be true when the collision term is switched on one has to take into account the nuclear matter potential energy of each particle in the process of determining the kinematical variables of the particles in the final state of a collision. To be precise, the in-vacuum energy conservation relation

$$\sum_i \sqrt{p_i^2 + m_i^2} \quad = \quad \sum_j \sqrt{p_j^2 + m_j^2}, \tag{4}$$

where indexes i and j run on the particles involved in the collision (or decay, reabsorption), has to be replaced by

$$\sum_i \sqrt{p_i^2 + m_i^2} + U_i \quad = \quad \sum_j \sqrt{p_j^2 + m_j^2} + U_j, \tag{5}$$

where now both indexes run over all the particles of the whole system present at the moment of the collision. This relation enforces energy conservation at a global level (GEC). The restriction of the relation above only to the particles involved in the collision will be called in the following local energy conservation (LEC), as only the total energy of the colliding particles is conserved in this case (global energy conservation is fulfilled also in this case if the meanfield potentials are momentum and isospin independent). In this case and with the simplifying assumption of momentum independent meanfield potentials the multiplicities of produced $\Delta^-, \Delta^0(1232)$ isobars increase with respect to the free case, the opposite holding true for the $\Delta^+, \Delta^{++}(1232)$ iso-quadruplet partners. This effect is a direct consequence of the attractive/repulsive nature of the proton/neutron isovector potentials. Consequently an enhancement of the PMR is expected, a conclusion that can however be altered in the realistic case of a momentum dependent symmetry potential. The situation when potential energy effects are neglected, as was the case with previous versions of the model, will be referred to in the following as in-vacuum energy conservation (VEC).

Energy conservation including in-medium baryon potential energy is enforced in the center of mass frame of the colliding nuclei, for all collision, decay and absorption processes, including thus elastic nucleon-nucleon scattering. The angular dependence of cross-sections is implicitly (slightly) modified in the iteration process of determining the correction to the total energy which originates from the change of potential energy between initial and final states and which leads to a different boost between the center of mass frame of the nuclei and the "modified" center of mass frame of the colliding baryons (in which the vacuum angular dependence of cross-section is used). In the case of momentum dependent potentials the determination of the allowed ranges for resonance's masses in the final state of NN→NR reactions becomes more laborious. The maximum allowed value of resonance's mass ($\mathrm{M}_{max}^{LEC,GEC}$) does not appear at zero relative momentum as is the case for the VEC scenario. However, for the case of the Gogny potential, $|\mathrm{M}_{max}^{LEC,GEC}$-M(k=0)$| \leq 0.5$ MeV and consequently the approximation $\mathrm{M}_{max}^{LEC,GEC}$=M(k=0) is a very good one due to the presence of Fermi motion allowing a considerable speed up of simulations; M(k=0) is the resonance's mass for the case of a zero relative momentum in the state (final or initial) which contains a resonance.

3. Results and Discussion

The aim of the present study is to prove that compatible constraints for the stiffness of asy-EoS can be extracted from elliptic flow and charged pion multiplicity ratio experimental data together with a good

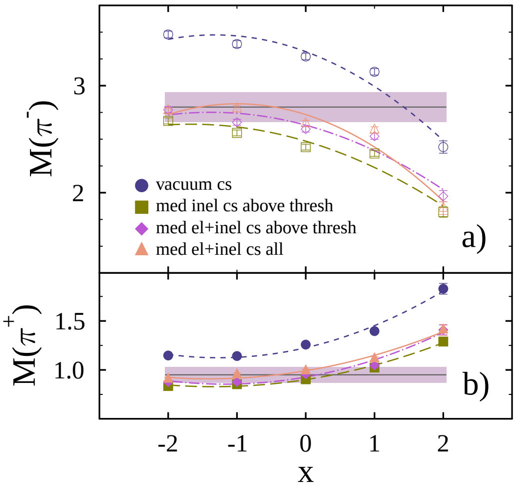

Figure 1: Impact of medium modified cross-sections on the π^-(top panel) and π^+(bottom panel) multiplicities for the GEC scenario in central ^{197}Au+^{197}Au collisions (b=0.0 fm) at a projectile energy T=400 MeV/nucleon. The experimental FOPI data [16] are represented by horizontal bands. Results are presented as a function of asy-EoS stiffness parameter x, corresponding values for L and K_{sym} can be found in Tab. (1).

description of individual elliptic flow and pion multiplicity values. To this end certain model parameters will be varied within accepted limits and in-medium modified values for both elastic and inelastic nucleon-nucleon scattering cross-sections will have to be adopted. This analysis has been performed for the GEC scenario since, within the current model, it represent the closest approximation to the real situation: it does conserve total energy but it violates causality since retardation effects are neglected.

For the standard soft value of the compressibility modulus K=210 MeV experimental values for π^- and π^+ multiplicities [16] in ^{197}Au+^{197}Au collisions at incident energies of 400 MeV/nucleon are overestimated by about 20% and 40% respectively. Choosing a stiffer EoS results in less nucleon-nucleon collisions and thus pion multiplicities decrease. At values of the compressibility modulus close to K=310 MeV the best description of the FOPI data is achieved, however the π^- and π^+ multiplicities cannot be equally good described for the same value of K. Additionally, for such a stiff EoS elliptic flow strength of protons is overestimated independent of the asy-EoS stiffness. This suggests the need for an in-medium modification of the inelastic channel cross-sections. It was found that the choice K=245 MeV coupled with medium modifications of both elastic and inelastic channels cross-sections allows a good reproduction of experimental pion multiplicities together with a fair one (within 15-20%)for elliptic flow values. For the latter ones an improvement of the description can be achieved by choosing an optical potential that becomes repulsive at higher momenta [39].

The impact of medium modified cross-sections on pion multiplicities (PM) is presented in Fig. (1) for central (b=0.0 fm) ^{197}Au+^{197}Au collisions at an projectile energy of 400 MeV/nucleon. PM are overestimated if vacuum cross-section are used. Including medium effects for the inelastic channels results in a visible decrease of PM, while the inclusion of medium effects on the elastic ones above pion production threshold pushes them up slightly. The impact of medium modifications of elastic nucleon-nucleon cross-sections below pion production threshold also results in a small increase of PM. These last two scenarios lead to PM which are within the 1σ bounds of the experimental data for values of the asy-EoS stiffness compatible to those extracted from elliptic flow data. Furthermore the PMR is almost independent on the cross-section scenario used (not shown), a conclusion which does not hold at much lower energies where symmetry energy and medium modifications of cross-sections effects on this observable are of comparable magnitude [40]. In this work, the medium modification of elastic and inelastic cross-sections was introduced via the effective mass scaling scenario of Ref. [41, 42, 43] which assumes that in-medium elastic cross-sections scale with the in-medium masses of the considered particles, an assumption which was extended here also to the inelastic channels.

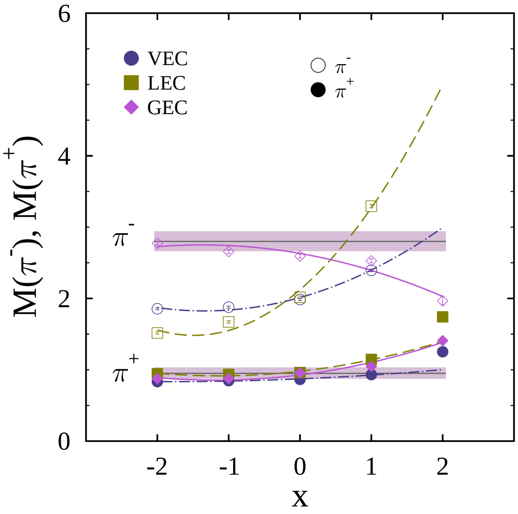

Figure 2: π^- and π^+ multiplicities for the three different energy conservation scenarios described in the text as a function of the asy-EoS stiffness parameter x.

A comparison of PM for the VEC, LEC and GEC scenarios is presented in Fig. (2) for the same reaction as above. The results for the LEC and VEC scenarios have been obtained by applying the respective energy conservation constraints, without any change of model parameters as compared to the GEC case. In all three cases the medium modified cross-sections have been used above pion threshold, while below this point only the inelastic channels have been modified. It is seen that the π^+ multiplicities change only for very soft choices of the asy-EoS. The difference between the three energy conservation scenarios is however dramatic for the π^-. It has been checked that the heavy-ion collision dynamics does not change when shifting from VEC to LEC and ultimately the GEC scenario, as the magnitude of threshold shifts in elastic nucleon-nucleon collisions is at the order of a few MeV. Consequently no extra isospin fractionation appears in the GEC scenario compared to the VEC one. The observed impact on pion multiplicities can be explained from the magnitude of the threshold shifts for resonances production alone. In the case of the π^+ meson the threshold correction in both the LEC and GEC is positive and almost independent on the asy-EoS stiffness leading to the observed slight or moderate increase of this observable relative to the VEC scenario. The π^- case is different: for the LEC scenario the threshold shifts starts at negative values for a stiff asy-EoS and increases towards softer asy-EoSes leading to the observed dependence of the multiplicities on the x parameter; for the GEC scenario the threshold shift starts at a value of about 60 MeV for the stiff asy-EoS and decreases with increasing x.

The PMR theoretical values for the three scenarios described above are compared with the experimental FOPI data for different values of the stiffness parameter x in Fig. (3). As a consequence of the differences observed for PM, in particular those of the π^- meson, the results for PMR for the GEC scenario differ substantially from those obtained if the either LEC or VEC is enforced. VEC favors an extremely soft asy-EoS and LEC leads to a somewhat stiffer one but still in the soft region. Constraints for the asy-EoS extracted using the GEC scenario range, in view of the not optimal experimental accuracy and the smaller sensitivity of the PMR to the asy-EoS stiffness in this region, from a stiff to a linear density dependence of SE, compatible at the softer limit with constraints extracted from nuclear structure measurements. It is noteworthy that constraints extracted from PMR using the GEC scenario are in complete agreement with constraints previously extracted from elliptic flow data [12]. The dependence of elliptic flow constraints on the energy conservation scenarios will be addressed below. The power-law parametrization of the symmetry potential has been implemented too in order to perform a comparison with the results of Song *et al.* [27]. Similar results (not shown) compared to the Gogny parametrization are obtained for PMR for both GEC and LEC scenarios, a slightly softer asy-EoS is however favored.

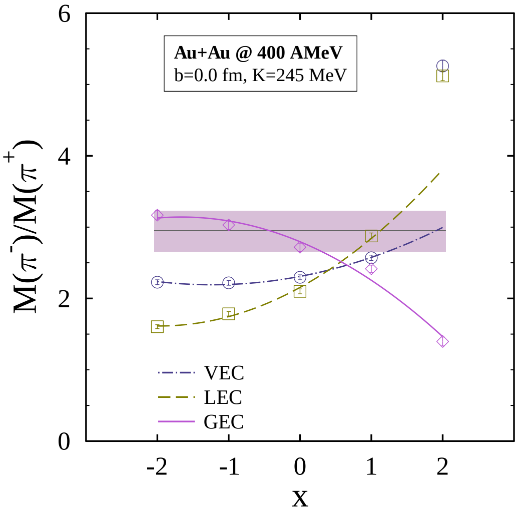

Figure 3: The same as Fig. (2) but for the PMR.

The energy dependence of the PMR ratio is addressed in Fig. (5), where results for projectile impact energies of 200, 250, 300 and 400 MeV/nucleon respectively are displayed for ^{197}Au+^{197}Au central collisions (b=0.0 fm). To produce the results in this figure heavy-ion collissions stopping time has been increased to 100 fm/c, as compared to 60 fm/c for the previously presented results, to account for a longer life-time of the high density fireball at lower impact energies. The sensitivity of PMR to asy-EoS stiffness increases towards lower energies, particularly impact energies below the vacuum pion production threshold, the ratio PMR(x=-2)/PMR(x=2) increasing by a factor of 2 between the extreme cases of T=400 MeV/A and T=200 MeV/A. Experimental data for impact energies as low a possible are thus clearly desirable. Progress in this direction will be accomplished in the near future by the SAMURAI Collaboration by measuring the PMR in ^{132}Sn+^{124}Sn and ^{105}Sn+^{112}Sn collisions close to threshold [20].

The results presented up to this point have been obtained with the choices for the isoscalar and isovector baryon potentials mentioned in Section 2. In view of the mentioned uncertainties we have studied the impact of variations of these scenarios on PMR, the results being presented in Fig. (5). The Δ(1232) isoscalar potential has been varied by 25% around its selected value. The impact on the PMR is moderate, as depicted in Fig. (5), a more attractive isoscalar potential resulting in a lower value for PMR and vice versa. Additionally, the strength of the isovector Δ(1232) potential has been altered. The Δ isobar potentials of Eq. (3) can be rewritten in the form

$$\begin{aligned} V_{\Delta^-} &= V_N + (3/2)\,V_v \\ V_{\Delta^0} &= V_N + (1/2)\,V_v \\ V_{\Delta^+} &= V_N - (1/2)\,V_v \\ V_{\Delta^{++}} &= V_N - (3/2)\,V_v \end{aligned} \tag{6}$$

where V_N is the isoscalar nucleon potential and V_v=δ, with the definition δ=(1/3)(V_n-V_p). By varying the magnitude of V_v different scenarios for the strength of the isovector baryon potential can be explored. The results for the choices V_v=0, δ, 2δ and 3δ have been plotted in Fig. (5). The fourth choice leads, in the case of a momentum independent potential, to no threshold effects. It is thus not surprising that in this case a result similar to the VEC scenario is obtained. For the intermediate case V_v=2δ the PMR shows no dependence on the stiffness of the asy-EoS, while for the first case, V_v=0, which neglects any isospin dependence of the baryon potentials (with the exception of nucleons), the largest dependence on the asy-EoS stiffness is observed. We conclude that the constraint extracted from PMR for the asy-EoS stiffness is highly sensitive to the strength of the iso-vector Δ potential. A trustworthy result cannot thus be obtained without a proper knowledge of this quantity and to a lesser extent of the isoscalar Δ potential.

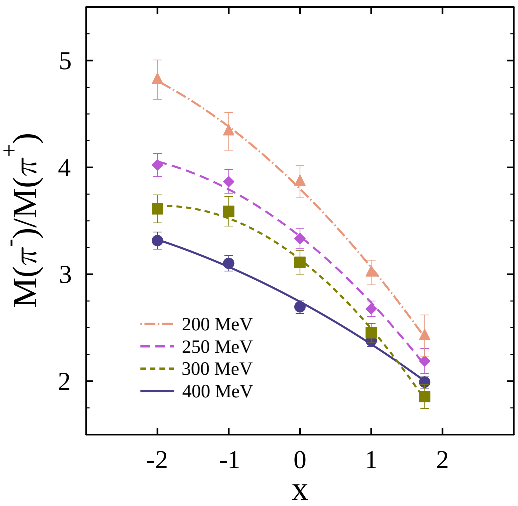

Figure 4: Dependence of the PMR on the asy-EoS stiffness for several values of the impact energy for central ^{197}Au+^{197}Au collisions.

It is tempting to extract constraints for the asy-EoS stiffness, for the sake of comparison with results available in the liturature, for the standard scenario for the $\Delta(1232)$ potential. Using the results from Fig. (5) and Tab. (1) one arrives at L=91±45 MeV and K_{sym}=28±291 MeV at 1σ uncertainty level. The result for the slope parameter L is in full agreement with constraints extracted from elliptic flow experimental data using any of the TuQMD, UrQMD or IBUU transport models. It is noteworthy that for no isovector Δ potential case (V_v=0) a result which falls on top of nuclear structure measurements is obtained for the slope parameter: L=61±22 MeV and K_{sym}=-163±146 MeV. In contrast, a comparison between experimental data and the theoretical results for the scenarios with stronger isovector Δ potentials (V_v=2δ, 3δ) may suggest that they are unrealistic.

We conclude by displaying the impact of the three energy conservation scenarios (VEC, LEC and GEC) on the neutron-proton elliptic flow ratio in ^{197}Au+^{197}Au collisions at a collision energy of 400 MeV/nucleon and integrated impact parameter (b≤7.5 fm). Due to the fact that for elastic collisions the threshold shifts amount, on average, only a few MeV, the impact of LEC and respectively GEC on elliptic flow values of neutrons and protons is small, resulting in a slight impact on the extracted constrains for asy-EoS from this observable. An identical statement holds true for neutron-proton elliptic flow differences. There is however a noticeable impact on flows in the case when medium effects on cross-sections are introduced for the elastic channels below pion production threshold. The impact on neutron-proton elliptic flow ratio is also in this case within the uncertainty of the extracted asy-EoS constraints. A more pronounced impact is noticed on neutron-proton elliptic flow differences. The density and asymmetry dependence of elastic and inelastic nucleon-nucleon cross-section is thus a topic that necessitates further study. We can however conclude that for the GEC scenario compatible constraints for the asy-EoS stiffness can be extracted from pion multiplicity and elliptic flow observables.

The presented model lacks a few ingredients that may prove important. First on the list is the in-medium pion potential, which may have an important impact on pion multiplicities close to threshold [18] given its estimated S wave magnitude from chiral perturbation theory [44] or from the explanation of the existence of pionic atoms [45]. Within the scenario of global energy conservation two-body collisions loose their local character and effects such as retardation may be of relevance. Such an expectation is supported by the fact that the ratio of the size of the high density nuclear matter fireball and its lifetime is of the order of the speed of light. Thirdly, given the size of the threshold shifts of pion production threshold of a few tens of MeV and the sensitivity of the PMR to small variations, it may be also expected that, for accurate predictions, isospin breaking effects in the $\Delta(1232)$ isospin quadruplet (both mass and decay width) could have an impact, even though a secondary one. Work to include these presently omitted contributions is planned.

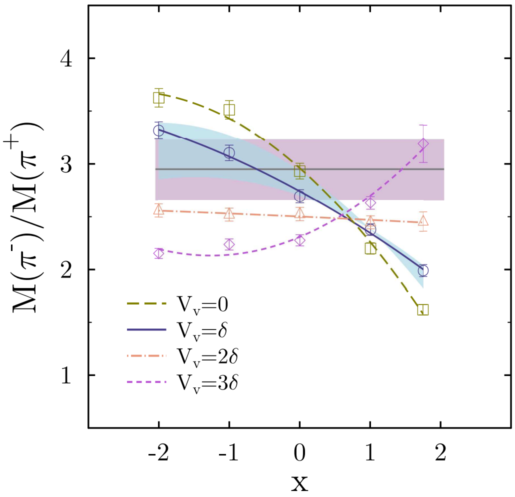

Figure 5: The impact of the isoscalar (band) and isovector (curves) in-medium Δ potentials on PMR for central ^{197}Au+^{197}Au collisions at a projectile incident energy of 400 MeV/nucleon. The experimental value is depicted by the horizontal band.

It would be of interest to extend the current model to include the strangeness production channels and reexamine [33, 46] or extract constraints of the high density dependence of the isoscalar and isovector components of the nuclear matter equation of state using presently or in the near future available experimental information gathered by the KaoS [47], FOPI [48] and HADES [49] Collaborations from GSI.

4. Conclusions

An upgraded version of the Tübingen QMD transport model, which allows the conservation of energy in intermediate energy heavy-ion collisions at an event by event basis, has been presented. The conservation of energy in two-body collisions has been implemented at local and, alternatively, at global level by taking into account in the energy conservation constraint for two-body collisions of the in-medium potential energies of the propagating particles, which has as result a shift of particle production thresholds and effective energies at which elastic collisions take place inside dense nuclear matter. It has been shown that the impact of such effects on pion multiplicities and ratios is important when energy conservation is enforced at a global level and only moderately so for the local scenario. This results in very different extracted constraints for the asy-EoS stiffness using the vacuum, local and global scenarios respectively.

The impact on elliptic flow values of neutrons and protons is however small due to threshold shifts that amount on average only a few MeV, an order of magnitude smaller than in the pion production case. In the case of the global energy conservation scenario an almost perfect agreement between the constraints on the asy-EoS stiffness extracted from pion multiplicity and elliptic flow observable is achieved, compatible with a linear dependence on density of the symmetry energy above the saturation point.

An increasing sensitivity of the pion multiplicity ratio to the stiffness of the asy-EoS towards lower incident energy collisions is observed, in agreement with expectations. The effect increases by a factor of two at T=200 MeV/nucleon with respect to the lowest energy point for which experimental data for pion multiplicities are available, T=400 MeV/nucleon. Measurements at incident energies around and below pion production threshold are thus extremely desirable and promising.

The impact of the poorly known $\Delta(1232)$ in-medium potentials in the pion multiplicity ratio has been investigated. The impact of the isoscalar part when its strength is modified by 25% from the usual one, equal to that of the nucleon, is moderate. The pion multiplicity ratio is however demonstrated to be highly sensitive to the isovector part of the $\Delta(1232)$ potential, constraints for the asy-EoS stiffness ranging from very stiff to extremely soft can be extracted depending on the choice made for this quantity. The observed sensitivity hinders at present the extraction of meaningful constraints for the density dependence

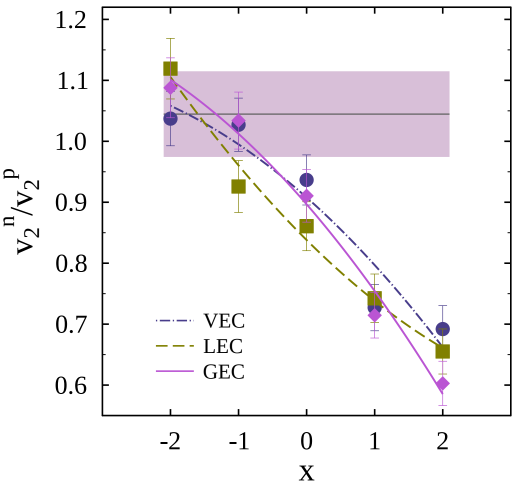

Figure 6: The impact of the different energy conservation scenarios on the neutron-proton effective flow ratio. The horizontal band represents the experimental FOPI-LAND results listed in Ref. [12]. Using one centrality bin for the experimental data analysis results in a lower value, by about 3%, for the neutron-proton elliptic flow ratio [50]. Preliminary results of the ASYEOS Collaboration favor a lower value for this observable too [19].

of the symmetry energy above saturation using the charged pion multiplicity ratio, in spite of its proven dependence on it.

Progress on the theoretical side, by extracting information on the isovector part of the Δ(1232) potential either from ab-initio calculations or from phenomenological descriptions using the same transport model for reactions in which it plays a role (e.g. pion-nucleus scattering) can alleviate the noted problem. In the worst case scenario the realistic isovector Δ potential may turn out to be such that it cancels out the sensitivity of the pion multiplicity ratio to the stiffness of the asy-EoS. Even in such a case, the experimental measurement of this observable will be extremely valuable since, due to the many effects that impact it, it will allow a precise test of our understanding of the hadronic interactions in the intermediate energy range.

Further work to include so far neglected potentially important contributions, like the in-medium pion potential, retardation effects and possibly even isospin breaking effects in the Δ(1232) isospin quadruplet, and an extension of the model to include strangeness production channels are planned.

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