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Qualifying/Placement Exam, Part-A
Fall 2011

Put your **Student Number** on every sheet of this 6 problem Exam -- NOW

You have 2 hours to complete the 6 problems on Part-A of the exam. Show your work! Full credit will not be given for answers without justification. Some partial credit may be earned for the correct procedure, even if the correct answer is not achieved. Answers must be in the spaces provided. The **BACK** of the problem page may be used for lengthy calculations. Do not use the back of the previous page for this purpose!

You may need the following constants:

$k_e = 8.99 \times 10^9 \text{ Nm}^2/\text{C}^2$	permittivity of free space
$\sigma = 5.7 \times 10^{-8} \text{ Wm}^{-2}\text{K}^{-4}$	Stefan - Boltzmann constant
$k = 1.4 \times 10^{-23} \text{ J/K}$	Boltzmann constant
$\hbar = 1.05 \times 10^{-34} \text{ J} \cdot \text{s}$	Planck's constant
$c = 3.0 \times 10^8 \text{ m/s}$	speed of light

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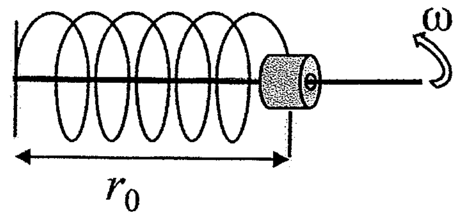

A1. A bead of mass m on a horizontal wire (gravity plays no role in this problem) slides without friction. An ideal spring, spring constant k, and rest length r_0 is attached to the bead and to one end of the wire. If the bead is displaced from $r = r_0$ and let go, it will oscillate with a frequency $\omega_0 = \sqrt{k/m}$. Suddenly the wire starts rotating in the horizontal plane with an angular velocity $\omega < \omega_0$ around the end where the spring is fixed.

a) [5 pts] Use Lagrange's equation to get a differential equation for r.

b) [2 pts] Find the new equilibrium position, $r = r_P$, along the rotating wire.

c) [3 pts] Find $r(t)$, if the initial conditions are $\frac{dr(0)}{dt} = 0$, and $r(0) = r_0$.

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A2. A uniform steel ball of mass M and radius R (hence, moment of inertial $I = \frac{2}{5}MR^2$) rolls without slipping down a ramp that makes an angle θ with the horizontal. It starts from rest.

a. [4 pts] Calculate how far it has traveled as a function of time.

b. [3 pts] Calculate the component of force on the ball due to the ramp in the direction perpendicular to the ramp.

c. [3 pts] Calculate the component of force on the ball due to the ramp in the direction parallel to the ramp

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A3. A particle is dropped into a hole drilled straight through the center of the Earth, mass M, radius R, with a uniform density.

a) [5 pts] Neglecting rotational effects and air friction, show that the particles' motion is simple harmonic.

b) [5 pts] Find the oscillation period τ in terms of the Earth's mass and radius.

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A4. Starting from Maxwell's equations,

a. [6 pts] derive the wave equation for an electromagnetic wave, and

b. [4 pts] express the speed of light in terms of ε_0 and μ_0.

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A5. An isolated uniform conducting sphere of radius 10 cm is charged to a potential (with respect to ∞) of -10 kV.

a) [4 pts] Find the electric field at the surface of the sphere.

b) [4 pts] Find the energy stored in the system.

c) [2 pts] Find the excess of number of electrons ($e = 1.6 \times 10^{-19}$ C) over protons in the sphere.

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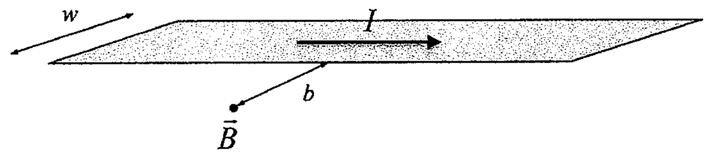

A6. [10 pts] A very thin and very long strip of metal of width w, carries a current I along the length of the strip. Assuming that the current is uniformly distributed across the width, find the magnetic field, $\vec{B}$, in the plane of the strip at a distance b from the near edge.

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Qualifying/Placement Exam, Part-B
Fall 2011

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You may need the following constants:

$k_e = 8.99 \times 10^9 \, \mathrm{Nm^2/C^2}$	permittivity of free space
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B1. The ground state of the one-dimensional harmonic oscillator has a normalized wave function given by $\psi(x) = \left(\frac{\alpha}{\pi}\right)^{1/4} e^{-\alpha x^2/2}$, with $\alpha = m\omega/\hbar$.

a. In this state, find the expectation value of the following operators:

i) X; ii) P iii) X^2 iv) P^2

b. Interpret these results in terms of Heisenberg's Uncertainty Relation.

These integrals may be of use in solving this problem:

$$\int_{-\infty}^{\infty} e^{-\beta x^2} dx = \left(\frac{\pi}{\beta}\right)^{1/2} \quad , \quad \int_{-\infty}^{\infty} x^2 e^{-\beta x^2} dx = \frac{1}{2}\left(\frac{\pi}{\beta^3}\right)^{1/2}$$

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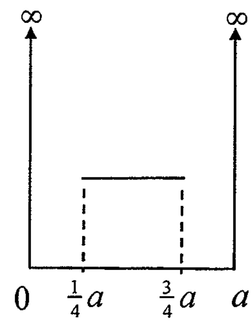
B2. A particle of mass m moving in the infinite potential well shown in the figure has an initial wave function $\psi(x,0)$ given by

$$\psi(x,0) = \begin{cases} A & \text{if } \frac{1}{4}a \le x \le \frac{3}{4}a \\ 0 & \text{otherwize} \end{cases}$$

where A is a real constant.



a) [2 pts] Determine A by normalizing $\psi(x,0)$.

b) [3 pts] At $t = 0$, find $\langle x \rangle$ and $\langle x^2 \rangle$.

c) [2 pts] At $t = 0$, what is the probability that the particle will be found in the interval $\frac{1}{2}a \le x \le a$?

d) [3 pts] If the energy of the particle is measured, what is the probability that the results will be $E = 9\pi^2\hbar^2/2ma^2$, i.e., the energy of the $n = 3$ eigenstate.

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B3. The Hamiltonian H for a system is represented by the matrix

$$H = \begin{pmatrix} 2a & 0 & a \\ 0 & 2a & 0 \\ a & 0 & 2a \end{pmatrix}.$$

where $a > 0$.

a) [3 pts] What are the possible energies of the system?

b) [2 pts] What is the energy eigenvalue associated with each of the (normalized) eigenvectors of H:

$$e_1 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad , \quad e_2 = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \quad , \quad e_3 = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} \quad ?$$

c) [3 pts] What is the average energy one would measure for the system described by H, in the initial state

$$\psi(0) = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \; ?$$

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B4. Consider the light patterns created by a green laser illuminating a single slit of 40 μm width, or a pair of slits of the same width and spaced 250 μm apart.

i) [3 pts] For the pair of slits, if the width of each slit is made 20% smaller, what is the change in the angle of the first interference maximum?

a) 40% decrease b) 20% increase c) 40% increase d) 20% decrease e) no change

ii) [3 pts] For the pair slits, if the spacing between the slits is made 20% smaller, what is the change in the angle of the first diffraction minimum?

a) 40% decrease b) 20% increase c) 40% increase d) 20% decrease e) no change

iii) [4 pts] Standing on the surface of the moon, it is not possible to block the pinpoint of light of a star, e.g., Sirius, holding at arms length a wire with a diameter of 40 μm. What physical property of light prevents this?