SECRET STUDENT NUMBER: 145

DO NOT WRITE YOUR NAME OR STUDENT NUMBER ON ANY SHEET!

$$
\begin{aligned}
\vec{a}\times(\vec{b}\times\vec{c}) &= \vec{b}(\vec{a}\cdot\vec{c})-\vec{c}(\vec{a}\cdot\vec{b}),\\
\vec{a}\cdot(\vec{b}\times\vec{c}) &= \vec{b}\cdot(\vec{c}\times\vec{a})=\vec{c}\cdot(\vec{a}\times\vec{b}),\\
(\vec{a}\times\vec{b})\cdot(\vec{c}\times\vec{d}) &= (\vec{a}\cdot\vec{c})(\vec{b}\cdot\vec{d})-(\vec{a}\cdot\vec{d})(\vec{b}\cdot\vec{c}),\\
\nabla\times(\nabla\psi) &= 0,\\
\nabla\cdot(\nabla\times\vec{a}) &= 0,\\
\nabla\times(\nabla\times\vec{a}) &= \nabla(\nabla\cdot\vec{a})-\nabla^2\vec{a},\\
\nabla\cdot(\psi\vec{a}) &= \vec{a}\cdot\nabla\psi+\psi\nabla\cdot\vec{a},\\
\nabla\times(\psi\vec{a}) &= \nabla\psi\times\vec{a}+\psi\nabla\times\vec{a},\\
\nabla(\vec{a}\cdot\vec{b}) &= (\vec{a}\cdot\nabla)\vec{b}+(\vec{b}\cdot\nabla)\vec{a}+\vec{a}\times(\nabla\times\vec{b})+\vec{b}\times(\nabla\times\vec{a}),\\
\nabla\cdot(\vec{a}\times\vec{b}) &= \vec{b}\cdot(\nabla\times\vec{a})-\vec{a}\cdot(\nabla\times\vec{b}),\\
\nabla\times(\vec{a}\times\vec{b}) &= \vec{a}(\nabla\times\vec{b})-\vec{b}(\nabla\times\vec{a})+(\vec{b}\cdot\nabla)\vec{a}-(\vec{a}\cdot\nabla)\vec{b},\\
\nabla\cdot\vec{r} &= 3,\\
\nabla\times\vec{r} &= 0,\\
\nabla\cdot\hat{r} &= 2/r,\\
\nabla\times\hat{r} &= 0,\\
(\vec{a}\cdot\nabla)\hat{r} &= \frac{1}{r}[\vec{a}-\hat{r}(\vec{a}\cdot\hat{r})]=\frac{\vec{a}_\perp}{r}.
\end{aligned}
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\begin{aligned}
\int_V d^3r\ \nabla\cdot\vec{A} &= \int_S d\vec{S}\cdot\vec{A},\\
\int_V d^3r\ \nabla\psi &= \int_S \psi\ d\vec{S},\\
\int_V d^3r\ \nabla\times\vec{A} &= \int_S d\vec{S}\times\vec{A},\\
\int_V d^3r\ (\phi\nabla^2\psi+\nabla\phi\cdot\nabla\psi) &= \int_S \phi\ d\vec{S}\cdot\nabla\psi,\\
\int_V d^3r\ (\phi\nabla^2\psi-\psi\nabla^2\phi) &= \int_S (\phi\nabla\psi-\psi\nabla\phi)\cdot d\vec{S},\\
\int_S (\nabla\times\vec{A})\cdot d\vec{S} &= \oint d\vec{\ell}\cdot\vec{A},\\
\int_S d\vec{S}\times\nabla\psi &= \oint_C d\vec{\ell}\ \psi.
\end{aligned}
$$

$$
\begin{aligned}
\nabla^2 &= \partial_r^2+\frac{2}{r}\partial_r-\frac{\ell(\ell+1)}{r^2},\\
\nabla^2 &= \partial_\rho^2+\frac{1}{\rho}\partial_\rho-\frac{m^2}{r^2},\\
\nabla^2\left(\frac{1}{r}\right) &= -4\pi\delta(\vec{r}).
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\vec{a}\times(\vec{b}\times\vec{c}) &= \vec{b}(\vec{a}\cdot\vec{c})-\vec{c}(\vec{a}\cdot\vec{b}),\\
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\boldsymbol{\nabla}\times(\boldsymbol{\nabla}\psi) &= 0,\\
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\boldsymbol{\nabla}\times(\psi\vec{a}) &= \boldsymbol{\nabla}\psi\times\vec{a}+\psi\boldsymbol{\nabla}\times\vec{a},\\
\boldsymbol{\nabla}(\vec{a}\cdot\vec{b}) &= (\vec{a}\cdot\boldsymbol{\nabla})\vec{b}+(\vec{b}\cdot\boldsymbol{\nabla})\vec{a}+\vec{a}\times(\boldsymbol{\nabla}\times\vec{b})+\vec{b}\times(\boldsymbol{\nabla}\times\vec{a}),\\
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\boldsymbol{\nabla}\times(\vec{a}\times\vec{b}) &= \vec{a}(\boldsymbol{\nabla}\times\vec{b})-\vec{b}(\boldsymbol{\nabla}\times\vec{a})+(\vec{b}\cdot\boldsymbol{\nabla})\vec{a}-(\vec{a}\cdot\boldsymbol{\nabla})\vec{b},\\
\boldsymbol{\nabla}\cdot\vec{r} &= 3,\\
\boldsymbol{\nabla}\times\vec{r} &= 0,\\
\boldsymbol{\nabla}\cdot\hat{r} &= 2/r,\\
\boldsymbol{\nabla}\times\hat{r} &= 0,\\
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\int_V d^3r\ \boldsymbol{\nabla}\times\vec{A} &= \int_S d\vec{S}\times\vec{A},\\
\int_V d^3r\ (\phi\nabla^2\psi+\boldsymbol{\nabla}\phi\cdot\boldsymbol{\nabla}\psi) &= \int_S \phi\ d\vec{S}\cdot\boldsymbol{\nabla}\psi,\\
\int_V d^3r\ (\phi\nabla^2\psi-\psi\nabla^2\phi) &= \int_S (\phi\boldsymbol{\nabla}\psi-\psi\boldsymbol{\nabla}\phi)\cdot d\vec{S},\\
\int_S (\boldsymbol{\nabla}\times\vec{A})\cdot d\vec{S} &= \oint d\vec{\ell}\cdot\vec{A},\\
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\nabla^2 &= \partial_r^2+\frac{2}{r}\partial_r-\frac{\ell(\ell+1)}{r^2},\\
\nabla^2 &= \partial_\rho^2+\frac{1}{\rho}\partial_\rho-\frac{m^2}{r^2},\\
\nabla^2\left(\frac{1}{r}\right) &= -4\pi\delta(\vec{r}).
\end{aligned}
$$

$$
\begin{aligned}
\vec{\beta} &= \vec{v}/c, \quad \gamma=1/\sqrt{1-\beta^2}\\
x^\alpha &= L^\alpha_\beta x^\beta,\\
L^\alpha{}_\beta &= \begin{bmatrix} \gamma & \gamma v/c & 0 & 0\\ \gamma v/c & \gamma & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1 \end{bmatrix}\\
p^\alpha &= \begin{pmatrix} p_0\\ \vec{p}\end{pmatrix} = m\begin{pmatrix} u_0\\ \vec{u}\end{pmatrix} = m\begin{pmatrix} \gamma c\\ \gamma\vec{v}\end{pmatrix}
\end{aligned}
$$

$$
\begin{aligned}
F_{\alpha\beta} &= \partial_\alpha A_\beta-\partial_\beta A_\alpha\\
&= \begin{pmatrix} 0 & E_x & E_y & E_z\\ -E_x & 0 & -cB_z & cB_y\\ -E_y & cB_z & 0 & -cB_x\\ -E_z & -cB_y & cB_x & 0 \end{pmatrix},\\
F^{\alpha\beta} &= \begin{pmatrix} 0 & -E_x & -E_y & -E_z\\ E_x & 0 & -cB_z & cB_y\\ E_y & cB_z & 0 & -cB_x\\ E_z & -cB_y & cB_x & 0 \end{pmatrix}.\\
J^\alpha &= \begin{pmatrix} \rho\\ \vec{J}/c\end{pmatrix}\\
A^\alpha &= \begin{pmatrix} \Phi\\ c\vec{A}\end{pmatrix}
\end{aligned}
$$

$$
\begin{aligned}
m\frac{d}{d\tau}u^\alpha &= eF^{\alpha\beta}u_\beta,\\
\frac{d\vec{p}}{dt} &= e\vec{E} + e\vec{v}\times\vec{B},\\
\omega_c &= \frac{eB}{\gamma m},\\
\vec{D} &= \epsilon\vec{E},\quad \vec{B} = \mu\vec{H}\\
\nabla\cdot\vec{D} &= \rho,\\
(\nabla\times\vec{H}) - \partial_t\vec{D} &= \vec{J},\\
\nabla\cdot\vec{B} &= 0,\\
\partial_t\vec{B} + \nabla\times\vec{E} &= 0,\\
\oint_S \vec{D}\cdot d\vec{S} &= \int d^3r\ \rho,\\
\oint_S \vec{B}\cdot d\vec{S} &= 0,\\
\oint_C \vec{E}\cdot d\vec{\ell} &= -\int_S \frac{\partial\vec{H}}{\partial t}\cdot d\vec{S}\\
\oint_C \vec{H}\cdot d\vec{\ell} &= -\int_S \left(\frac{\partial\vec{D}}{\partial t} + \vec{J}\right)\cdot d\vec{S}\\
\partial_\alpha F^{\alpha\beta} &= J^\beta/\epsilon_0,\\
\partial_\alpha \tilde{F}^{\alpha\beta} &= 0
\end{aligned}
$$

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\begin{aligned}
\vec{E}(\vec{r}) &= \frac{1}{4\pi\epsilon_0}\int \rho(\vec{r}')\frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|^3}d^3r'\\
\vec{B}(\vec{r}) &= \frac{\mu_0}{4\pi}\int \vec{J}(\vec{r}')\times\frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|^3}d^3r'
\end{aligned}
$$

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\begin{aligned}
e^2 &= \frac{\hbar c}{137.036},\\
T^{\alpha\beta} &= \pi^\alpha\partial^\beta\phi - g^{\alpha\beta}\mathcal{L},\\
\pi^\alpha &\equiv \frac{\partial\mathcal{L}}{\partial(\partial_\alpha\phi)},\\
T^{00} &= \frac{1}{8\pi}(E^2+B^2),\\
T^{0i} &= \frac{1}{4\pi}\epsilon_{ijk}E_jB_k,\\
T^{ij} = -T^i{}_j &= \frac{1}{8\pi}(\delta_{ij}(E^2+B^2) - 2E_iE_j - 2B_iB_j),\\
\vec{E} &= -\nabla A_0 - \partial_t\vec{A} = -\nabla\Phi - \partial_t\vec{A},\quad \vec{B} = \nabla\times\vec{A},\\
\vec{S} &= \vec{E}\times\vec{B}.
\end{aligned}
$$

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\begin{aligned}
P_\ell(\cos\theta) &= \sqrt{\frac{4\pi}{2\ell+1}}Y_{\ell m=0}(\theta),\\
Y_{0,0} &= \frac{1}{\sqrt{4\pi}},\quad Y_{1,0} = \sqrt{\frac{3}{4\pi}}\cos\theta,\\
Y_{1,\pm1} &= \mp\sqrt{\frac{3}{8\pi}}\sin\theta e^{i\pm\phi},\quad Y_{2,0} = \sqrt{\frac{5}{16\pi}}(3\cos^2\theta - 1),\\
Y_{2,\pm1} &= \mp\sqrt{\frac{15}{8\pi}}\sin\theta\cos\theta e^{\pm i\phi},\quad Y_{2,\pm2} = \sqrt{\frac{15}{32\pi}}\sin^2\theta e^{\pm 2i\phi},\\
Y_{\ell -m}(\theta,\phi) &= (-1)^m Y^*_{\ell m}(\theta,\phi),\\
\delta_{\ell\ell'}\delta_{mm'} &= \int d\Omega\ Y_{\ell,m}(\theta,\phi)Y_{\ell',m'}(\theta,\phi),\\
P_0(x) &= 1,\quad P_1(x) = x,\\
P_2(x) &= \frac{1}{2}(3x^2-1),\quad P_3(x) = \frac{1}{2}(5x^3-3x),\\
P_\ell(x=1) &= 1,\quad \int_{-1}^{1}dx\ P_\ell(x)P_{\ell'}(x) = \frac{2}{2\ell+1}\delta_{\ell\ell'},\\
(3D)\quad \Phi &= \sum_{\ell m}\left(A_{\ell m}r^\ell + B_{\ell m}r^{-\ell-1}\right)Y_{\ell m}(\theta,\phi)e^{im\phi},\\
(2D)\quad \Phi &= A_0\ln(\rho) + \sum_m e^{im\phi}\left(A_m\rho^m + B_m\rho^{-m}\right),\\
\frac{1}{|\vec{r}-\vec{a}|} &= \sum_{\ell=0}^{\infty}\frac{a^\ell}{r^{\ell+1}}P_\ell(\cos\theta),\quad r>a\\
\Phi &= A_0J_0 = \sum_m e^{im\phi}\left(A_mJ_m(k\rho) + B_mN_m(k\rho)\right)e^{\pm kz},\\
\Phi &= \frac{q}{r} + \frac{\vec{p}\cdot\vec{r}}{r^3} + \sum_m\frac{4\pi}{5r^3}q_{2m}(r)Y_{2m}(\theta,\phi),\\
\vec{E} &= -\frac{1}{r^3}\vec{p} + 3\frac{\vec{p}\cdot\vec{r}}{r^5}\vec{r} + \cdots,\\
\Phi(r,\theta,\phi) &= \sum_{\ell m}\frac{4\pi}{(2\ell+1)r^{\ell+1}}q_{\ell m}(r)Y_{\ell m}(\theta,\phi),\\
q_{22} &= \sqrt{\frac{15}{32\pi}}\int d^3r\ \rho(\vec{r})(x-iy)^2 = \sqrt{\frac{15}{288\pi}}(Q_{11} - 2iQ_{12} - Q_{22}),\\
q_{21} &= -\sqrt{\frac{15}{8\pi}}\int d^3r\ \rho(\vec{r})(x-iy)z = -\sqrt{\frac{15}{72\pi}}(Q_{13} - iQ_{23}),\\
q_{20} &= \sqrt{\frac{5}{16\pi}}\int d^3r\ \rho(\vec{r})(3z^2-r^2) = \sqrt{\frac{5}{16\pi}}Q_{33},\\
Q_{ij} &\equiv \int d^3r\ (3r_ir_j - r^2\delta_{ij})\rho(\vec{r}),\\
U &= q\Phi_0 - \vec{p}\cdot\vec{E} - \frac{1}{6}Q_{ij}\partial_iE_j,
\end{aligned}
$$

$$\begin{aligned}
\nabla^2 A^\alpha &= -4\pi J^\alpha, \\
\vec{m} &= \frac{1}{2}\int d^3r\, \vec{r}\times\vec{J} = \frac{I}{2}\int \vec{r}\times d\vec{\ell}, \\
\vec{B} &= -\frac{\vec{m}}{r^3} + \frac{3\vec{r}}{r^5}(\vec{m}\cdot\vec{r}), \\
\mu_e &= g_e\frac{e\hbar}{2m_e}, \\
U &= \frac{(\vec{\mu}_N\cdot\vec{\mu}_e)}{r^3} - \frac{3(\vec{\mu}_N\cdot\vec{r})(\vec{\mu}_e\cdot\vec{r})}{r^5} - \frac{8\pi}{3}(\vec{\mu}_N\cdot\vec{\mu}_e)\delta^3(\vec{r}) \\
& \quad -e\frac{(\vec{\mu}_N\cdot\vec{L})}{mr^3}, \\
T_{00} &= \frac{1}{8\pi}\left(|\vec{E}|^2 + |\vec{B}|^2\right) \\
&= \frac{a_i^2+b_i^2}{8\pi} = \frac{|\vec{a}|^2}{4\pi}\cos^2(\vec{k}\cdot\vec{r}-\omega t), \\
T_{0i} &= \epsilon_{ijk}\frac{E_jB_k}{4\pi} \\
&= \hat{k}_i\frac{|\vec{a}|^2}{4\pi}\cos^2(\vec{k}\cdot\vec{r}-\omega t), \\
T^{ij} = -T^i{}_j &= \frac{1}{8\pi}(\delta_{ij}(E^2+B^2) - 2E_iE_j - 2B_iB_j). \\
&= \frac{1}{4\pi}\left\{|\vec{a}|^2\delta_{ij} - a_ia_j - b_ib_j\right\}\cos^2(\vec{k}\cdot\vec{r}-\omega t), \\
\omega_s &= \omega\sqrt{(1-v)/(1+v)}, \\
(TM)\quad E_z &= \psi(x,y)e^{-i\omega t+ik_zz}, \\
\nabla_t^2\psi &= -(\omega^2-c^2k_z^2)\psi, \quad \psi|_S = 0, \\
\vec{E}_t(x,y) &= \frac{ick_z}{(\omega^2-c^2k_z^2)}e^{-i\omega t+ik_zz}\boldsymbol{\nabla}_t\psi(x,y), \\
\vec{B}_t(x,y) &= \left(\frac{\omega}{ck_z}\right)\hat{z}\times\vec{E}_t, \\
(TE)\quad B_z &= \psi(x,y)e^{-i\omega t+ik_zz}, \\
(\hat{n}\cdot\boldsymbol{\nabla}_t)\psi(x,y)|_S &= 0, \\
\vec{B}_t(x,y) &= \frac{ick_z}{(\omega^2-c^2k_z^2)}e^{-i\omega t+ik_zz}\boldsymbol{\nabla}_t\psi(x,y), \\
\vec{E}_t(x,y) &= -\left(\frac{\omega}{k_z}\right)\hat{z}\times\vec{B}_t. \\
(TEM)\quad E_z &= B_z = 0, \quad \omega = ck_z, \\
\nabla_t^2\psi(x,y) &= 0 \\
\vec{E}_t &= (-\nabla_t\psi)e^{-i\omega t+ik_zz}, \quad \vec{E}_t\times\vec{S} = 0, \\
\vec{B}_t &= \pm\hat{z}\times\vec{E}_t/c.
\end{aligned}$$

$$\begin{aligned}
A^\alpha(x) &= \int d^4x'\,\frac{1}{|\vec{x}-\vec{x}'|}J^\alpha(x')\delta(x_0-x_0'-|\vec{x}-\vec{x}'|), \\
\vec{E} &= e\left\{\frac{\hat{n}\times[(\hat{n}-\vec{\beta})\times\dot{\vec{\beta}}]}{(1-\vec{\beta}\cdot\hat{n})^3|\vec{x}|}\right\}, \\
\vec{B} &= \hat{n}\times\vec{E}.
\end{aligned}$$

$$\begin{aligned}
P &= \frac{2e^2}{3c}|\dot{\vec{\beta}}|^2 \quad \text{(Non.Rel.)}, \\
\frac{dP}{d\Omega} &= \frac{e^2}{4\pi(1-\vec{\beta}\cdot\hat{n})^6}|(\hat{n}-\vec{\beta})\times\dot{\vec{\beta}})|^2, \\
P &= \frac{2}{3c}e^2\gamma^6\left[\dot{\beta}^2 - |\vec{\beta}\times\dot{\vec{\beta}}|^2\right], \\
\frac{dP}{d\Omega} &= \frac{e^2}{4\pi(1-\beta\cos\theta)^5}|\dot{\vec{\beta}}|^2\sin^2\theta \quad \text{(linear)}, \\
P &= \frac{2e^2\dot{\beta}^2}{3c}\gamma^6 \quad \text{(linear)}, \\
\frac{dP}{d\Omega} &= \frac{e^2}{4\pi(1-\beta n_\beta)^5}|\dot{\vec{\beta}}|^2\left((1-\beta n_\beta)^2 - (1-\beta^2)n_r^2\right) \quad \text{(circular)}, \\
P &= \frac{2}{3c}e^2\dot{\beta}^2\gamma^4 \quad \text{(circular)}.
\end{aligned}$$

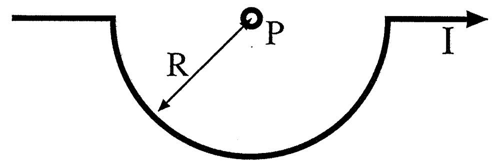
$$\begin{aligned}
\frac{dP}{d\Omega} &= \frac{1}{8\pi}\omega^4|\hat{n}\times\vec{p}|^2, \\
P &= \frac{\omega^4}{3}|\vec{p}|^2,
\end{aligned}$$

$$\begin{aligned}
\text{(Thomson)}\quad \sigma &= \frac{8\pi e^4}{3m^2}, \\
\frac{\Delta\lambda}{\lambda} &= \frac{\hbar\omega}{m}(1-\cos\theta_s).
\end{aligned}$$

electron	$-2.00231930436182 \pm 0.00000000000052$
muon	$-2.0023318418 \pm 0.0000000013$
proton	$5.585694702 \pm 0.000000017$
neutron	$-3.82608545 \pm 0.00000090$

SHORT ANSWER SECTION

1. (5 pts) A current I is flowing through an infinitesimally thin wire made of two straight lines and a half circle joining them as shown in Fig. 1. The radius of the circle is R. Find the strength and the direction of the magnetic field B at the center of the half circle, (P).



2. (5 pts) Which of the following are odd under parity? Circle the answers.

   (a) $\vec{A}$ (the vector potential)
   (b) A_0 (the electric potential)
   (c) $\vec{E}$ (the electric field)
   (d) $\vec{B}$ (the magnetic field)
   (e) $\vec{E} \times \vec{B}$
   (f) $|\vec{B}|^2 - |\vec{E}|^2$
   (g) $|\vec{E}|^2 + |\vec{B}|^2$
   (h) $\vec{E} \cdot \vec{B}$
   (i) $J \cdot A$ (J is the electric current density)

LONG ANSWER SECTION

3. (25 pts)Consider a relativistic proton beam being transported through a beam line with a kinetic energy E and a current I along the x axis. The beam is a DC beam (that is, not bunched, sometimes referred to as a "coasting beam") with a transverse radius of a. The protons are uniformly distributed within $r < a$. Here, define the frame K' as moving together with the protons along the x axis, that is, the frame K' is the rest frame of the protons and the coordinates in the frame K' have the superscripts of $'$. The frame K is defined as the laboratory frame (or fixed to earth).

   (a) (5 pts) What is the velocity, β, of the proton beam? $(\beta = v/c)$

   (b) (5 pts) What is the charge density, $\rho(r)$, of the beam?

   (c) (5 pts) Write down the charge density ρ' in the K' frame in terms of the charge density ρ in the K frame.

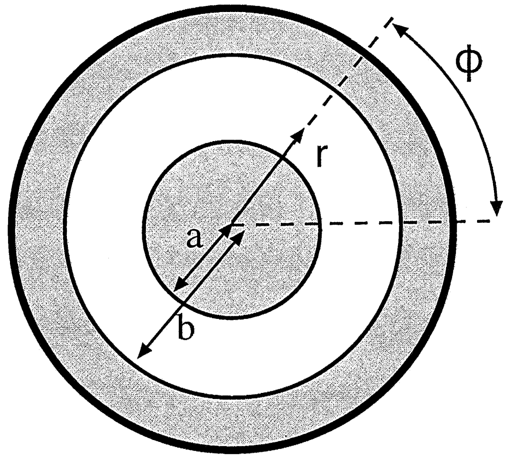
   (d) (5 pts) Find the electric fields E'_r in the K' frame as a function of r' for both $r' < a$ and for $r' > a$.

   (e) (5 pts) Write down the electric and magnetic fields, both E_r and B_ϕ, in the K frame in terms of $\xi \equiv E'_r$, and β.

Extra workspace for #3

4. (15 pts) A charge Q is placed a distance a from the center of a spherical conducting shell of radius R, where the charge is inside the conductor ($a < R$) along the x axis. Show how an image charge can be used to describe the potential inside the conductor. Give both the charge q and its position x.

Extra work space for #4



5. (30 pts) A cylindrical cavity is designed to work with a transverse electromagnetic (TEM, $E_z = B_z = 0$) mode. The design consists of a coaxial transmission line along the z axis featuring two concentric circular cylinders of copper. See the figure for the definitions of an inner radius a and an outer radius b. In between the two cylinders is vacuum. The cylinder extends from $z = -L/2$ to $z = +L/2$, and is capped at both end by conductors. For each question below, consider the solution with the fewest nodes (radial, tangential and along the z axis).

   (a) (5 pts) What is the angular frequency ω for the TEM mode? Give answer in terms of a, b and L.

   (b) (5 pts) Sketch both the electric and magnetic field lines in the Figure for $z = 0$.

   (c) (10 pts) If the maximum electric field is E_0, write down the electric and magnetic field components, $E_r(r, \phi, z, t)$, $E_\phi(r, \phi, z, t)$, $H_r(r, \phi, z, t)$ and $H_\phi(r, \phi, z.t)$.

   (d) (5 pts) Find the electric current I that travels along the z axis through the inner conductor.

   (e) (5 pts) Remove the the end caps, and consider an infinitely long cavity. If E_0 is the maximum strength of the electric field, what is the power of a TEM wave with longitudinal wave number k_z?

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Extra work space for #5

6. (20 pts) Three point charges are located at the origin $(x = 0, y = 0, z = 0)$, at $(x = 0, y = 0, z = +a)$, and at $(x = 0, y = 0, z = -a)$ as shown in Fig. 5. The charge at the origin is $-2q$, while the other two charges are $+q$.

   (a) (5 pts) Write down the potential as a sum, expanding in powers of $1/r$, and using Legendre polynomials, $P_\ell(\cos\theta)$. Here, θ is the azimuthal angle between the z axis and the observation point.

   (b) (5 pts) Repeat, but in powers of r, i.e. for short distances.

   (c) (5 pts) Keeping the product of qa^2 finite, find the form of the potential, $V(r, \theta)$, in the limit of $a \to 0$.

   (d) (5 pts) Which multipole is qa^2, monopole, dipole, quadrupole or others?

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Extra work space for #6