PHY820/422 Final Exam

and

Classical Mechanics Subject Exam

December 13, 2017

Name: ....

If you are a graduate student taking the Subject Exam, DO NOT WRITE YOUR NAME.
A number is assigned to each graduate student.

1. A particle of mass m moves in one dimension under the influence of the potential

$$V = -\frac{k}{x} + \frac{A}{x^2}$$

where both A and k are positive constants.

   (a) Describe the qualitative nature of the motion for positive and negative total energies ($E > 0$ vs. $E < 0$).

   (b) Find the equilibrium point, and the frequency of small oscillations around equilibrium. Does the frequency of small oscillations depend on the amplitude of oscillations? Explain why or why not.

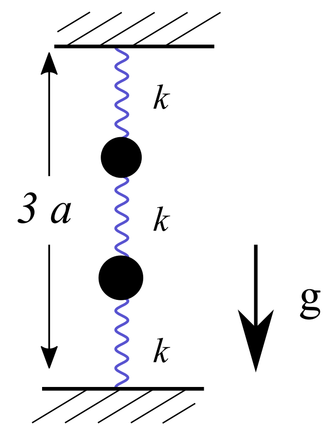
2. Two particles of equal mass m move vertically at the junction of three springs as shown in the figure. The springs all have unstretched lengths equal to a. The total height of the system is $3a$. All the spring constants are k.

   (a) Assume that there is no force of gravity ($g = 0$). Find the eigenfrequencies and normal modes of the system.

   Now assume that the motion takes place in the presence of the gravitational force (g is a nonzero constant).

   (b) Find the equilibrium positions of the two particles.

   (c) What are the eigenfrequencies and normal modes of the system in the presence of gravity? Compare your answer to part (a).



You do NOT need to do any calculations for this problem. Justify your answers in words.

3. (a) What is the total cross section for a repulsive potential described by ($V_0 > 0$ is a positive constant)

$$V(r) = \begin{cases} V_0, & r < a \\ 0, & r > a \end{cases}$$

(b) What is the total cross section for an attractive potential described by ($V_0 > 0$ is a positive constant)

$$V(r) = \begin{cases} -V_0, & r < a \\ 0, & r > a \end{cases}$$

(c) What is the total cross section for the potential ($V_0 > 0$ is a positive constant)

$$V(r) = \begin{cases} 0, & r < a \\ V_0, & r > a \end{cases}$$

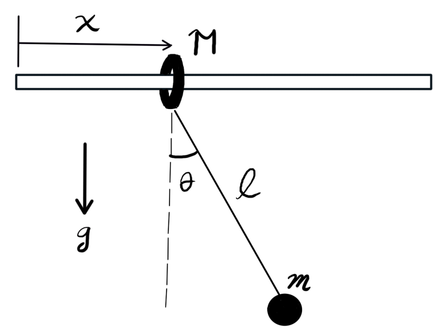
4. A bead of mass M is threaded on a horizontal rod. A simple pendulum of length l with a mass m attached to its end is suspended from the bead as shown in the figure. The motion takes place under the influence of gravity with the gravitational constant g.

   Denote by x the position of the bead on the rod, and by θ the angle of the pendulum from the vertical axis.

   (a) Write down the Lagrangian for this system.

   (b) Use the Euler-Lagrange equation to obtain the equations of motion.

   (c) Identify a cyclic (ignorable) coordinate, if any, and the corresponding conserved generalized momentum. What does the conserved quantity physically represent?



5. Consider a system with two degrees of freedom x and y whose dynamics is governed by the Lagrangian

$$L = \dot{x}^2 + x^2\dot{y}^2$$

   (a) Derive the equations of motion for both degrees of freedom.
   (b) Derive the Hamiltonian $H(x, y, p_x, p_y)$ from the Lagrangian L.
   (c) Derive the equations of motion from the Hamilton's equations, and show that they are consistent with the Euler-Lagrange equations.

This question is only intended for graduate students and/or those who are taking the subject exam.

6. Consider a system of two degrees of freedom x and y whose dynamics is governed by the Hamiltonian

$$H = \frac{(p_x + at + bx + cy)^2}{2m} + \frac{p_y^2}{2m}$$

with m the mass, and a, b and c constant coefficients.

(a) Using the Hamilton's equations, find the equations of motion (expressed in terms of x, y and their time derivatives).

(b) Show that the equations of motion can be brought into the form

$$m\ddot{\mathbf{r}} = \mathbf{E} + \mathbf{v} \times \mathbf{B}$$

where $\mathbf{r} = x\hat{\mathbf{x}} + y\hat{\mathbf{y}}$ is the position vector, and $\mathbf{E} = E_x\hat{\mathbf{x}} + E_y\hat{\mathbf{y}}$ and $\mathbf{B} = B\hat{\mathbf{z}}$ are vectors which can be interpreted as electric and magnetic fields. ($\hat{\mathbf{x}}$, $\hat{\mathbf{y}}$, and $\hat{\mathbf{z}}$ are the unit vectors along the x, y, and z axis, respectively.) Find the components of $\mathbf{E}$ and $\mathbf{B}$ in terms of the constants a, b and c.