Student No.: ________________________

Qualifying/Placement Exam, Part-A
10:00 – 12:00, January 13, 2015, 1400 BPS

Put your **Student Number** on every sheet of this 6 problem Exam -- NOW

You have 2 hours to complete the 6 problems on Part-A of the exam. Show your work! Full credit will not be given for answers without justification. Some partial credit may be earned for the correct procedure, even if the correct answer is not achieved. Answers must be in the spaces provided. The **BACK** of the problem page may be used for lengthy calculations. *Do not use the back of the previous page for this purpose*!

You may need the following constants:

$k_e = 8.99\times10^9\ \mathrm{Nm^2/C^2}$	permittivity of free space
$\sigma = 5.7\times10^{-8}\ \mathrm{Wm^{-2}K^{-4}}$	Stefan-Boltzmann constant
$k = 1.4\times10^{-23}\ \mathrm{J/K}$	Boltzmann constant
$\hbar = 1.05\times10^{-34}\ \mathrm{J\cdot s}$	Planck's constant
$= 6.58\times10^{-16}\ \mathrm{eV\cdot s}$	"
$c = 3.0\times10^{8}\ \mathrm{m/s}$	speed of light
$e = 1.602\times10^{-19}\mathrm{C}$	charge of the electron

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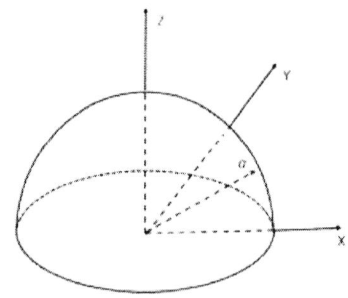
1. [10 pts] Calculate the center of mass of a uniform half sphere with a radius of a, as shown in the figure.



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2. [10 pts] Consider the Lagrangian $L = \left(\dot{x}^2 + \dot{y}^2\right)/2 - \left(\omega_1^2 x^2 + \omega_2^2 y^2\right)/2 + \alpha xy$. Find the normal modes and eigenfrequencies of this system.

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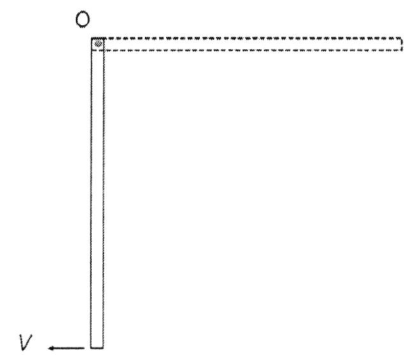
3. [10 pts] Without friction, a uniform bar with a mass m and length L pivots about the bar at point O. Initially placed horizontally, the bar swings back and forth after being released, as shown in the figure.



a) [5 pts] Determine the velocity, v, of the free end of the bar when the bar reaches the vertical orientation.
b) [5 pts] Determine the force between the bar and the point O, when the bar is in the vertical orientation.

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4. [10 pts] Two EM-waves are given by the equations of the electric field $\mathbf{E}(x,t)$:

Wave 1: $E_y = E_0 \sin(kx - \omega t)$, $E_z = 4E_0 \sin(kx - \omega t)$

Wave 2: $E_y = -E_0 \cos(kx + \omega t)$, $E_z = E_0 \sin(kx + \omega t)$

Assume $\omega/k = c$, and $E_x = 0$

For **EACH** wave:

a) [2 pts] Describe the polarization as linear, circular, elliptical, or unpolarized.
b) [3 pts] Determine the equations describing the magnetic field $\mathbf{B}(x,t)$.

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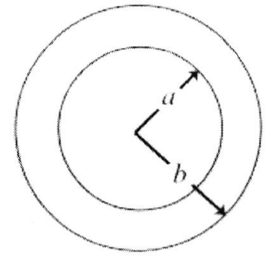
5. [10 pts] Two concentric metallic spherical shells maintained at a constant electric potential difference, V, form an annulus with inner radius a, and outer radius b, as shown in the figure. To obtain the *minimum* electric field at the inner surface, what is the ratio of b/a?



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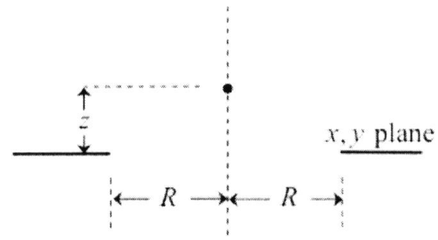

6. [10pts] From a very large flat sheet lying in the x, y plane that carries a uniform + charge density, σ, a hole of radius R is cut out, as shown in the figure.

a) [5pts] At a distance, z, along a line passing perpendicularly through the center of the hole, what is the magnitude of the electric field?

b) [5pts] If a particle with charge $-q$ and mass m is released from rest at the point $z_0 << R$ along the line, show that the particle undergoes an oscillatory motion and determine its oscillation frequency, ω.

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Qualifying/Placement Exam, Part-B
2:00 – 4:00, January 13, 2015, 3239 BPS (HEP conf. Rm.)

Put your **Student Number** on every sheet of this 6 problem Exam -- NOW

You have 2 hours to complete the 6 problems on Part-B of the exam. Show your work! Full credit will not be given for answers without justification. Some partial credit may be earned for the correct procedure, even if the correct answer is not achieved. Answers must be in the spaces provided. The **BACK** of the problem page may be used for lengthy calculations. *Do not use the back of the previous page for this purpose*!

You may need the following constants:

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$e = 1.602\times 10^{-19}\mathrm{C}$	charge of the electron

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1. [10 pts] A plane wave is incident on a double slit (slits are labeled 1 & 2) with a distant detector that can move perpendicular to the direction of the slits along a straight line parallel to the plane and whose coordinate is labeled y. If only one slit is open, at the position y, the wave function generated by the open slit is:

$$\text{slit-1: } \psi_1 = \sqrt{\tfrac{1}{2}}e^{-y^2/2}e^{i(\omega t - ay)}, \quad \text{or slit-2: } \psi_2 = \sqrt{\tfrac{1}{2}}e^{-y^2/2}e^{i(\omega t - ay - by)}.$$

If both slits are open, what is the intensity pattern observed by the distant detector as a function of its position y?

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2. [10 pts] A particle of mass m moves in a one-dimensional square-well potential, given by

$$V(x) = \left\{ \begin{array}{ll} 0 & \text{for } -a/2 \le x \le a/2 \\ \infty & \text{for } |x| > a/2 \end{array} \right\}.$$

a) [3 pts] Write the wave function and energy of the ground state.

b) [7 pts] If the particle is in the ground state, what are the possible results of measuring the momentum of the particle; and what are the probabilities for those results?

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3. [10 pts] A spin-1/2 electron in a uniform magnetic field along the z-axis has a diagonalized Hamiltonian represented by the matrix

$$H = \begin{pmatrix} E & 0 \\ 0 & -E \end{pmatrix},$$

with eigenvectors $\psi_+ = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\psi_- = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$.

In this basis, the spin operator along the x-axis can be represented by

$$S_x = \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$$

a) [3 pts] Show that the S_x has eigenvalues $\pm\hbar/2$, and obtain the eigenvectors φ_+ and φ_-.
b) [2 pts] Suppose at time $t = 0$, S_x is measured to have a value of $+\hbar/2$. What is the state vector of the system as a function of the time for $t > 0$?
c) [5 pts] What is the expectation value of S_x as a function of time?

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4. [10 pts] Initially at rest, an electron is accelerated across a potential difference of 420 kV. Note: the rest mass of the electron is 511 keV/c^2.

At the end of the acceleration, what are the following parameters of the electron?

a) [2 pts] kinetic energy
b) [2 pts] total energy
c) [3 pts] momentum
d) [3 pts] de Broglie wavelength

5. [10 pt] Observers on earth see a spaceship traveling past at a speed of $v = 0.5c$ towards a star, which is at a distance, $d = 10$ light-years, away. They calculate that the spaceship will reach the star in a time, $t = d/v = 20$ years.

a) [4 pts] How long will it take to reach the star, as seen by a person riding on the spaceship?
b) [4 pts] What is the distance to the star, as seen by a person riding on the spaceship?
c) [2 pts] What is the speed that the earth passes by, as seen by the person on the spaceship?

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6. [10 pts] If a copper block with heat capacity C is changed reversibly from temperature T_i to temperature T_f, the change of entropy is

$$(\Delta S)_{\text{one block}} = \int_{S_i}^{S_f} dS = \int_{T_i}^{T_f} C\,dT / T = C\ln(T_f / T_i).$$

Now consider two identical copper blocks with initial temperatures T_1 and T_2. Suppose they are brought into contact and allowed to come to equilibrium at $(T_1 + T_2)/2$.

a) [5 pts] Calculate the change of entropy, ΔS, of the complete system

b) [5 pts] Sketch a qualitatively correct graph of ΔS vs. T_2, for $0 < T_2 < 3T_1$.