Qualifying/Placement Exam
9:00 am, August 21, 2001

Name: ____________________ Student No.: ______________________

Put your NAME on every sheet of this 12 problem Exam -- NOW

You have 3 hours to complete the 12 problems on this exam. Show your work! Full credit will not be given for answers without justification. Some partial credit may be earned for the correct procedure, even if the correct answer is not achieved. Answers must be in the spaces provided. The **BACK** of the problem page may be used for lengthy calculations. Do not use the back of the previous page for this purpose!

Problem 1

(10 points) In orbit 200 km above the surface of the Earth, an astronaut *feels* weightless. In 50 words or less, describe why the astronaut feels this way. Be sure to mention all significant external (and internal) forces acting on (and in) the body of the astronaut.

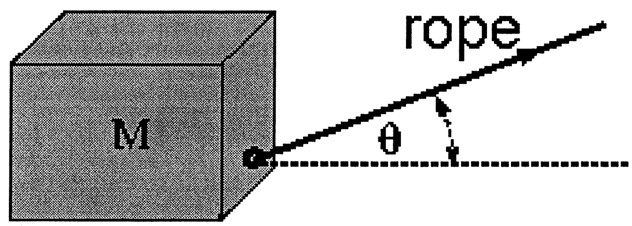
Problem 2

Consider the behavior of N ideal springs with spring constants, k_1, k_2, k_3 , ..., k_N, and the same unstressed length. In terms of the individual spring constants, **derive** the combined spring constant (a) k_P , for a parallel connection, and (b) k_S , for a series connection of the springs. (Note: the problem is analogous to the behavior of series and parallel circuit elements.) **(Show your work below)**

(a) Parallel springs (5 points)

(b) Series springs (5 points)

Problem 3



(10 points) Mechanical Engineering students attempt to pull a large crate of mass 231 kg on a level driveway, as shown in the figure above. The coefficient of static friction μ is 0.610. They use a rope of dubious quality to pull the crate as indicated in the figure. At what angle (in degrees) with respect to the ground should they pull to minimize the chance that the rope will break while they attempt to get the crate to start sliding.

(Hint: The maximum magnitude of the static frictional force is given by the product of the normal force and μ.)

Problem 4

A small sphere, mass 2×10^{-3} kg, hangs by a thread making an angle of 10° with the vertical, between two parallel vertical plates 0.05 m apart. The charge on the sphere is 6×10^{-9} C.

(a) (5 points) Below, draw a free body diagram showing the relevant forces acting in this situation.

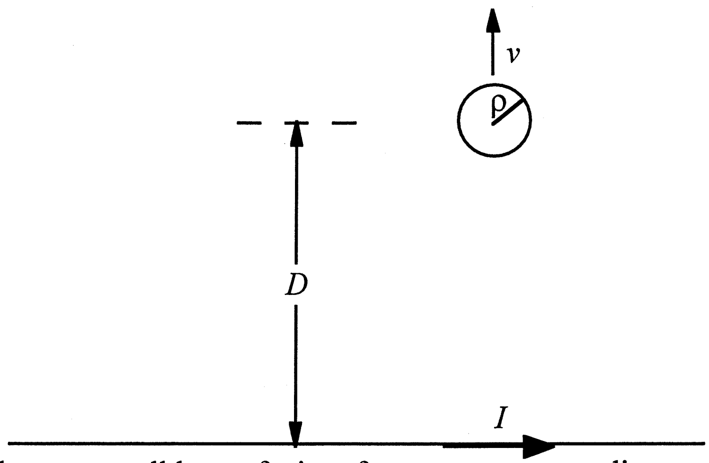
(b) (5 points) Calculate the potential difference between the plates (Show all work, and one significant figure in the answer will suffice.)

Problem 5

(10 points) The plates of a parallel plate, air-gap capacitor, charged to 1.4 V, are initially separated by 1.000 mm, but move closer together by 1.0 μm as a result of their mutual attraction. If the area of each plate is 0.55 cm^2, how much work was done by the electrostatic forces? Neglect fringing field effects.

(Hints: The movement of the plates does not significantly change the electric field. In MKS units, $\varepsilon_0 = 8.85 \times 10^{-12}$ F/m.)

Problem 6



As shown in the figure above, a small loop of wire of radius ρ, is in the same plane as a very long straight wire which carries a constant current *I*. The resistance of the small loop is *R*.

(a) (4 points) If the center of the loop is a distance *D* from the long wire, find an expression for the magnetic flux through the loop. Assume $\rho << D$.

(b) (5 points) Now the loop is moved away, along a line perpendicular to the long wire, with a constant speed *v*. As a function of the distance *D*, find an expression for the current *i* in the loop, due to electromagnetic induction.

(c) (1 point) Is the current in the loop clockwise or counterclockwise?

Problem 7

The flux (power per area per unit wavelength), $\Phi_{\lambda}(T)$, of the radiation from a unit surface area of a black body is given by Planck's radiation law,

$$\Phi_{\lambda}(T) = \frac{2\pi hc^2}{\lambda^5}\frac{1}{e^{hc/k\lambda T}-1}.$$

(a) (5 points) Show that the maximum flux occurs at a wavelength, λ_m, such that $\lambda_m T = \text{constant}$ (in terms of fundamental constants)
(Note the equation $5(1-e^x)+xe^x = 0$, has the numerical solution, $x = 4.96511...$).

(b) (5 points) Show that the total flux, P, emitted by a unit area of a black body is given by $P = \sigma T^4$, where σ is the Stefan-Boltzmann constant (we will not require the evaluation of any <u>dimensionless</u> integrals)

Problem 8

(a) (4 points) Obtain an expression for the change in entropy of an ideal monatomic gas when it is heated at constant volume from the temperature T_1 to T_2. (Hint: the internal energy of an ideal gas is $\frac{3}{2}NkT$.)

(b) (4 points) Repeat for the case when the volume increases at constant pressure.

(c) (2 points) In which case is the change in entropy larger, and why?

Problem 9

Bound physical systems have a characteristic energy for their energy levels (e.g., energy of first excited state above the ground state).

(a) (4 points) From the smallest (#1) to the largest (#4), order the characteristic energy for the energy levels of the following bound systems:

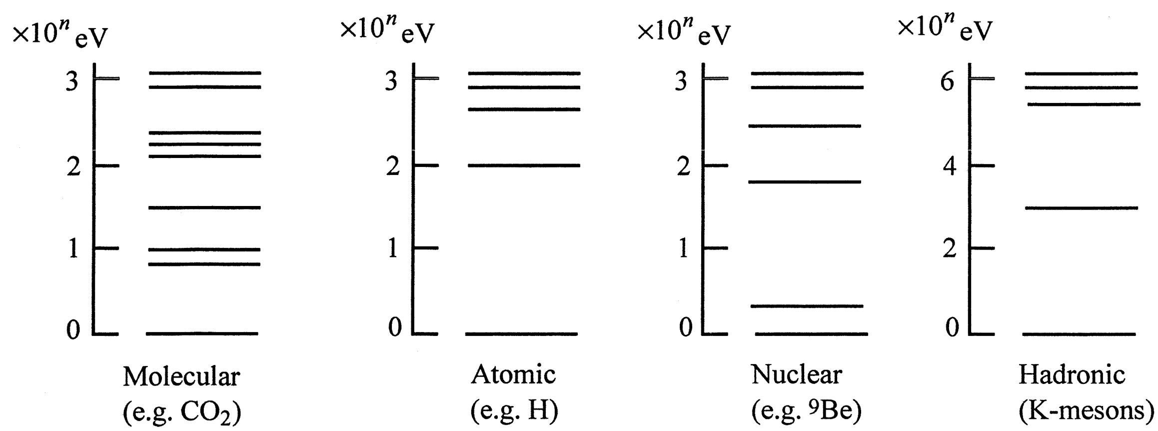
hadrons (Hadronic) _____

molecules (Molecular) _____

nuclei (Nuclear) _____

atoms (Atomic) _____

Energy Levels in Physical Systems



(b) (4 points) Sketched in the figure above are the energy levels characteristic of the various physical systems, expressed in units of $\times 10^n$ eV, with an unknown integer exponent n. Provide the appropriate integer exponent (give only one) for each of these systems. (note: Boltzmann's constant $k = 8.62\times 10^{-5}$ eV / K , and Planck's constant, $h = 4.14\times 10^{-15}$ eV$\cdot$s)

Molecular: $n =$ ____

Atomic: $n =$ ____

Nuclear $n =$ ____

Hadronic $n =$ ____

(c) (2 points) The masses, 2 MeV, 50 MeV, 300 MeV, 1.5 GeV, 5.0 GeV, and 175 GeV, are representative of what elementary particle system? _____________

Problem 10

(10 points)

We wish to produce ^{64}Cu by bombarding a thin sheet of natural copper (with natural abundance of 69% ^{63}Cu, and 31% ^{65}Cu) in a uniform flux of low-energy neutrons.

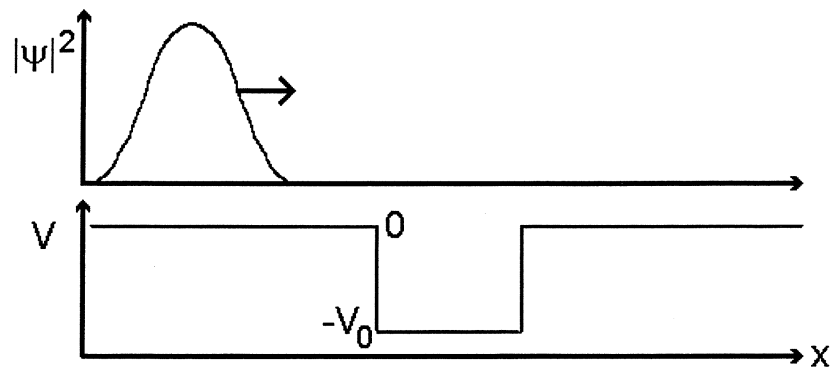
The cross section for $^{63}\mathrm{Cu} + \mathrm{n} \rightarrow {}^{64}\mathrm{Cu}$, is $\sigma = 4.5 \times 10^{-24}$ cm^2.

Find the production rate of ^{64}Cu (in number of nuclei produced per second), if the copper is 1.0 cm^2 in area and 0.01 cm thick, and is in a reactor where the uniform neutron flux is 1.0×10^{12} cm^{-2} s^{-1} (assume that the target is thin enough that attenuation of neutrons crossing the sheet is negligible).

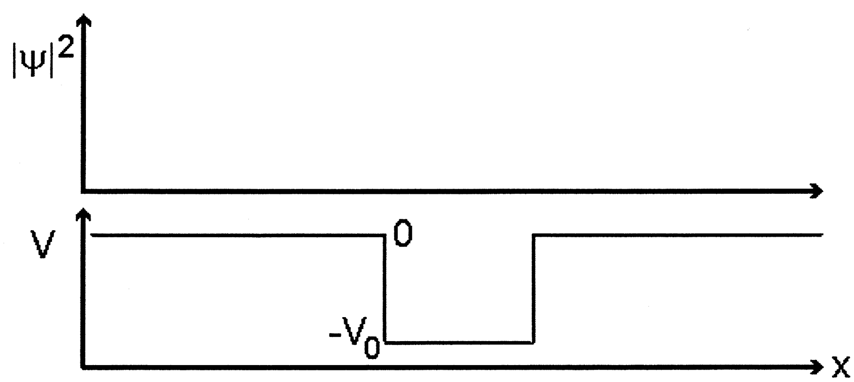
Note: natural copper has a density of 8.96 g / cm^3 and atomic mass 63.55. Avogadro's number is, $N_A = 6.02 \times 10^{23}$.

Problem 11

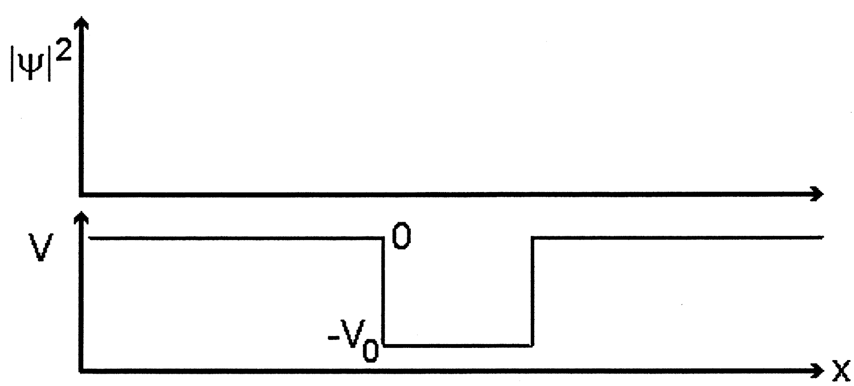
The figure shows a Gaussian wave packet $|\psi(x)|^2$ at time t = 0, incident from the left on a one-dimensional square-well potential (both plots share the same x-axis). The **mean energy** of the packet is $\frac{1}{2}V_0$.



(a) (5 points) Below make a qualitative sketch of $|\psi(x)|^2$ at time $t = t_0$, when the center of the wave packet passes over the square well. (5 points)



(b) (5 points) Below make a qualitative sketch of $|\psi(x)|^2$ at time $t = 2t_0$. (5 points)



Problem 12

(a) (5 points) Use the Lorentz transformation of energies, $E' = \gamma\left(E + \beta pc\right)$, to derive the "Doppler" shift in frequency if light is viewed from a reference frame moving with velocity $\beta = \frac{v}{c}$, toward the light source.

(b) (5 points) How fast must you move toward a red light ($\lambda = 650$ nm) for it to appear green ($\lambda = 525$ nm) ?