Name: _____ Student No.: _____

Qualifying/Placement Exam
9:00 am, January 13, 2004

Put your NAME on every sheet of this 12 problem Exam -- NOW

You have 3 hours to complete the 12 problems on this exam. Show your work! Full credit will not be given for answers without justification. Some partial credit may be earned for the correct procedure, even if the correct answer is not achieved. Answers must be in the spaces provided. The **BACK** of the problem page may be used for lengthy calculations. Do not use the back of the previous page for this purpose!

You may need the following constants:

$k_e = 8.99 \times 10^9 \text{ Nm}^2 / \text{C}^2$	permittivity of free space
$\sigma = 5.7 \times 10^{-8} \text{ Wm}^{-2}\text{K}^{-4}$	Stefan - Boltzmann constant
$k = 1.4 \times 10^{-23} \text{ J / K}$	Boltzmann constant
$\hbar = 1.05 \times 10^{-34} \text{ J} \cdot \text{s}$	Planck's constant
$c = 3.0 \times 10^8 \text{ m / s}$	speed of light

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1. [10 pts] A solid sphere of mass M and radius R rolls without slipping down an inclined plane, which makes angle θ with the horizontal. What is the acceleration of the center of mass of the sphere? (The moment of inertia of a solid sphere about its center of mass is $I = \frac{2}{5}MR^2$.)

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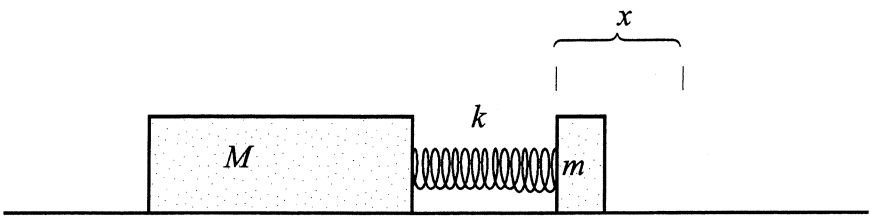

2. Consider a massless coil spring, spring constant k, compressed by a distance, x, between a large mass, M, and a small mass, m, as shown in the figure above. The masses are at rest when the spring is released.

a) [4 pts] In the laboratory frame, what is the kinetic energy gained by each mass when leaving the spring? (Hint: use the expression for KE in terms of the momentum, p.)

b) [4 pts] Use a non-relativistic transformation of velocities to express these kinetic energies in a frame moving with a speed u, parallel to the surface.

c) [2 pts] Show that the potential energy stored by a spring is a non-relativistic invariant.

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3. A spring of equilibrium length, L_0, hangs vertically. When a mass, M, is attached to the bottom of the spring, it stretches to a new length, L.

a)[6 pt] The mass is then set in oscillation. What is the period of oscillation for the mass?

b) [4 pt] If the mass were instead attached to two springs identical to the above spring, both hanging vertically in parallel, what would the new period of oscillation be?

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4. [10 pts] In the *x,y* plane a point dipole $\vec{p}_1 = p_0\hat{j}$ is at (0,0), and another point dipole $\vec{p}_2 = p_0\hat{j}$ is at $(s\frac{\sqrt{2}}{2}, s\frac{\sqrt{2}}{2})$. How much external work must you do to invert $\vec{p}_2$ from its initial orientation to $\vec{p}_{2,final} = -p_0\hat{j}$, if $\vec{p}_1$ is held fixed? (Hint: The electric field of a point electric dipole $\vec{p}$ is $\frac{1}{4\pi\varepsilon_0 r^3}\left[3(\vec{p}\cdot\hat{r})\hat{r} - \hat{p}\right]$, where $\hat{r}$ and r are the unit vector and distance from the dipole. The potential energy of a dipole in an applied field, $\vec{E}$, is $-\hat{p}\cdot\vec{E}$.)

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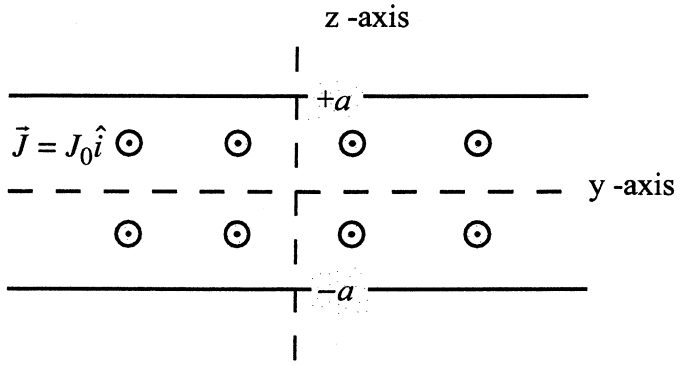
5. The figure below shows a conducting slab which extends between $-a \le z \le +a$ and is infinite in the x and y directions. There is a constant volume current density, $\vec{J} = J_0\hat{i}$ (Amperes/meter^2) flowing in the entire slab.



(a) [6 pts] What is the magnetic field $\vec{B}$ in the region, $-a \le z \le +a$?

(b) [2 pts] What is the magnetic field $\vec{B}$ in the region, $z \ge +a$?

(c) [2 pts] What is the magnetic field $\vec{B}$ in the region, $z \le -a$?

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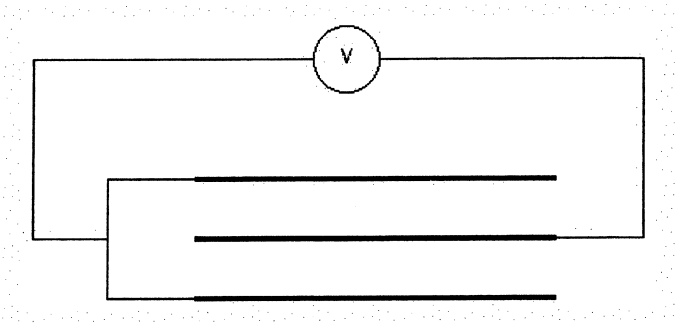

6. A capacitor is made by rolling up three sheets of aluminum foil, with a 0.05 mm thick sheet of paper with dielectric constant = 3.5 between each layer of foil. A wire joins the two outer sheets of aluminum, as in the equivalent circuit shown.

a) [6 pts] What area of foil is needed to produce a capacitance of 20 μF?

b) [4 pts] If the paper can stand an electric field of 14 kV/mm, what is the maximum voltage the capacitor can take?

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7. Astronauts find some rocks in an asteroid belt. When analyzed, the rocks show rubidium-80 and strontium-80 present in the ratio of 4:1. It is known that rubidium-80 decays to the stable strontium-80 with a half-life of 4.7×10^{10} years. No strontium would have been present in this type of rock unless it came from radioactive decay of rubidium-80. Calculate how old the rocks are.

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8. Recent observations of neutrino flavor (e, μ, τ) oscillations show that the mass (mc^2) of the heaviest neutrino is likely to be close to its minimum possible value of 0.05 eV.

(a) [2 pts] What is the ratio of the rest mass of the electron to that of such a neutrino?

(b) [6 pts] For such a neutrino with an energy of 1.0 MeV, by how much would its speed differ from the speed of light?

(c) [2 pts] What kinetic energy must an electron have for its speed to be the same as a 1.0 MeV neutrino?

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9. [10 pts] The decay sequence for the isotope Uranium-235 (A=235, Z=92) ends with the stable nucleus Lead-207 (A=207, Z=82). The decay proceeds through the emission of α particles (Helium nuclei) and β particles (electrons).

a) [4 pts] What are the atomic number (Z) and mass number (A) for the α and β particles?

b) [6 pts] What are the total number of α and β particles emitted per nucleus in the decay sequence?

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10. An electron (mass m) is confined to a three-dimensional infinite square well (a box), so that its coordinates are restricted to $0 \le x, y, z \le L$. Its energy eigenfunctions have the form:

$$\psi(x) = N \sin(k_x x) \sin(k_y y) \sin(k_z z) \ .$$

a) [4 pts] What are the possible values of k_x, k_y, and k_z?

b) [6 pts] What are the three lowest energy eigenvalues and their degeneracies? (Don t forget the electron spin.)

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11. One mole of an ideal gas initially at a temperature of 300 K and pressure of 1 atm compressed isothermally until its volume is reduced by a factor of 10.

a) [3 pts] What is the final pressure of the gas?

b) [4 pts] What is the work done by the gas?

c) [3 pts] How much heat is transferred in the process? Is the transfer of heat to or from the gas?

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12. In the questions below, assume the earth is in thermal equilibrium and radiates energy into space at the same rate at which it receives it from the sun (use the Earth orbit mean radius, $a = 1.5 \times 10^{11}$ m, and luminosity of the sun, $L = 3.9 \times 10^{26}$ W). The answers can be shown to not depend on the earth s radius, R.

(a) [4 pts] At what orbit radius around the sun would the oceans (if just water) freeze?

(b) [4 pts] At what orbit would they boil?

(c) [2 pts] Compare the fraction change in orbit radius for the two extremes.