Qualifying Exam

Jan 7, 1999

Exam number:

You have 3 hours to complete the 12 problems on this exam. Do all problems. Show your work! Full credit will not be given for answers without justification. Some partial credit may be earned for the correct procedure, even if the correct answer is not achieved. While waiting to begin, please enter your name and student number on the lines below.

Name: ______________________________ Student No.: ____________________

List of subject areas

Subject area	Problem numbers
Mechanics	1, 2, 3
Electricity and Magnetism	4, 5, 6
Modern Physics	7, 8, 9, 10
Thermodynamics	11, 12

Do not turn this page and start the exam until you are told to begin.

Exam #

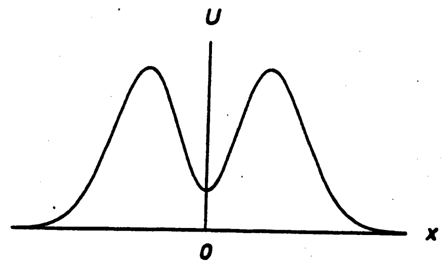
1. The potential energy function for a *classical* particle of mass m is shown in the figure, and is given by the expression

$$U(x) = \frac{1}{2}\left(k_0 x_0^2 + kx^2\right) e^{-(x/x_0)^2},$$

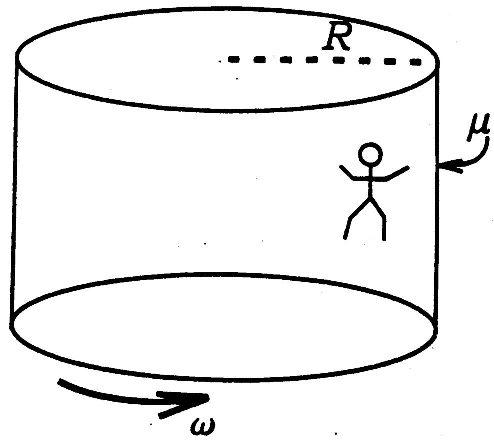
where $k > k_0 > 0$.

(a) *(5 points)* If the particle is initially at the origin, what is the minimum energy, classically, that it must have to escape to infinity?

(b) *(5 points)* What is the frequency for small oscillations of the particle in the potential well about the origin?



2. An amusement park ride consists of a cylindrical room of radius R which rotates about its axis with angular speed ω. A person stands against the inner wall of the room and is held by friction to the wall, after the floor of the room is lowered. If the coefficient of static friction between the person and the wall is μ, what is the minimum angular speed that the room must have so that the person does not slide down the wall?



3. (a) *(4 points)* What test will determine if an external force acts on a system of two particles?

(b) *(3 points)* Test the following one-dimensional system for the existence of an external force: particles of mass m_1 and m_2 are observed to follow the paths

$$\begin{aligned} x_1(t) &= A_1 \sin \omega t + L + v_1 t \, , \\ x_2(t) &= A_2 \sin \omega t + v_2 t \end{aligned}$$

respectively, where constants A_1, A_2, v_1, v_2, L, are arbitrary except that $m_1 A_1 = -m_2 A_2$.

(c) *(3 points)* Test this system again if the paths of the two particles are

$$\begin{aligned} x_1(t) &= A_1 \sin \omega t \\ x_2(t) &= A_2 \sin(\omega t + \phi), \end{aligned}$$

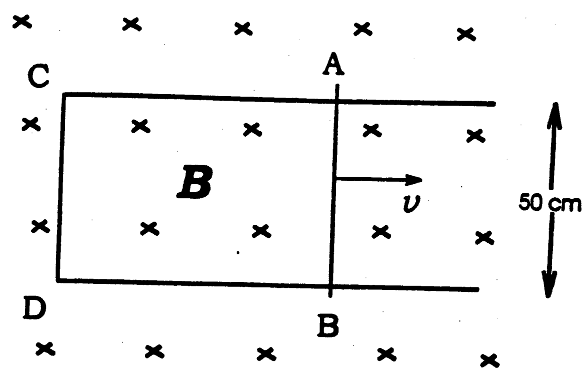
where A_1 and A_2 are related as before and $\phi \neq 0$.

4. In the figure, a conducting rod AB is in contact with metal tracks CA and DB, which are immersed in a uniform magnetic field of $B = 0.5\,\mathrm{T}$ in the direction perpendicular to the paper. (ACDB is a continuous conductor.)

(a) *(4 points)* If the rod moves toward the right with a speed of $4.0\,\mathrm{m/s}$, find the magnitude and direction of the induced emf.

(b) *(3 points)* If the circuit has a resistance of $0.20\,\Omega$ when the rod is at a certain position, find the force exerted on the rod by the magnetic field.

(c) *(3 points)* Compare the power of the external force and the power lost in Joule heating. Ignore friction.

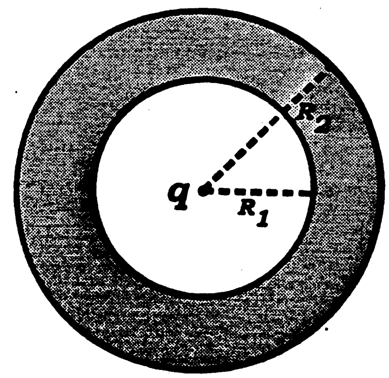


5. In the figure, a point charge $q = 4 \times 10^{-10}$ C is placed at the center of a conducting spherical shell which has inner and outer radii of $R_1 = 2$ cm and $R_2 = 3$ cm respectively.

(a) *(5 points)* Find the electric field $\vec{E}$ as a function of r, the radial distance from the center.

(b) *(4 points)* Find the electric potential of the conducting sphere.

(c) *(1 point)* If the point charge is moved 1 cm from the center, then what is the electric potential of the conducting sphere?



6. (a) *(3 points)* Write Maxwell's equations for electric and magnetic fields in a metal with conductivity σ. (The dielectric permittivity and magnetic permeability may be approximated by the vacuum values ϵ_0 and μ_0.) Assume the charge density ρ is 0; the current density $\vec{J}$ is $\sigma\vec{E}$.

(b) *(5 points)* There exists a solution of the equations in which the electric field has the form

$$\vec{E}(\vec{x}, t) = E_0 e^{i(kz-\omega t)}\hat{e}_y$$

where $\hat{e}_y$ is a unit vector in the y-direction. Derive the relation between k and ω. (Hint: $\nabla \times (\nabla \times \vec{E}) = \nabla(\nabla \cdot \vec{E}) - \nabla^2 \vec{E}$.)

(c) *(2 points)* If $\sigma \neq 0$ then k has a nonzero imaginary part. What does this imply, physically, for the solution in (b)?

7. The first excited state of ^{57}Fe has an excitation energy of 14.4 keV and a mean lifetime of 141 ns.

(a) *(2 points)* What is the width ΔE of the excited state? Express it in eV if you can.

(b) *(5 points)* When the state decays to the ground state by emission of a photon, what is the recoil energy of the ^{57}Fe atom?

(c) *(3 points)* The photon cannot be reabsorbed by another ^{57}Fe atom in its ground state. Why not? (Assume the two atoms are both at rest initially.)

8. This problem does not require great precision in your answers. Use your judgement to decide if one or two significant digits is appropriate.

The Stefan-Boltzmann constant of proportionality is $\sigma = 5.67 \times 10^{-8}\,\mathrm{W\,m^{-2}\,K^{-4}}$. The surface temperature of the Sun is 5800 K, its radius is 6.96×10^{8} m, while the radius of the Earth's orbit is 1.5×10^{11} m.

(a) *(5 points)* Assuming that the sun radiates like a black body, estimate (to just 1 sig. digit) the thermal energy flux emitted from its surface.

(b) *(5 points)* Estimate the energy flux of sunlight arriving on the solar panel of a satellite in low Earth orbit, and the amount of energy available in 6 minutes on a surface area of $1\,\mathrm{m}^2$ facing the sun.

9. Write the two forms of the Heisenberg uncertainty principle involving momentum, energy, time, and position, of a particle moving in the x-direction. Show that for a photon the two forms are equivalent.

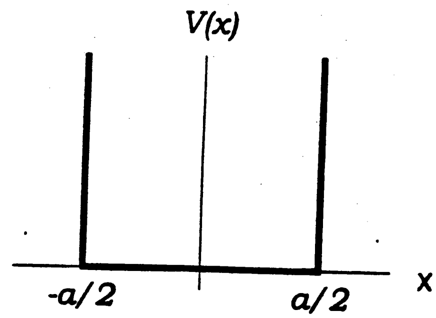
10. A particle of mass m moves in a one-dimensional square-well potential, given by

$$V(x) = \begin{cases} 0 & \text{for } -a/2 \leq x \leq a/2 \\ \infty & \text{for } |x| > a/2 \end{cases}$$

(a) *(3 points)* Write the wave function and energy of the ground state.

(b) *(3 points)* Write the wave function and energy of the first excited state.

(c) *(4 points)* If the particle is in the ground state, calculate the probability for the particle to be in the region $-a/4 \leq x \leq a/4$.



11. In the figure, a thermally conducting container with volume V_0 contains ideal gas of pressure p_0. If the gas is slowly compressed to a volume $V_0/2$ by a piston, and the temperature of the surroundings remains constant in the process, then calculate: (a) the work done by the external force; (b) the heat exchange with the surroundings; (c) the change of entropy of the gas.

12. Consider a gas of noninteracting atoms in a magnetic field $B\hat{e}_z$ in the z-direction. Each atom has total spin 1/2. Each atom has a permanent dipole moment, with magnitude m_0, in the direction of the spin; for spin up $\vec{m} = m_0\hat{e}_z$, and for spin down $\vec{m} = -m_0\hat{e}_z$. The magnetic energy of the atom is $-m_0B$ for spin up, and $+m_0B$ for spin down.

(a) *(7 points)* Determine the mean dipole moment of an atom, as a function of the temperature T. (Recall that the probability for a system to be in a state with energy E is proportional to $e^{-E/kT}$.)

(b) *(3 points)* Sketch a graph of the mean dipole moment versus T.