Student No.: ______________________

Qualifying/Placement Exam, Part-A
12:30 – 2:30, January 9, 2017, 3239 BPS

Put your **Student Number** on every sheet of this 6 problem Exam -- NOW

You have 2 hours to complete the 6 problems on Part-A of the exam. Show your work! Full credit will not be given for answers without justification. Some partial credit may be earned for the correct procedure, even if the correct answer is not achieved. Answers must be in the spaces provided. The **BACK** of the problem page may be used for lengthy calculations. *Do not use the back of the previous page for this purpose*!

You may need the following constants:

$k_e = 8.99\times10^{9}\ \mathrm{Nm^2/C^2}$	permittivity of free space
$\sigma = 5.7\times10^{-8}\ \mathrm{Wm^{-2}K^{-4}}$	Stefan-Boltzmann constant
$k = 1.4\times10^{-23}\ \mathrm{J/K}$	Boltzmann constant
$\hbar = 1.05\times10^{-34}\ \mathrm{J\cdot s}$	Planck's constant
$= 6.58\times10^{-16}\ \mathrm{eV\cdot s}$	"
$c = 3.0\times10^{8}\ \mathrm{m/s}$	speed of light
$e = 1.602\times10^{-19}\mathrm{C}$	charge of the electron

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1. [10 pts]

a) [7 pts] Find the Fourier transform $g(y)$ of $f(x)$ where

$$f(x)=\begin{cases} 1, & |x|<a \\ 0, & |x|>a \end{cases}.$$

b) [3 pts] Sketch the functions $f(x)$ and $g(y)$ for $a = 3$.

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2. [10 pts] A cycloid is given by the parametric equations:

$$x = 3(\theta - \sin\theta) \quad y = 3(1 - \cos\theta) \quad 0 \le \theta \le 2\pi.$$

a) [7 pts] Calculate the area under the cycloid arc, showing your work.

b) [3 pts] Make a sketch of the cycloid.

Hint: The following trigonometric identity may be useful:

$$\cos^2\theta = \frac{1}{2}(1 + \cos 2\theta).$$

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3. [10 pts] Consider the function $f(\phi), -\pi < \phi < \pi$, with

$$f(\phi) = \begin{cases} 1, & -\pi/2 < \phi < \pi/2 \\ 0, & \pi/2 < \phi < 3\pi/2 \end{cases}$$

If f is expressed as a sum,

$$f(\phi) = \sum_{n=0}^{\infty} a_n \cos n\phi + b_n \sin n\phi,$$

(a) [5 pts] List all n for which $a_n = 0$ and all n for which $b_n = 0$.

(b) [5 pts] Find all other a_n and b_n.

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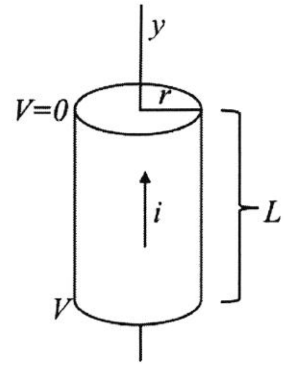
4. [10 pts] A potential difference, V, is applied across a cylindrical conductor of radius r, length L, and resistance R, as shown in the figure below. As a result, a current, i, is flowing through the conductor, which gives rise to a magnetic field, B. The conductor is placed along the y-axis, and the current is flowing in the positive y-direction. Assume that the electric field is uniform throughout the conductor.



The Poynting vector is: $\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B}$.

a) [5 pts] Find the magnitude and the direction of the Poynting vector at the surface of the conductor of the static electric and magnetic fields.

b) [5 pts] Show that $\int \vec{S}\bullet d\vec{A} = i^2R$, where the integral is over the cylinder's surface.

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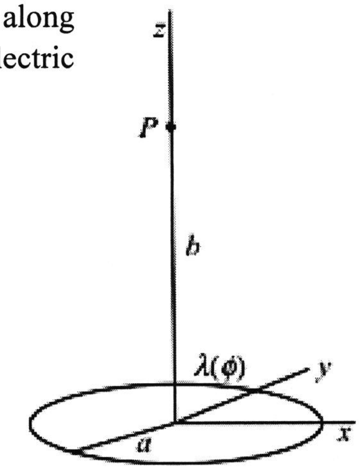
5. [10 pts] A thin ring of radius a, in the x-y plane, has a ϕ-dependent charge density, $\lambda(\phi) = \lambda_0 \cos\phi$, with $\phi=0$ on the x-axis. At a point P up a distance b along the z-axis, perpendicular to the ring and through its center, find the electric field, **E**. (Hint: you may need the identity $\cos 2x = \cos^2 x - \sin^2 x$)



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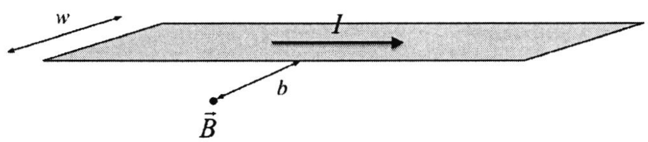
6. [10 pts] A very thin and very long strip of metal of width w, carries a current I along the length of the strip, as shown in the figure below.



Assuming that the current is uniformly distributed across the width, find the magnetic field, $\vec{B}$, in the plane of the strip at a distance b from the near edge.