Student No.: ______________________

Qualifying/Placement Exam, Part-A
10:00 – 12:00, Jan. 3, 2019

Put your **Student Number** on every sheet of this
6 problem Exam -- NOW

You have 2 hours to complete the 6 problems on Part-A of the exam. Show your work! Full credit will not be given for answers without justification. Some partial credit may be earned for the correct procedure, even if the correct answer is not achieved. Answers must be in the spaces provided. The **BACK** of the problem page may be used for lengthy calculations. *Do not use the back of the previous page for this purpose*!

You may need the following constants:

$k_e = 8.99\times 10^9\ \mathrm{Nm^2/C^2}$	electric force constant
$\sigma = 5.7\times 10^{-8}\ \mathrm{Wm^{-2}K^{-4}}$	Stefan-Boltzmann constant
$k = 1.4\times 10^{-23}\ \mathrm{J/K}$	Boltzmann constant
$\hbar = 1.05\times 10^{-34}\ \mathrm{J\cdot s}$	Planck's constant
$= 6.58\times 10^{-16}\ \mathrm{ev\cdot s}$	"
$c = 3.0\times 10^8\ \mathrm{m/s}$	speed of light
$e = 1.602\times 10^{-19}\ \mathrm{C}$	charge of the electron

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1. [10 pts] Using contour integration techniques, evaluate the integral

$$f(t) = \int_{-\infty}^{\infty} d\omega \frac{e^{-i\omega t}}{\omega - \omega_0 + i\gamma}.$$

Where the integral is along the real axis, and t, ω_0, and γ are real constants and with $\gamma > 0$.
Be sure to consider the integral for both a) $t > 0$ and b) $t < 0$. [5 pts each]
Hint: Apply the residue theorem around any poles.

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2. [10 pts] Calculate the first three non-zero terms in the Taylor series around $x = 0$, for

a) [3 pts] $\frac{1}{\sqrt{1+x^2}}$; b) [3 pts] $\frac{\sin x}{x}$; c) [4 pts] $\frac{1-\cos x}{x^2}$.

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3. [10 pts]

a) [5pts] Calculate the inverse of the matrix $A = \begin{pmatrix} 2 & 3 \\ 6 & 4 \end{pmatrix}$

b) [5pts] Using matrix operations, solve for x and y in these simultaneous equations:

$$4x + 5y = 23$$
$$6x - 2y = 6$$

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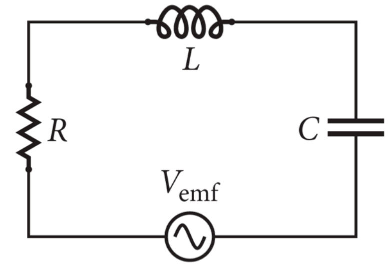

4. [10 pts] In the RLC circuit shown, a resistor $R = 20.0\,\Omega$, an inductor L = 10.0 mH, and a capacitor $C = 5.00\,\mu\text{F}$ are connected in series with an AC power source for which $V_{\text{emf}} = 10.0$ V (rms) at a frequency $f = 100.$ Hz.

Hints: The voltage across the inductor *leads* the voltage across the resistor by 90°, and the voltage across the capacitor *trails* the voltage across the resistor by 90°. Impedances of the inductor and capacitor are ωL and $1/\omega C$, respectively, where $\omega = 2\pi f$. Note: $A_{\max} = \sqrt{2}A_{\text{rms}}$.

a) [3 pts] Calculate the amplitude (maximum) of the current, $I_{\max}$, through the circuit.

b) [3 pts] Calculate the phase between the source current and voltage.

c) [3 pts] Calculate the *maximum* voltage across each component.

d) [1 pt] Show that the maximum voltages of part c) are consistent with the maximum voltage across the series combination.

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5. [10 pts] Beginning with Maxwell's Equations in a vacuum,

a) [8 pts] derive the wave equations for an electromagnetic wave.
b) [2 pts] express the speed of light in terms of ε_0 and μ_0 .

Hint: Derive any identities needed.

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6. [10 pts] A neutral conducting sphere of radius R is placed in a region of uniform electric field, such as $\mathbf{E} = E\,\hat{z}$ before introducing the sphere. Find the induced dipole moment of the sphere.

Hint: Match the boundary conditions in the general solutions of Laplace's equation for the potential in spherical coordinates:

$$\Phi(\bar{r}) = \sum_{\ell \geq 0} P_\ell(\cos\theta)\left\{\frac{A_\ell}{r^{\ell+1}} + B_\ell r^\ell\right\}$$

$$P_0(x) = 1, \quad P_1(x) = x, \quad P_2(x) = \left(3x^2 - 1\right)\big/2, \quad \ldots$$