PHY-841: CLASSICAL ELECTRODYNAMICS - I

Final Subject Exam: Total = 100 points

May 1, 2006; room 1420 BPS; 10:00 a.m. - 1:00 p.m.

1. /20/ A relativistic proton (rest energy mc^2 =938 MeV) with total energy $E = 4.7$ GeV is elastically scattered by a proton at rest. The energy of one of the protons after the collision is measured to be $E' = 2$ GeV.

   a. Find the angle in the laboratory frame between the directions of motion of the protons after the collision.

   b. What is the minimum possible value of this angle, and at what energy E' would it be observed?

2. /20/ A very long solid cylinder of radius R is charged by a volume charge density ρ which is uniform along the length and the radius of the cylinder but depends on the azimuthal angle α,

$$\rho = \rho_0 \cos\alpha.$$

   Find the electrostatic potential and the electric field vector inside and outside the cylinder.

   The Laplace operator in cylindrical coordinates is

$$\nabla^2 = \frac{\partial^2}{\partial z^2} + \frac{1}{r}\,\frac{\partial}{\partial r}\left(r\,\frac{\partial}{\partial r}\right) + \frac{1}{r^2}\,\frac{\partial^2}{\partial \alpha^2}. \qquad (1)$$

3. /20/ An infinitely long wire carrying a steady current I is placed near a square circuit of linear dimension a. The wire is parallel to one side of the square.

   (a). The magnetic flux through the area of the square is measured to be equal to Φ. What is the distance from the current to the closest side of the square?

   (b). In the same geometry, let there be in addition a steady current I' in the square circuit. What is the force (magnitude and direction) acting on the square?

4. /20/ Consider steady electric currents $\mathbf{j}$ in the finite volume V within an insulating surface.

   a. Show that the energy U_m of the magnetic field in the entire space can be expressed as

   $$U_m = \frac{1}{2c} \int dV \, (\mathbf{j} \cdot \mathbf{A}), \tag{2}$$

   where $\mathbf{A}$ is the vector potential of the magnetic field created by the currents and the integral is restricted to the finite volume of the currents.

   b. Is the explicit appearance of the vector potential in eq. (2) compatible with gauge invariance?

5. /20/ *a*. A uniform static magnetic field $B = B_z$ is applied in the region $y > 0$. A nonrelativistic proton with velocity $v = v_y$ enters this region. Find the proton energy loss by radiation during its flight in the region of the magnetic field.

   b. Compare this loss with the initial kinetic energy of the proton for $v/c = 0.1$ and $B = 1$ T.