8.311: Electromagnetic Theory **Final Exam 5/12/05**

Time: 9:30-11 Your Name: ______________________________

1. Coaxial transmission line.

Consider a half-infinite coaxial cable made of a cylindrical conducting shell of inner radius a and a wire of radius b on the axis inside, connected at the end to a source of voltage $V(t) = V_0 \cos \omega t$. The frequency ω is such that only the TEM mode is excited: $\omega \ll c/a$, c/b.

a) [10pt] Find the EM fields $\mathbf{E}(\mathbf{r}, t)$, $\mathbf{B}(\mathbf{r}, t)$ at a distance x from the voltage source.

b) [10pt] Find the energy flow by evaluating the flux of Poynting vector through the cable cross-section at a distance x from the voltage source. Show that at $x = 0$ the energy flux has the form $Z\,(V(t))^2$. Find Z, the transmission line impedance.

2. EM waves in a gas of polar molecules.

The response of a polar molecule to a time-dependent electric field $E(t)$ is described by an equation

$$\dot{\mathbf{d}} = -\frac{1}{\tau}\left(\mathbf{d} - \alpha_0 \mathbf{E}(t)\right) \tag{1}$$

where $\mathbf{d}$ is the molecule average dipole. Here α_0 is static polarizability, and τ is the relaxation time parameter.

a) [10pt] Consider the response of a single molecule $d(\omega) = \alpha(\omega)E(\omega)$ to a time-dependent field $E(\omega) = E_0 \exp(-i\omega t)$. Find the complex polarizability $\alpha(\omega)$. Plot the real and imaginary part $\alpha'(\omega)$ and $\alpha''(\omega)$.

b) [10pt] For a monochromatic EM wave of frequency ω, find the wavevector k. Use complex permittivity $\epsilon(\omega)$ obtained from $\alpha(\omega)$ in the dilute gas approximation ($n\alpha \ll 1$, $\epsilon(\omega) \approx 1$).

Find the EM absorption length L as a function of frequency. Sketch the $L(\omega)$ dependence.

3. Dipole radiation.

Consider a *nonrelativistic* electron moving along the z axis as $z(t) = a \cos \omega_0 t + b \cos 2\omega_0 t$. The EM field at a large distance R away from the origin can be obtained from the dipole radiation field $\mathbf{E}_r = \frac{1}{Rc^2}\mathbf{n} \times (\mathbf{n} \times \ddot{\mathbf{d}})$, $\mathbf{B}_r = \mathbf{n} \times \mathbf{E}_r$.

a) [10pt] Find the time-averaged radiated power angular distribution dP/do. Use the angle θ between the radiation direction $\mathbf{n}$ and the z axis.

b) [10pt] Find the total time-averaged radiated power P.

c) [10pt] Find the radiation frequency spectrum $dP/d\omega$.

4. Relativistic motion in parallel E and B fields.

Consider a *relativistic* electron moving in parallel E and B fields, $\mathbf{E}$, $\mathbf{B} \parallel \hat{\mathbf{z}}$, spatially uniform and constant. Initial velocity of the electron is perpendicular to E and B: $(p_x, p_y)|_{t=0} = (p_0, 0)$.

a) [10pt] Write down the relativistic equations of motion for electron momentum components. Show that $p_x^2 + p_y^2$ is a constant of motion, and find the kinetic energy as a function of time.

b) [10pt] Find p_x and p_y as a function of time.

c) [10pt] Determine the motion, find the electron trajectory $x(t)$, $y(t)$, $z(t)$.