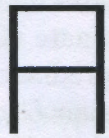
PHY-820 - CLASSICAL MECHANICS

Final Exam

Problem 1 - 10 Points



Consider your desired grade dangling in front of you as shown above. The letter is made up of six identical thin rods that are attached rigidly to each other. Each of the six rods has mass m and length l. The letter is suspended from the midpoint of the top rod and is free to swing left and right.

a) [2pts] Determine the center of mass of the letter.

b) [2pts] Determine the moment of inertia of the letter around the suspension point.

c) [2pts] Set up the Lagrangian of the system using the generalized coordinate of the angle θ of the letter with the vertical.

d) [2pts] Derive the equations of motion and the stationary point.

e) [2pts] Linearize the equations of motion around the stationary point and find the frequency of small oscillations.

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Problem 2 - 10 Points

Consider a roller coaster in which a car of mass m is attached to a frictionless three-dimensional track. The track has a circular footprint with radius R, i.e. the x and y coordinates defining the horizontal plane satisfy $x^2 + y^2 = R^2$. The vertical z coordinate of the track is made to depend on the azimuthal angle θ, measured in radians, as $z = h \cdot (1 + \sin\theta)$.

a) [2pts] Determine the Lagrangian L in terms of θ and $\dot{\theta}$.

b) [2pts] Derive the equations of motion.

c) [2pts] Determine the generalized momentum p_θ, and express $\dot{\theta}$ by p_θ.

d) [2pts] Determine the Hamiltonian H of the system.

e) [2pts] Are p_θ and/or the Hamiltonian H conserved, why or why not?

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Problem 3 - 10 Points

Consider a hollow spherical shell of radius R. On the inside of the spherical shell, a particle of mass m is moving without friction under the influence of gravity which is of uniform magnitude and pointing vertically downward.

a) [2pts] Assume that the scale of the total energy K is such that the particle at rest in the lowest point of the sphere corresponds to $K = 0$. What is the minimum total energy K_{min} the particle needs to have so as to never detach from the shell, regardless of its orbit?

b) [3pts] Assume that the energy of the particle is greater than K_{min}. Determine the Lagrangian of the motion using spherical coordinates r, θ, φ.

c) [2pts] Determine the generalized momenta p_θ and p_φ. Determine which if any of them are conserved, and give a simple explanation.

d) [3pts] Determine the equations of motion.

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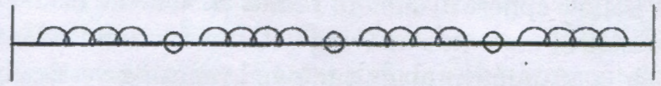
Problem 4 - 10 Points



Three identical masses m are sliding without friction on a rod as shown. They are connected with each other and with walls on both ends of the rod through springs of spring constant k and relaxed length l. The distance between the two walls is $4l$, which leads to an obvious equilibrium position of potential energy 0. As generalized coordinates, use the displacements from the respective equilibrium positions

a) [2pts] Determine the kinetic and potential energy and the Lagrangian

b) [2pts] Determine the equations of motion, linearize, and express them in matrix form

c) [4pts] Determine the eigenmodes and eigenfrequencies of the system

d) [2pts] Discuss qualitatively the meaning of the eigenmodes and the associated frequencies, and in particular the relative sizes of the frequencies.