# Symmetry Breaking and Correlations in Nuclei

## I – Spontaneous Symmetry Breaking and Restoration in Finite Systems

# Nuclear Many-Body Correlations

**short-range**
(hard repulsive core of the NN-interaction)

**long-range**
nuclear resonance modes (giant resonances)

...vary smoothly with nucleon number! Can be included implicitly in an effective Energy Density Functional.

**collective correlations**
large-amplitude soft modes:
(center of mass motion, rotation, low-energy quadrupole vibrations)

...sensitive to shell-effects and strong variations with nucleon number! Cannot be included in a simple EDF framework.

# Spontaneous Symmetry Breaking

Spontaneous Symmetry Breaking (SSB) ⇒ the ground state of a QM many-body system has a symmetry that is lower than the symmetry of the underlying Hamiltonian. The system lowers its energy through spontaneous symmetry breaking, resulting in a state of lower symmetry and higher order.

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Consider a system whose Lagrangian $\mathscr{L}$ is invariant under some symmetry transformations. For example, $\mathscr{L}$ might be spherically symmetric, i.e. invariant under spatial rotations.

The ground state of the system:

1) unique and invariant under the symmetry transformations of $\mathscr{L}$

2) degenerate and the corresponding eigenstates are not invariant, but transform linearly amongst themselves under the symmetry transformations of $\mathscr{L}$.
⇒ there is no unique ground state.

If we arbitrarily select one of the degenerate states as the ground state, then the ground state no longer shares the symmetries of $\mathcal{L}$ ≡ SPONTANEOUS SYMMETRY BREAKING

The Goldstone model: $$\mathscr{L}(x) = [\partial^\mu \phi^*(x)][\partial_\mu \phi(x)] - \mu^2 |\phi(x)|^2 - \lambda |\phi(x)|^4$$

→ complex scalar field: $$\phi(x) = \frac{1}{\sqrt{2}}[\phi_1(x) + \mathrm{i}\phi_2(x)]$$

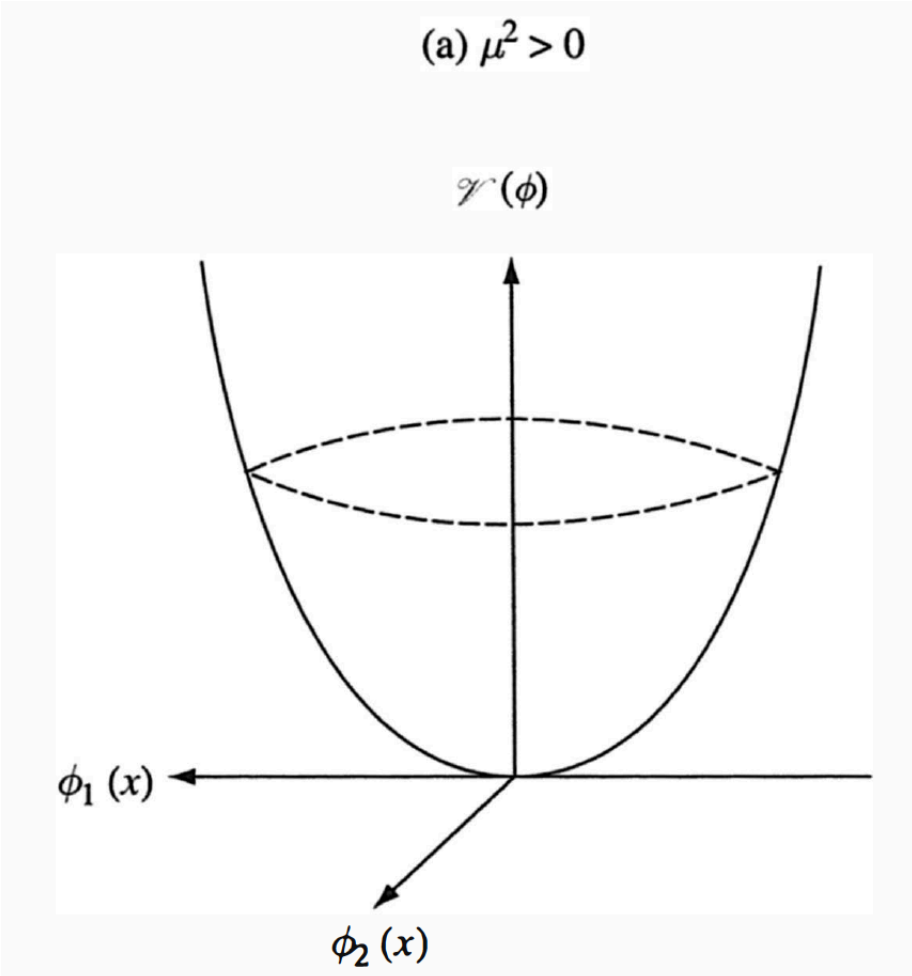
→ relativistic notation: $$\frac{\partial \phi}{\partial x^\mu} \equiv \partial_\mu \phi \equiv \phi_{,\mu} \qquad \frac{\partial \phi}{\partial x_\mu} \equiv \partial^\mu \phi \equiv \phi_{,}{}^{\mu}$$

The Lagrangian density is invariant under the global U(1) phase transformations:

$$\phi(x) \to \phi'(x) = \phi(x)\mathrm{e}^{\mathrm{i}\alpha}, \qquad \phi^*(x) \to \phi^{*\prime}(x) = \phi^*(x)\mathrm{e}^{-\mathrm{i}\alpha}$$

The potential energy density of the field: $$\mathscr{V}(\phi) = \mu^2 |\phi(x)|^2 + \lambda |\phi(x)|^4$$

For the energy of the field to be bounded from below: λ > 0. Minimum of the potential ?

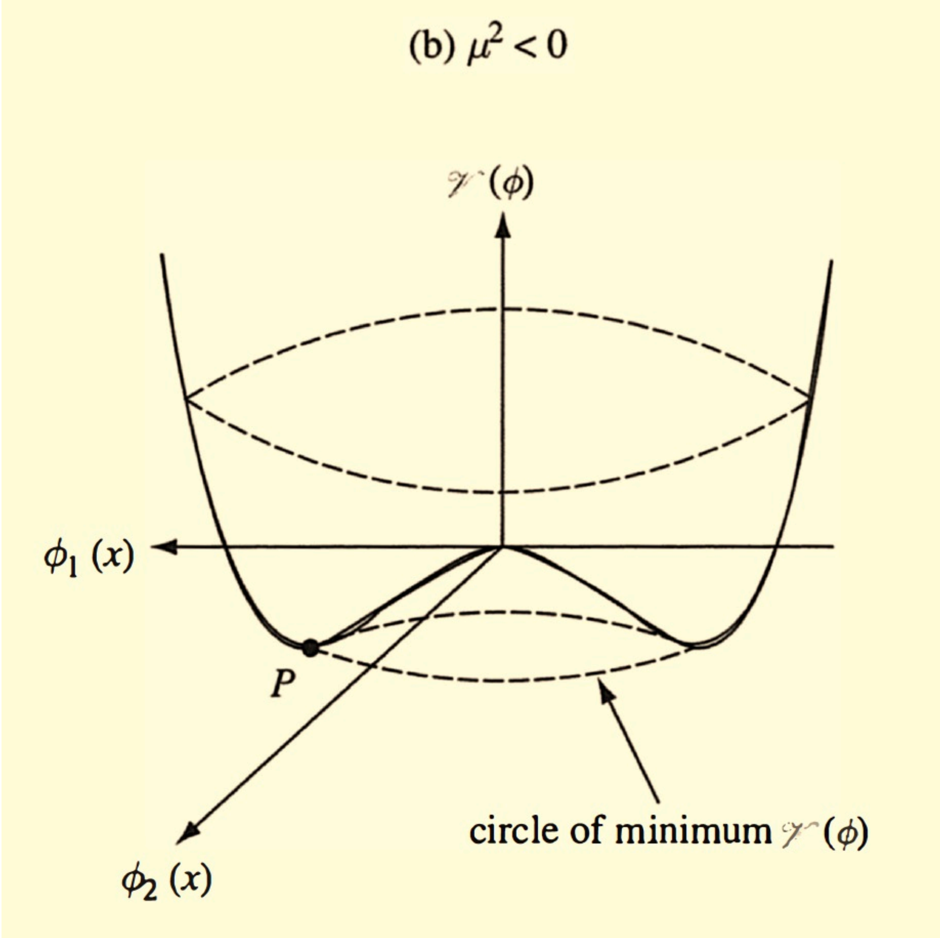



$$\mathscr{V}(\phi) = \mu^2|\phi(x)|^2 + \lambda|\phi(x)|^4$$

The potential has an absolute minimum for the unique value ϕ(x) = 0 ⇒ normal modes of oscillation about the stable equilibrium position.

In quantum field theory the state of lowest energy is the vacuum | 0 >. In this case the ground state is unique and the expectation value of the field ϕ(x) vanishes:

$$\langle 0|\phi(x)|0\rangle = 0$$




Local maximum for ϕ(x) = 0 and minimum for:

$$\phi(x) = \phi_0 = \left(\frac{-\mu^2}{2\lambda}\right)^{1/2} e^{i\theta}, \qquad 0 \leq \theta < 2\pi$$

⇒ the state of lowest energy is not unique.
It is determined by the choice of ϑ, e.g. ϑ=0:

$$\phi_0 = \left(\frac{-\mu^2}{2\lambda}\right)^{1/2} = \frac{1}{\sqrt{2}}v \quad (>0)$$

The symmetry is spontaneously broken because the ground state does not share the symmetry of the Lagrangian and the field ϕ(x) does not vanish in the vacuum state:

$$\langle 0|\phi(x) 0\rangle = \phi_0$$

$\phi_0$ → complex order parameter.

⇒ introduce two real fields σ(x) i η(x): $$\phi(x) = \frac{1}{\sqrt{2}}[v + \sigma(x) + i\eta(x)]$$

In terms of these fields, the Lagrangian density:

$$\begin{aligned}\mathscr{L}(x) = &\frac{1}{2}[\partial^\mu \sigma(x)][\partial_\mu \sigma(x)] - \frac{1}{2}(2\lambda v^2)\sigma^2(x) \\ &+ \frac{1}{2}[\partial^\mu \eta(x)][\partial_\mu \eta(x)] \\ &- \lambda v \sigma(x)[\sigma^2(x) + \eta^2(x)] - \frac{1}{4}\lambda[\sigma^2(x) + \eta^2(x)]^2\end{aligned}$$

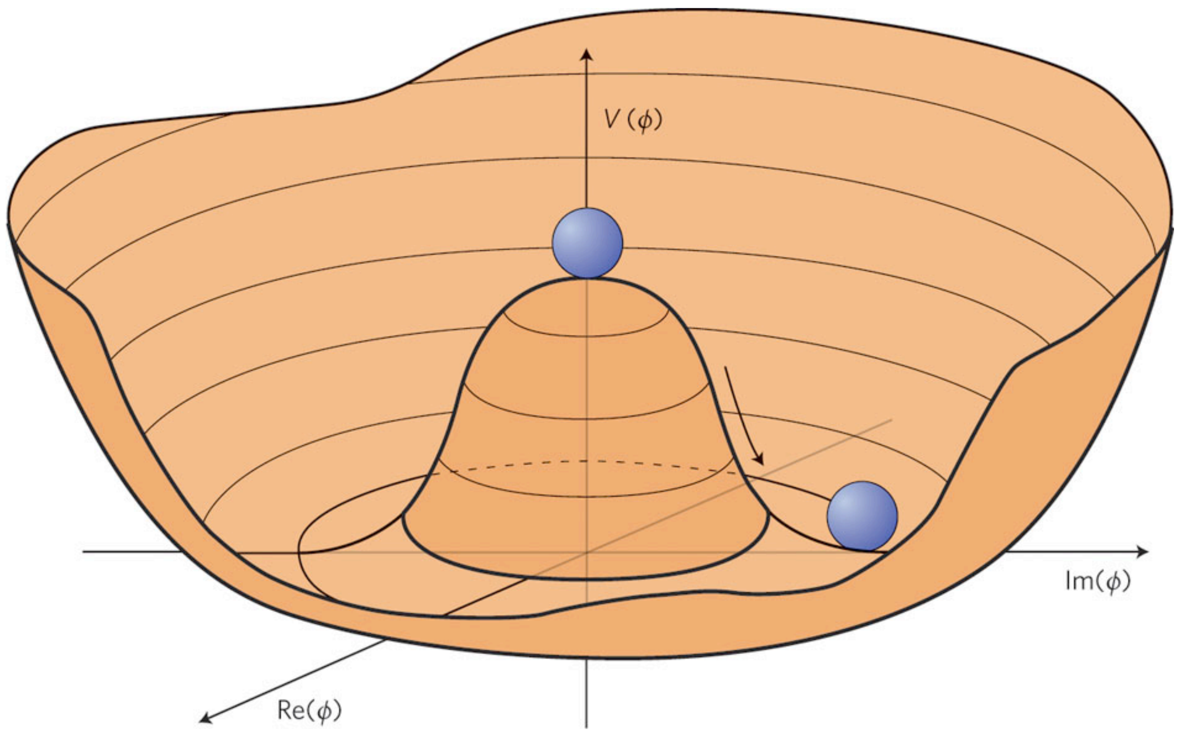
$\mathscr{L}_0$ free Lagrangian

interaction terms

On quantization, both fields lead to neutral spin-0 particles: the σ boson with the (real positive) mass $(2\lambda v^2)^{1/2}$ and the η boson of mass 0.

$\sigma(x)$ → represents a displacement in the radial plane in which the potential energy density increases quadratically with $\sigma$.

$\eta(x)$ → represents a displacement along the valley of minimum potential energy = constant, so that the corresponding quantum excitations: $\eta$ bosons – are massless.




The zero mass of the Goldstone bosons is a consequence of the degeneracy of the vacuum.

# SSB in finite systems with a small number of particles

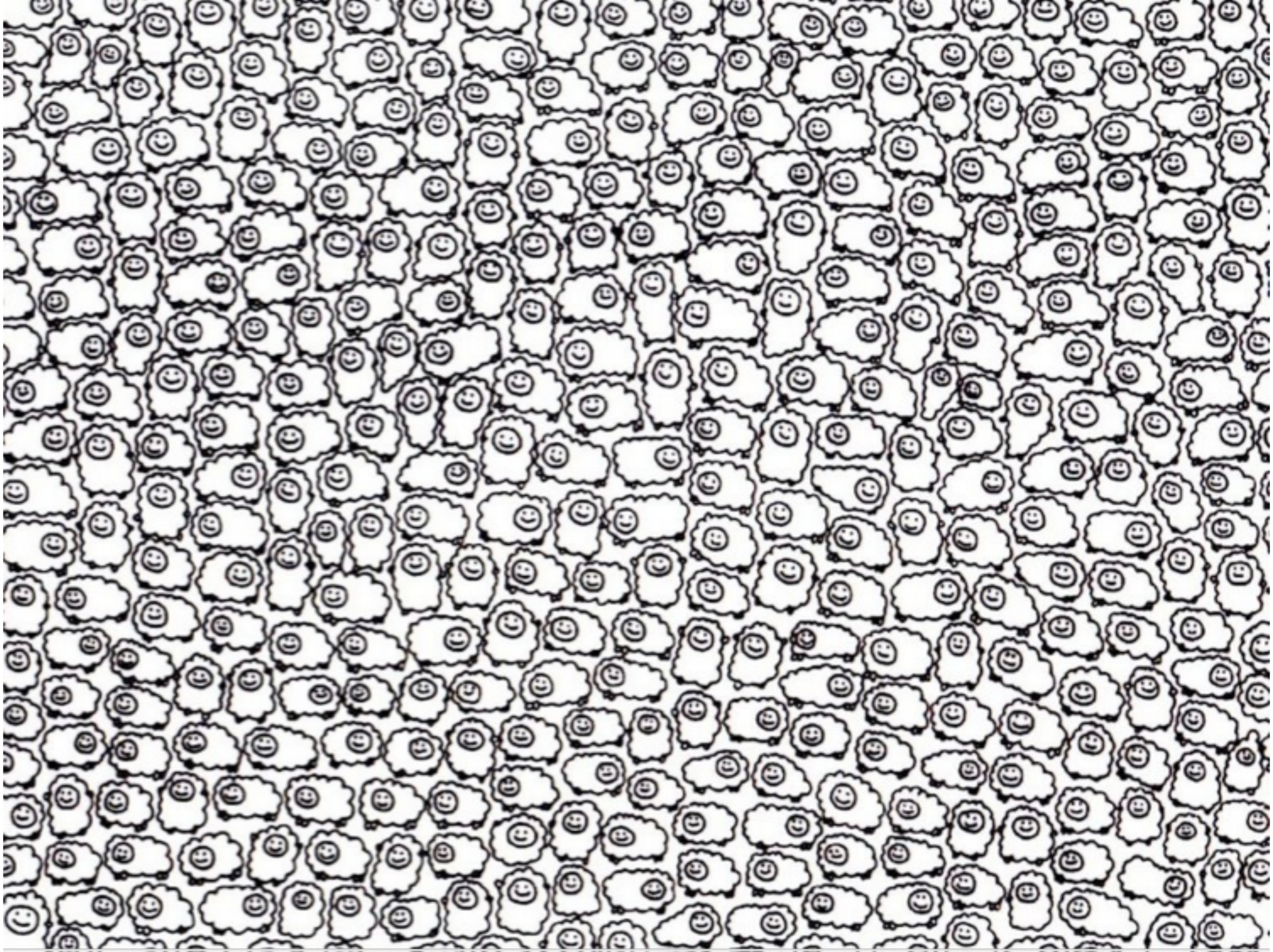
Two-step method:

1) symmetry breaking at the mean-field level. The system lowers the energy by including static correlations. The many-body wave function is a single Slater determinant associated with "central mean-field". Examples for nuclei:

→ translational invariance of the Hamiltonian ⇒ localized solutions
→ rotational invariance ⇒ deformed nucleonic density
→ particle number invariance ⇒ pairing correlations
→ reflection symmetry ⇒ octupole deformation of the nucleonic density

2) Subsequent restoration of the broken symmetry via projection techniques ⇒beyond the mean-field approximation the projected many-body wave function is a linear superposition of Slater determinants and it preserves all the symmetries of the original many-body Hamiltonian ⇒ dynamical correlations additionally lower the energy of the system.

*Important!* The lower symmetry found at the broken symmetry does not disappear, it becomes *intrinsic* or *hidden*.



# Collective correlations

# Restoration of Broken Symmetries

A self-consistent mean-field wave function breaks necessarily several symmetries of the nuclear Hamiltonian (translational, rotational).

**EXAMPLE:** the only translational invariant product wave functions are products of plane waves, but they cannot be used to describe strong correlations between nucleons and their clustering into a finite nucleus.

## Symmetry of a Hamiltonian and Broken Symmetry

→ symmetry of the Hamiltonian of the system:

$$U H U^{+} = H \qquad\qquad [H, U] = 0$$

-unitary transformation: preserves the norm of state vectors and the matrix elements of observables.

-symmetry group of H: the eigenvectors of H are classified according to the irreducible representation of the symmetry group.

State of broken symmetry (deformed state): cannot be classified according to an irreducible representation of the symmetry group of the Hamiltonian H.

$U(\alpha)$ → set of unitary operators (representing the symmetry group of the Hamiltonan). The parameter α can be discrete or continuous.

$$|\Phi\alpha\rangle = U(\alpha)|\Phi\rangle$$

$$\begin{aligned}\langle\Phi\alpha|H|\Phi\alpha\rangle &= \langle\Phi|U^{+}(\alpha)HU(\alpha)|\Phi\rangle \\ &= \langle\Phi|H|\Phi\rangle \qquad \forall\alpha\end{aligned}$$

⇒ all deformed states $|\Phi\,\alpha\rangle$ are DEGENERATE.

# Symmetries of the Hartree-Fock field

$|\Phi>$ → independent-particle state with the associated single-particle density ρ

$$\rho_{ij} = \langle\Phi|a_j^+ a_i|\Phi\rangle \equiv \langle\Phi|\rho|\Phi\rangle$$

→ consider a unitary transformation: $$|\overline{\Phi}\rangle = U|\Phi\rangle$$

⇒ $$\bar{\rho}_{ij} \equiv \langle\overline{\Phi}|a_j^+ a_i|\overline{\Phi}\rangle$$

→ with: $U^+ a_j^+ U = \sum_k U_{jk}^* a_k^+$ $\qquad$ $U^+ a_i U = \sum_k U_{ik} a_k$

⇒ $$\bar{\rho} = U\rho U^+$$

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→ in the HF approximation: $E[\rho] = \langle\Phi|H|\Phi\rangle$

→ if $\quad UHU^+ = H$ ⇒ $$E[\bar{\rho}] = E[\rho]$$

$$E[\bar{\rho}] = E[\rho] \quad \Rightarrow \quad h[\bar{\rho}] = U h[\rho] U^{+}$$

transformation of the Hartree-Fock hamiltonian

1) the density matrix ρ is invariant under the transformation U:

$$\bar{\rho} = \rho \quad \Longrightarrow \quad h[\bar{\rho}] = h[\rho]$$

U represents a ***self-consistent symmetry*** of the HF hamiltonian.

2) $\bar{\rho} \neq \rho \qquad [h[\rho], U] \neq 0$

U represents a ***broken symmetry*** of the HF hamiltonian.

***Example:*** translational symmetry is always broken by the HF potential of a bound finite system.

# Symmetries in the presence of pairing fields

In the Hartree-Fock-Bogoliubov (HFB) approximation the quasiparticle vacuum is characterized by the generalized density matrix:

$$\mathcal{R} = \begin{pmatrix} \rho & \kappa \\ -\kappa^* & 1\!\!1 - \rho^* \end{pmatrix}$$

→ unitary transformation:

$$|\overline{\Phi}\rangle = U|\Phi\rangle$$

$$\begin{aligned} \bar{\rho}_{ij} &= \langle\overline{\Phi}|a_j^+ a_i|\overline{\Phi}\rangle = (U\rho U^+)_{ij} \\ \bar{\kappa}_{ij} &= \langle\overline{\Phi}|a_j a_i|\overline{\Phi}\rangle = (U\kappa\tilde{U})_{ij} \end{aligned}$$

$$\bar{\mathcal{R}} = \mathcal{U}\mathcal{R}\mathcal{U}^+ \qquad \mathcal{U} = \begin{pmatrix} U & 0 \\ 0 & U^+ \end{pmatrix}$$

If U represents a symmetry of the Hamiltonian H ⇒ $E[\bar{\mathcal{R}}] = E[\mathcal{R}]$

⇒ $\mathcal{H}[\bar{\mathcal{R}}] = \mathcal{U}\mathcal{H}[\mathcal{R}]\mathcal{U}^{+}$ transformation of the HFB Hamiltonian

1) self-consistent symmetry of the HFB Hamiltonian

$$[\mathcal{R}, \mathcal{U}] = 0 \qquad \bar{\mathcal{R}} = \mathcal{R} \qquad \mathcal{U}\mathcal{H}\mathcal{U}^{+} = \mathcal{H}$$

2) broken symmetry

$$[\mathcal{R}, \mathcal{U}] \neq 0 \qquad \bar{\mathcal{R}} \neq \mathcal{R} \qquad \mathcal{U}\mathcal{H}\mathcal{U}^{+} \neq \mathcal{H}$$

The pairing field breaks the invariance with respect to the transformation induced by the operator:

$$U = e^{iN\phi} \qquad N = \sum_i a_i^{+} a_i$$

⇒ $$\bar{\mathcal{R}} = \mathcal{U}\mathcal{R}\mathcal{U}^{+} = \begin{pmatrix} \rho & \kappa e^{2i\phi} \\ -\kappa^{*} e^{-2i\phi} & \mathbb{1} - \rho^{*} \end{pmatrix}$$

# Broken symmetries in finite systems

In ***finite systems*** broken symmetries arise only as a result of approximations (variational principle applied to a restricted set of trial wave functions).

A ***broken symmetry*** implies a ***degeneracy*** of the solutions of variational equations.

$$|\Phi\alpha\rangle \equiv U(\alpha)|\Phi\rangle$$

***SYMMETRY RESTORATION*** → the new wave function is a linear superposition of the degenerate deformed states.

$$|\psi\rangle = \int d\alpha\, f(\alpha)|\Phi\alpha\rangle$$

The ***minimization of the energy*** with respect to the expansion coefficients f(α) is equivalent to the ***projection*** of states of good symmetry from the deformed state |Φ >. The resulting states can be classified according to the ***irreps*** of the symmetry group.

**EXAMPLE:** ***parity*** - discrete broken symmetry

**|Φ >** a normalized state that is not an eigenstate of the **parity operator Π**. **[H,Π]=0** implies that the linearly independent states **|Φ > and Π|Φ > are degenerate.**

→ new trial function: $$|\psi(\lambda)\rangle = |\Phi\rangle + \lambda\Pi|\Phi\rangle$$

→ parameter to be evaluated by minimizing the energy expectation value

$$E(\lambda) = \frac{\langle\Psi(\lambda)|H|\Psi(\lambda)\rangle}{\langle\Psi(\lambda)|\Psi(\lambda)\rangle} = \langle H\rangle \frac{1+\lambda^2+2\lambda\langle H\Pi\rangle/\langle H\rangle}{1+\lambda^2+2\lambda\langle\Pi\rangle}$$

$$\frac{dE(\lambda)}{d\lambda} = 0 \quad \Longrightarrow \quad [\langle H\Pi\rangle - \langle H\rangle\langle\Pi\rangle](1-\lambda^2) = 0$$

If $|\Phi>$ is neither an eigenstate of the Hamiltonian H, nor of the parity operator:

$$\langle H\Pi\rangle \neq \langle H\rangle\langle\Pi\rangle \quad \Longrightarrow \lambda = \pm 1$$

→ parity eigenstates:

$$|\Psi_\pm\rangle = \frac{1}{\sqrt{2}}(1 \pm \Pi)|\Phi\rangle \quad \Longrightarrow \quad \Pi|\Psi_\pm\rangle = \pm|\Psi_\pm\rangle$$

The same states are obtained by simply acting with the projection operators:

$$P_\pm = \frac{1}{\sqrt{2}}(1 \pm \Pi)$$

on the deformed state $|\Phi>$. The degeneracy of the deformed states $|\Phi>$ and $\Pi|\Phi>$ has been removed:

$$E_+ - E_- = 2\frac{\langle H\Pi\rangle - \langle H\rangle\langle\Pi\rangle}{1 - \langle\Pi\rangle^2}$$

# Non-conservation of particle number

**|Φ >** a normalized state, not an eigenstate of the **particle number operator N**.

$$[H,N]=0 \quad \Longrightarrow \quad e^{-i\alpha N}|\Phi\rangle \quad \alpha\in[0,2\pi]$$

**degenerate states!**

→ new function:

$$|\Psi\rangle = \int_0^{2\pi}\frac{d\alpha}{2\pi}f(\alpha)e^{-i\alpha\hat{N}}|\Phi\rangle \equiv \int_0^{2\pi}\frac{d\alpha}{2\pi}f(\alpha)|\Phi\alpha\rangle$$

def. $\hat{N}=N-\bar{n},\qquad |\Phi\alpha\rangle\equiv e^{-i\alpha\hat{N}}|\Phi\rangle$

The projection on states with good particle number is equivalent to the requirement that the energy:

$$E = \frac{\langle\Psi|H|\Psi\rangle}{\langle\Psi|\Psi\rangle}$$

is stationary with respect to variations of ***f*(α)*** and ***f(α)***.

$$= \frac{\int_0^{2\pi}d\alpha\int_0^{2\pi}d\alpha' f^*(\alpha)\langle\Phi\alpha|H|\Phi\alpha'\rangle f(\alpha')}{\int_0^{2\pi}d\alpha\int_0^{2\pi}d\alpha' f^*(\alpha)\langle\Phi\alpha|\Phi\alpha'\rangle f(\alpha')}$$

$$\int_0^{2\pi} \frac{d\alpha'}{2\pi} \langle \Phi | e^{i\hat{N}(\alpha-\alpha')}(H-E)|\Phi\rangle f(\alpha') = 0$$

**Hill-Wheeler equation**

the solutions are eigenstates of the particle number operator!

Fourier transform:

$$f(\alpha) = \sum_{n=0}^{\infty} f_n e^{i(n-\bar{n})\alpha} \quad \Longrightarrow \quad f_n = \int_0^{2\pi} \frac{d\alpha}{2\pi} e^{-i(n-\bar{n})\alpha} f(\alpha)$$

HW equation:

$$\sum_{n=0}^{\infty} f_n \langle \Phi | (H-E) P_n | \Phi \rangle e^{i(n-\bar{n})\alpha} = 0$$

where:

$$P_n \equiv \int_0^{2\pi} \frac{d\alpha}{2\pi} e^{-i(N-n)\alpha}$$

→ operator projecting onto states with particle number n.

The Hill-Wheeler equation is valid for all angles α:

$$f_n \langle \Phi | (H - E) P_n | \Phi \rangle = 0$$

nonvanishing coefficients exist only if the energy E equals:

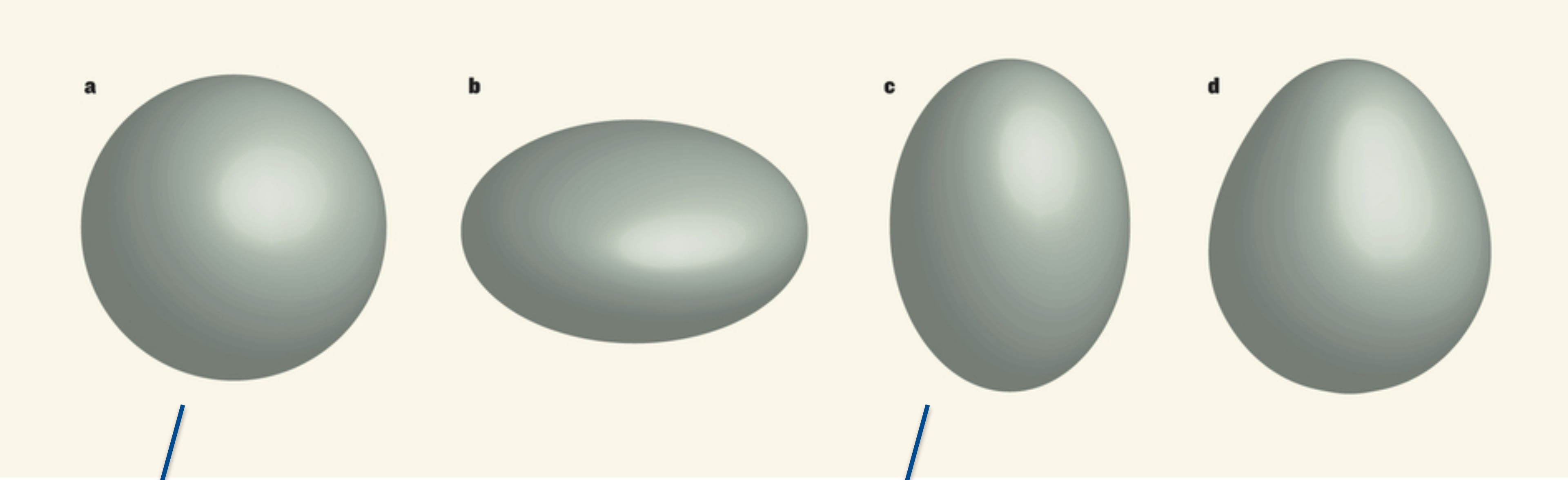
$$E_n = \frac{\langle \Phi | H P_n | \Phi \rangle}{\langle \Phi | P_n | \Psi \rangle}$$

The solution of the HW equation is the projected state:

$$|\Psi\rangle = f_n |\Psi_n\rangle, \qquad |\Psi_n\rangle \equiv P_n |\Phi\rangle$$

⇒ $f_n$ is a normalization constant.

# Nuclear deformation




$$^{20}\mathrm{O} \Longrightarrow \nu(1d5/2)^4 \qquad\qquad ^{20}\mathrm{Ne} \Longrightarrow \nu(1d5/2)^2\ \pi(1d5/2)^2$$

Deformation-driving part of the effective interaction → T=0 $Q_p \cdot Q_n$ force

→ deformation results from the coupling of nuclear surface oscillations to the motion of individual (valence) nucleons. The particle-vibration coupling leads to the *nuclear Jahn-Teller effect* ⇒ the lowest-energy intrinsic state is not an eigenstate characterized by the symmetry group of the total Hamiltonian.

## The nuclear Jahn-Teller effect

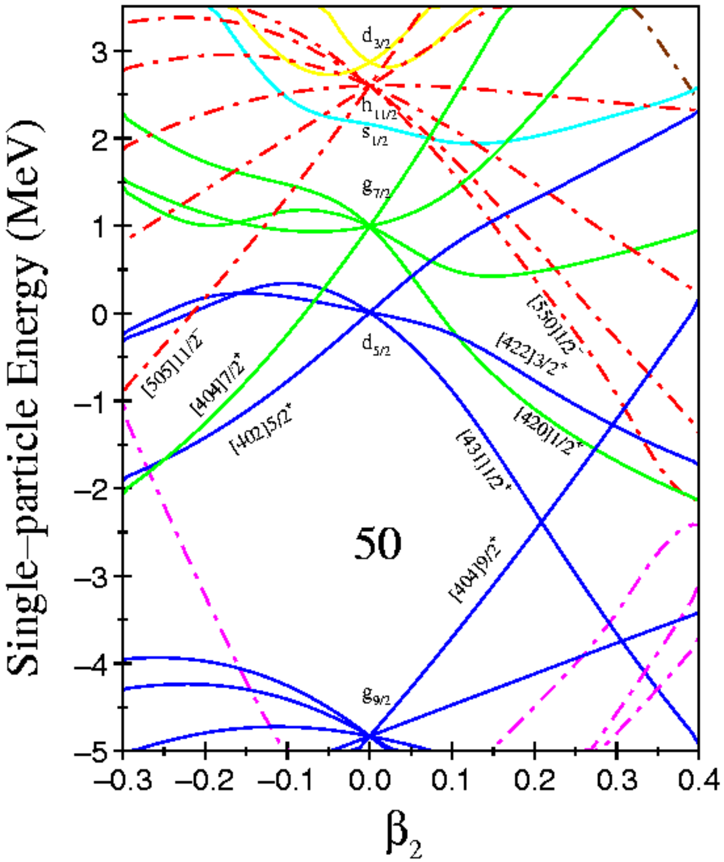
$\{\vec{Q}\} \equiv \{Q_j\}$ *SLOW* (collective) coordinates

$\{\vec{x}\} \equiv \{x_j\}$ *FAST* (noncollective) coordinates

$$H = T_{\vec{Q}} + T_{\vec{x}} + V(\vec{Q}, \vec{x})$$

⇒ at a given point in the collective space one solves the eigenproblem for the non-collective Hamiltonian:

$$[T_{\vec{x}} + V(\vec{Q}, \vec{x})]\psi_n(\vec{x}; \vec{Q}) = E_n(\vec{Q})\psi_n(\vec{x}; \vec{Q})$$




The total wave function:

$$\Psi = \sum_n \psi_n(\vec{x};\vec{Q})\chi_n(\vec{Q})$$

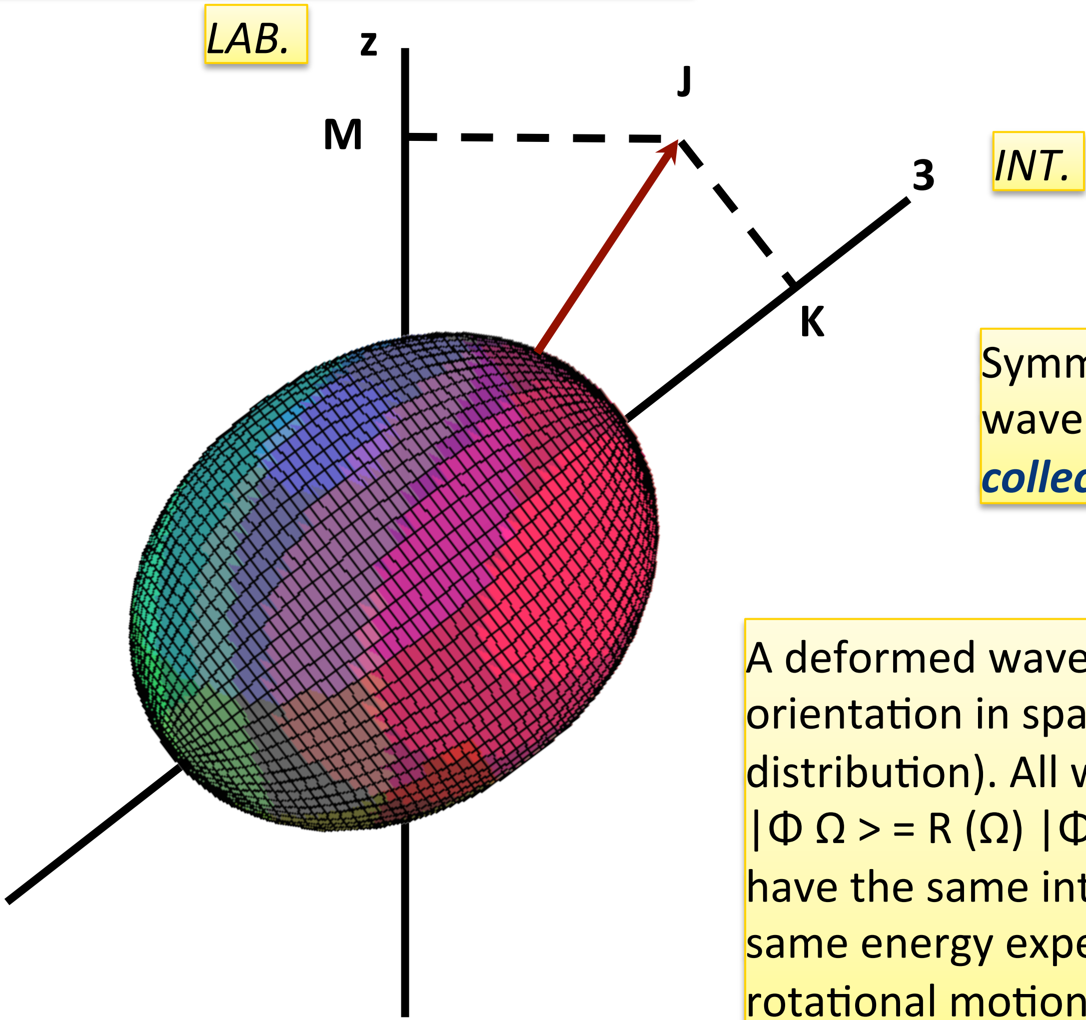
collective wave function that corresponds to the effective potential!

$$\sum_m [\delta_{mn} T_{\vec{Q}} + V_{mn}(\vec{Q})]\chi_m(\vec{Q}) = E_n \chi_n(\vec{Q})$$

The effective collective potential $V_{mn}(\mathbf{Q})$ contains the coupling term between different single-particle states:

$$\sum_m < n|T_{\vec{Q}}|m > \chi_m(\vec{Q})$$

# Angular momentum projection




Symmetry violation in many-particle wave functions can be related to ***collective motion***.

A deformed wave function $|\Phi>$ defines a fixed orientation in space (principal axes of the mass distribution). All wave functions:

$|\Phi\,\Omega> = R(\Omega)\,|\Phi>$

have the same internal structure and yield the same energy expectation value ⇒ collective rotational motion approximately preserves the intrinsic structure.

Angular momentum operators in the laboratory and intrinsic frames

INT. $\hat{e}_a \quad (a = 1, 2, 3): \quad \hat{e}_a \cdot \hat{e}_b = \delta_{ab}, \quad \hat{e}_a \times \hat{e}_b = \epsilon_{abc}\hat{e}_c$

LAB. $\hat{u}_i \quad (i = x, y, z): \quad \hat{e}_a = \tilde{\mathcal{R}}_{ai}(\Omega)\hat{u}_i, \quad \Omega = \{\alpha, \beta, \gamma\}$

Euler angles

The Euler angles are dynamical variables which specify the orientation of the intrinsic frame.

Def. intrinsic angular momentum operators: $I_a = \hat{e}_a \cdot \vec{J}$

$$[I_a, J_i] = 0 \quad \forall a, i$$

$$[I_a, I_b] = -i\epsilon_{abc} I_c$$

$$\vec{I}^{\,2} = \sum_a I_a^2 = \sum_i J_i^2 = \vec{J}^{\,2}$$

$$\vec{I}^{\,2} = \vec{J}^{\,2}, I_3, J_z$$

set of commuting operators,
can be diagonalized simultaneously.

$$\begin{aligned} \vec{I}^{\,2}|IKM\rangle &= I(I+1)|IKM\rangle \\ I_3|IKM\rangle &= K|IKM\rangle \quad -I \le K \le I \\ J_z|IKM\rangle &= M|IKM\rangle \quad -I \le M \le I \end{aligned}$$

$$\sum_{IKM} |IKM\rangle\langle IKM| = 1, \qquad \langle IKM|I'K'M'\rangle = \delta_{II'}\delta_{KK'}\delta_{MM'}$$

The states |IKM > can be represented by the wave functions < Ω|IKM >, which depend on the Euler angles Ω = {α,β,γ}. With the definition of the state |Ω >:

$$|\Omega\rangle = \mathcal{R}(\Omega)|\Omega = 0\rangle$$

$$\begin{aligned} \langle\Omega|IKM\rangle &= \langle\Omega = 0|\mathcal{R}^{+}(\Omega)|IKM\rangle \\ &= \sum_{I'K'M'} \langle\Omega = 0|I'K'M'\rangle\langle I'K'M'|\mathcal{R}^{+}(\Omega)|IKM\rangle \end{aligned}$$

The rotation does not change the intrinsic angular momenta => K=K' and I=I'. If the Euler angles are chosen in such a way that the INT and LAB frames coincide for Ω=0:

$$\langle \Omega = 0 | IKM \rangle = c_I \delta_{KM} \qquad c_I = \sqrt{(2I+1)/8\pi^2}$$

$$\begin{aligned} \langle \Omega | IKM \rangle &= \sqrt{(2I+1)/8\pi^2} D^I_{MK}(\Omega) \\ &= \sqrt{(2I+1)/8\pi^2} e^{i\alpha M} d^I_{MK}(\beta) e^{i\gamma K} \end{aligned}$$

$$\begin{aligned} \langle IKM | I'K'M' \rangle &= \frac{2I+1}{8\pi^2} \int d\Omega D^{I'}_{M'K'}(\Omega) D^{I^*}_{MK}(\Omega) \\ &= \delta_{II'} \delta_{KK'} \delta_{MM'} \end{aligned}$$

# Variational principle and angular momentum projection

Deformed state $|\Phi>$, not an eigenstate of $\vec{J}^{\,2},\ J_3$

$$\mathcal{R}(\alpha,\beta,\gamma) = e^{-i\alpha J_z} e^{-i\beta J_y} e^{-i\gamma J_z}$$

$$[H,\mathcal{R}] = 0 \quad \Longrightarrow \quad |\Phi\Omega\rangle = \mathcal{R}(\Omega)|\Phi\rangle$$ degenerate states

→ new trial function: $|\Psi\rangle = \int d\Omega f(\Omega)|\Phi\Omega\rangle \equiv \int d\Omega f(\Omega)\mathcal{R}(\Omega)|\Phi\rangle$

The weight function f(Ω) is determined by requiring that the energy expectation:

$$E = \frac{\langle\Psi|H|\Psi\rangle}{\langle\Psi|\Psi\rangle}$$ is stationary with respect to variations of f* and f.

Hill-Wheeler equation

$$\int d\Omega' \langle\Phi\Omega|H - E|\Phi\Omega'\rangle f(\Omega') = 0$$

The solutions of the HW equation are eigenstates of the operators $\vec{J}^{\,2}, \; J_3$

$$f(\Omega) = \sum_{IMK} \frac{2I+1}{8\pi^2} f^I_{MK} D^I_{MK}(\Omega) \Rightarrow f^I_{MK} = \int d\Omega f(\Omega) D^{I^*}_{MK}(\Omega)$$

$$|\Psi\rangle = \sum_{IMK} f^I_{MK} P^I_{MK} |\Phi\rangle$$

where:

$$P^I_{MK} = \frac{2I+1}{8\pi^2} \int d\Omega D^I_{MK}(\Omega) \mathcal{R}(\Omega)$$

$$\left(P^I_{MK}\right)^+ = P^I_{KM} \qquad \left(P^I_{MK}\right)^+ P^{I'}_{M'K'} = \delta_{II'} \delta_{MM'} P^I_{KK'}$$

not quite a projector!

with: $[H, P^I_{MK}] = 0$

HW equation:

$$\sum_{K'} \langle \Phi | (H - E) P^I_{KK'} | \Phi \rangle f^I_{MK'} = 0$$

⇒ eigenvalues determined by the equation:

$$\det \left[ \langle \Phi | (H - E) P^I_{KK'} | \Phi \rangle \right] = 0$$

a) the HW equation is equivalent to the diagonalization of the hamiltonian in the basis $P^I_{MK} | \Phi \rangle$
b) H does not connect states with I≠I', and the eigenvalues do not depend on M

⇒ eigenstates:

$$|\Psi_{IM}\rangle = \sum_K f^I_{MK} P^I_{MK} | \Phi \rangle$$

In cases when the wave function |Φ> has axial symmetry ⇒ K=0 and the coefficients are determined by the normalization of |Ψ>. In general the Hamiltonian has to be minimized with respect to the coefficients f.

# Projection before and after variation

How do we determine the deformed (symmetry-violating) intrinsic function $|\Phi>$ ?

**i) Variation before the projection (VBP)**

$|\Phi>$ is determined by the variational principle:

$$\delta \frac{\langle\Phi|H|\Phi\rangle}{\langle\Phi|\Phi\rangle} = 0$$

The deformed solution is a superposition of eigenstates of the corresponding symmetry operator (for example, angular momentum). The wave function:

$$|\Psi_I\rangle = P^I|\Phi\rangle$$

is no longer a product wave function, but a complicated superposition of Slater determinants. It contains many more correlations than the function $|\Phi>$.

This method violates the variational principle, because we do not vary the projected wave function. It does not allow for changes in the self-consistent mean-field for different values of I (within a rotational band).

## ii) Variation after projection (VAP)

$$\Rightarrow \quad \delta \frac{\langle \Psi_I | H | \Psi_I \rangle}{\langle \Phi_I | \Psi_I \rangle} = \delta \frac{\langle \Phi | P^I H P^I | \Phi \rangle}{\langle \Phi | P^I P^I | \Phi \rangle} = 0$$

⇒ minimize the expectation value of the projected energy $P^I H P^I$ within the set of product wave functions |Φ >.

This method corresponds to a double variation, using the ansatz:

$$|\Psi\rangle = \int d\Omega f(\Omega) \mathcal{R}(\Omega) |\Phi\rangle$$

and varying the energy with respect to both the weight function f(Ω) and the generating function |Φ >.

**Much more complicated than VBP!**