# *Phenomenological Effective Potentials*

## *Hands-On Exercises*

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# Hartree-Fock Approximation in Various Representations

Ex.1 We write a generic two-body Hamiltonian in configuration space

$$\hat{H} = \sum_{ab} t_{ab} c_a^\dagger c_b + \frac{1}{4} \sum_{abcd} \bar{v}_{abcd} c_a^\dagger c_b^\dagger c_d c_c, \tag{1}$$

where $t_{ab}$ are the matrix elements of the kinetic energy operator, and $\bar{v}_{abcd}$ the non-antisymmetrized matrix elements of some non-local two-body potential $\hat{V}$. Express this Hamiltonian using field operators $c_x^\dagger$, $c_x$, with $x \equiv (\boldsymbol{r}, \sigma, \tau)$.

Ex.2 Assume the potential $\hat{V}$ is local. Starting from the expression of $\hat{H}$ with field operators obtained previously, calculate the HF energy as function of the density matrix (in coordinate space).

Ex.3 Introduce the spin-isospin expansion of the one-body density matrix $\rho(x, x')$ and of the two-body potential $\hat{V}$, and use them to rewrite the HF energy.

# Proton and Neutrons

Ex.4 Recall that the one-body density matrix is written

$$\rho_{ij} = \frac{\langle \Phi | c_j^\dagger c_i | \Phi \rangle}{\langle \Phi | \Phi \rangle}, \quad \rho(x, x') = \frac{\langle \Phi | c(x') c(x) | \Phi \rangle}{\langle \Phi | \Phi \rangle} \tag{2}$$

What can we say of the isospin of states $i$ and $j$ (and $x$ and $x'$) if there is no neutron-proton mixing?

Simplify the expression of the spin-isospin expansion of the one-body density matrix $\rho(x, x')$ in such a case by introducing proton, neutron and total densities.

Ex.5 Also simplify the expression of the HF energy in the case of no proton-neutron mixing assuming a local, finite-range interaction.

# Pairing

Ex.6 From the generic expression of the pairing field in configuration space

$$\Delta_{ab} = \frac{1}{2} \sum_{abcd} \bar{v}_{abcd} \kappa_{cd}, \tag{3}$$

derive the pairing field in coordinate space and show that it is (i) non-local, (ii) of ”pure exchange” character.

Ex.7 We consider the following pairing force

$$\begin{aligned} \hat{V}(x_1, x_2, x_1', x_2') &= \delta(x_1' - x_1)\delta(x_2' - x_2)\delta_{\tau_1\tau_2} \\ &\times V_{\tau_1}^0 \left[ 1 - \gamma \frac{\rho(\boldsymbol{R})}{\rho_0} \right] \delta(\boldsymbol{r}_1 - \boldsymbol{r}_2), \quad \boldsymbol{R} = \frac{\boldsymbol{r}_1 + \boldsymbol{r}_2}{2} \end{aligned} \tag{4}$$

Calculate the pairing energy.

Ex.8 Consider homogeneous nuclear matter (NM) at density $\rho$. At the HF approximation (independent particles), NM is an ideal Fermi gas characterized by the Fermi momentum $k_F$ ($\rho = \frac{2}{3\pi^2}k_F^3$), and the HF wave functions are $\phi_{\boldsymbol{k}}(\boldsymbol{r}, \sigma) = \frac{1}{(2\pi)^3}e^{i\boldsymbol{k}\cdot\boldsymbol{r}}\chi_\sigma$

Calculate the binding energy per nucleon in nuclear matter for the Skyrme pseudopotential (at the HF approximation).

Ex.9 Derive the contribution to the energy density coming from the $t_2$ term,

$$\hat{V}_2(x_1, x_2) = t_2\left(1 + x_2\hat{P}_\sigma\right)\hat{\boldsymbol{k}}' \cdot \delta(\boldsymbol{r}_1 - \boldsymbol{r}_2)\hat{\boldsymbol{k}} \quad (5)$$

with $\hat{\boldsymbol{k}} = (\boldsymbol{\nabla}_1 - \boldsymbol{\nabla}_2)/2i$; $\hat{\boldsymbol{k}}'$ has a similar expression but acts on the left. [Hints: Use the relation

$$\boldsymbol{\nabla}^2\rho(\boldsymbol{r}) = 2\sum_{ac}\rho_{ac}\phi_a^*\boldsymbol{\nabla}^2\phi_c + 2\tau(\boldsymbol{r}) \quad (6)$$