Quantum Gravity in the Lab

Matrix Quantum Mechanics meets Quantum Computing

Enrico Rinaldi
University of Michigan + (Quantum Computing + Theoretical Quantum Physics Lab. + iTHEMS) @ RIKEN
2021-10-26 NSCL/FRIB Theory Seminar

[My personal website]

Short self-intro

who am I?

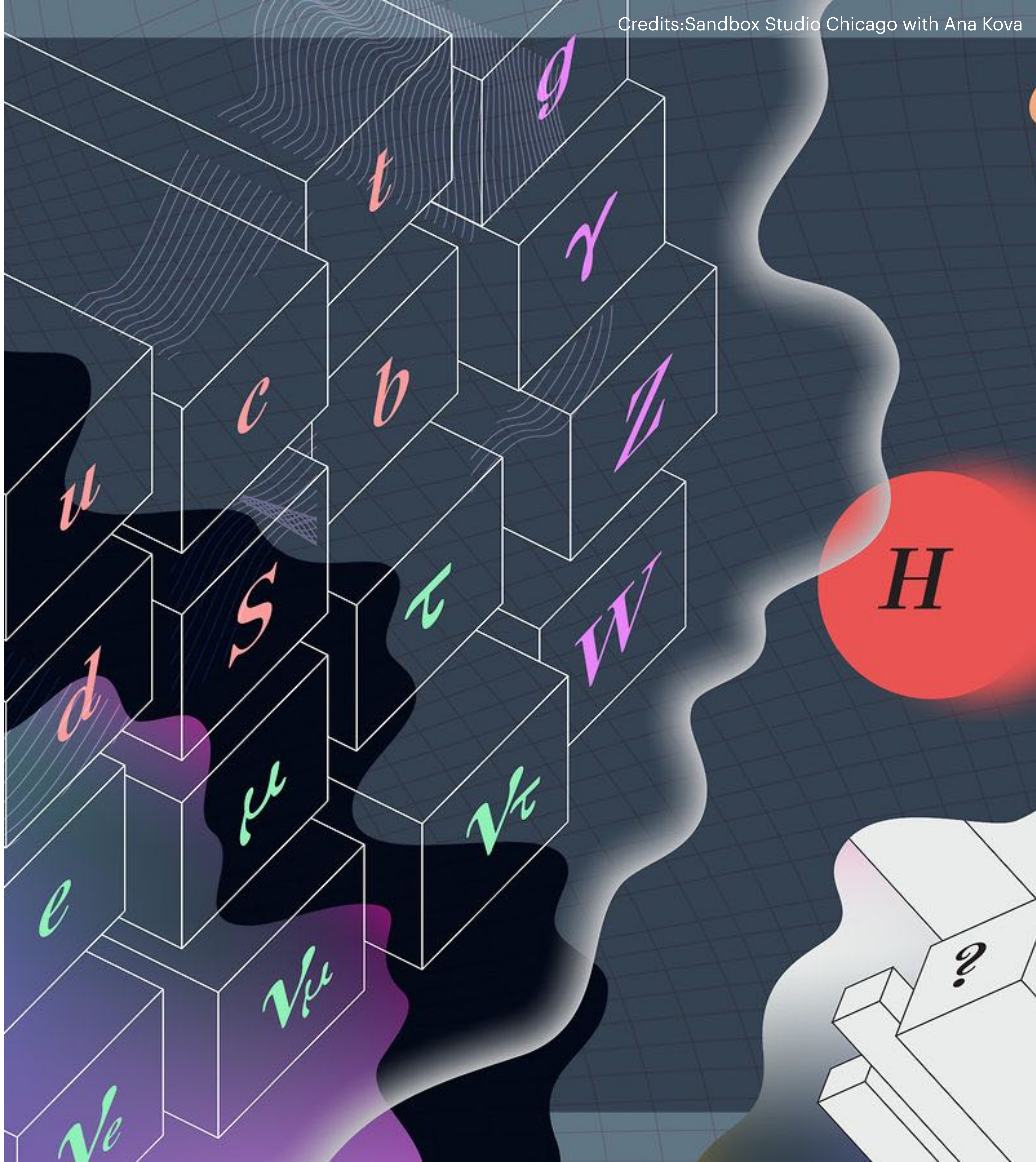
- I am a computational physicist
- Worked on simulations for particle physics and dark matter models using TOP500 HPC systems
- *"Interdisciplinary science is all you need"*©
- Currently in Tokyo @ RIKEN Quantum Computing Center
- Previously @ AI startup in Tokyo (better view from the office 😅)

Understanding Gravity

- Einstein's General Relativity is at the heart of GPS technology
- In 2017 LIGO won the Nobel Prize for the detection of gravitational waves from black hole mergers
- In 2018 the Event Horizon Telescope produced the first "image" of the supermassive black hole in the Milky Way

Quantum Field Theory

- The **Standard Model of particle physics** is our most precise description of the subatomic world
- It is a Quantum Field Theory, a very complicated many-body quantum system obeying the rules of quantum mechanics



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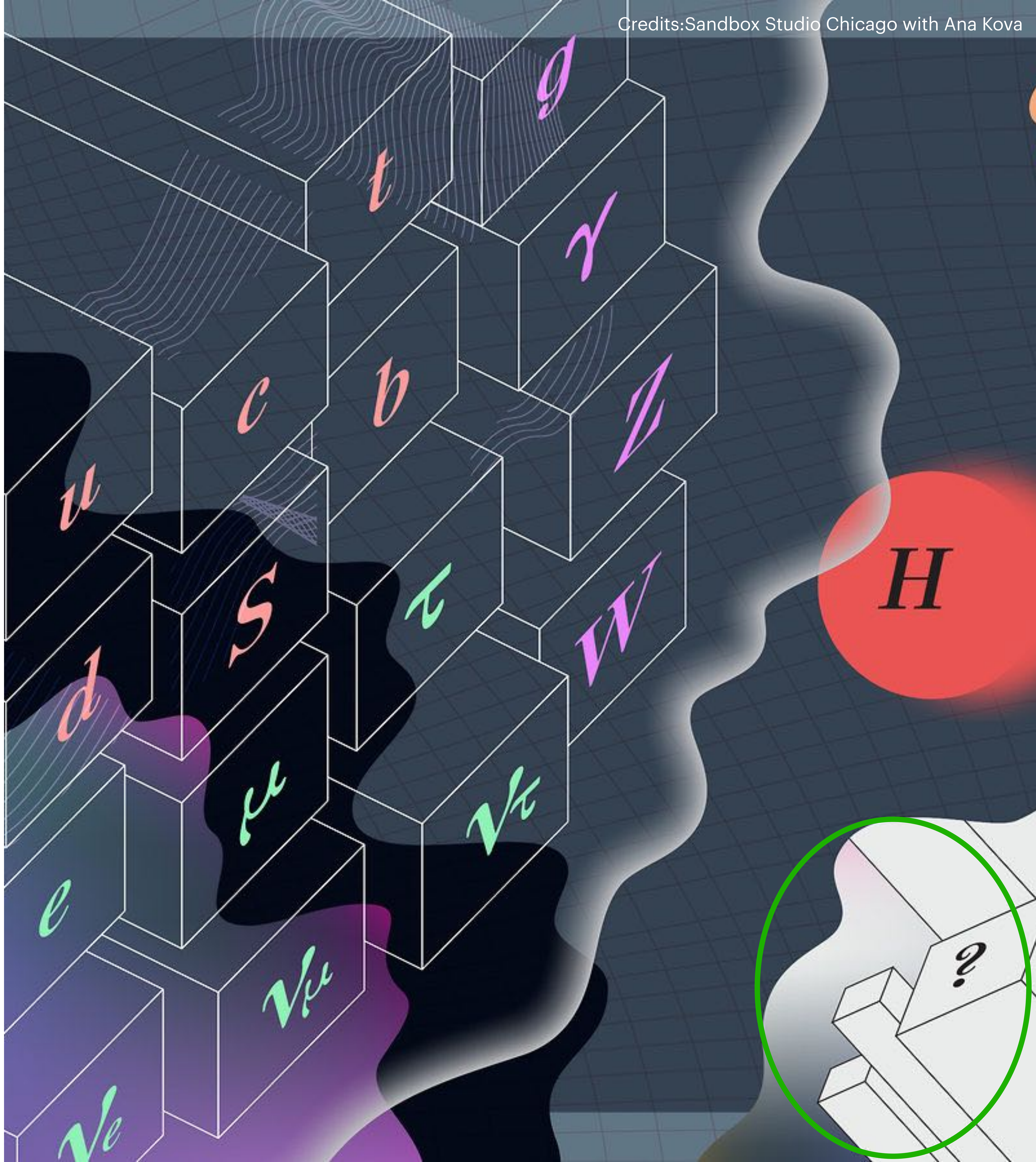

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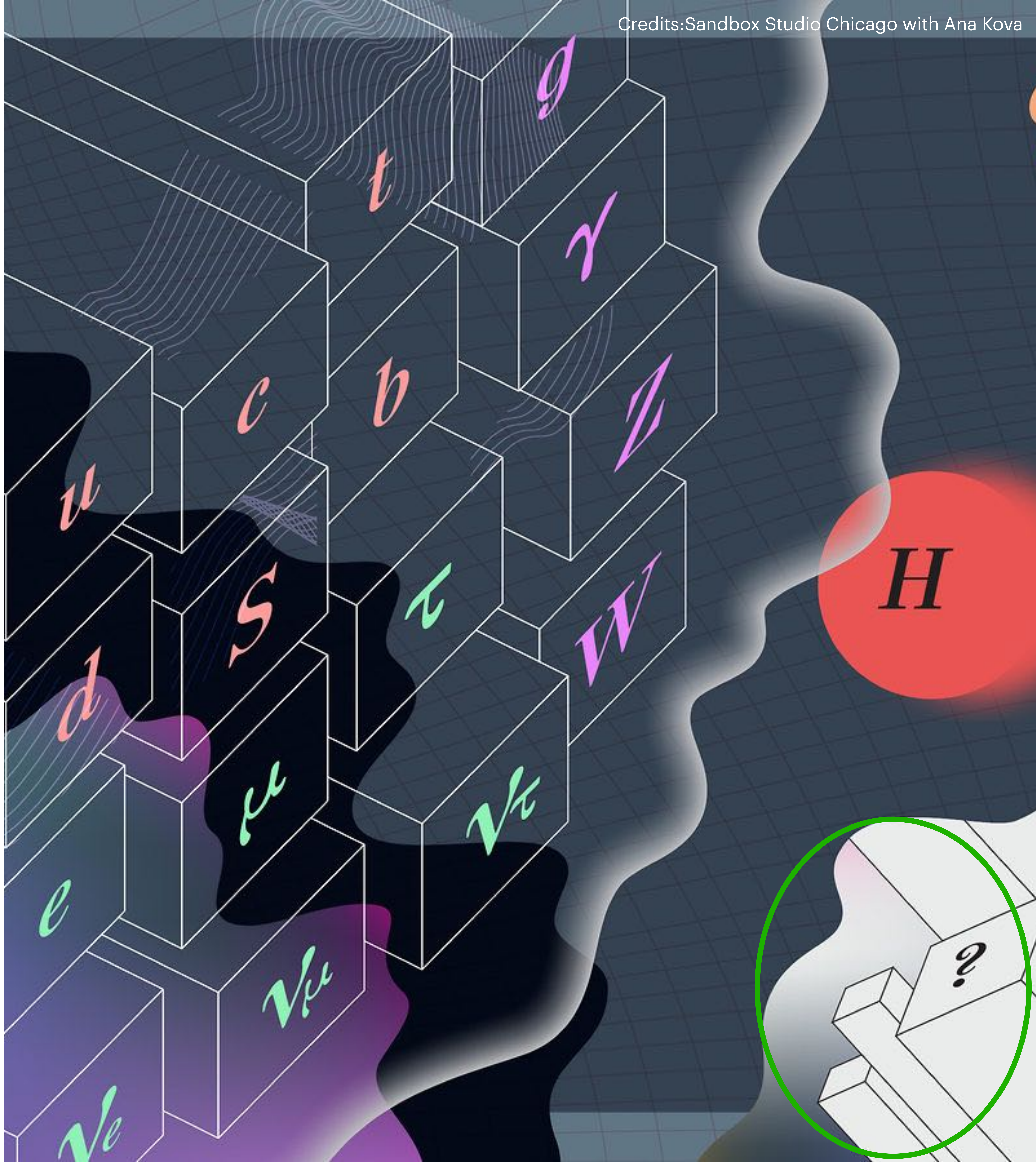
  - What is the quantum theory for gravity?




Credits: Sandbox Studio, Chicago with Ana Kova
quantum mechanical process
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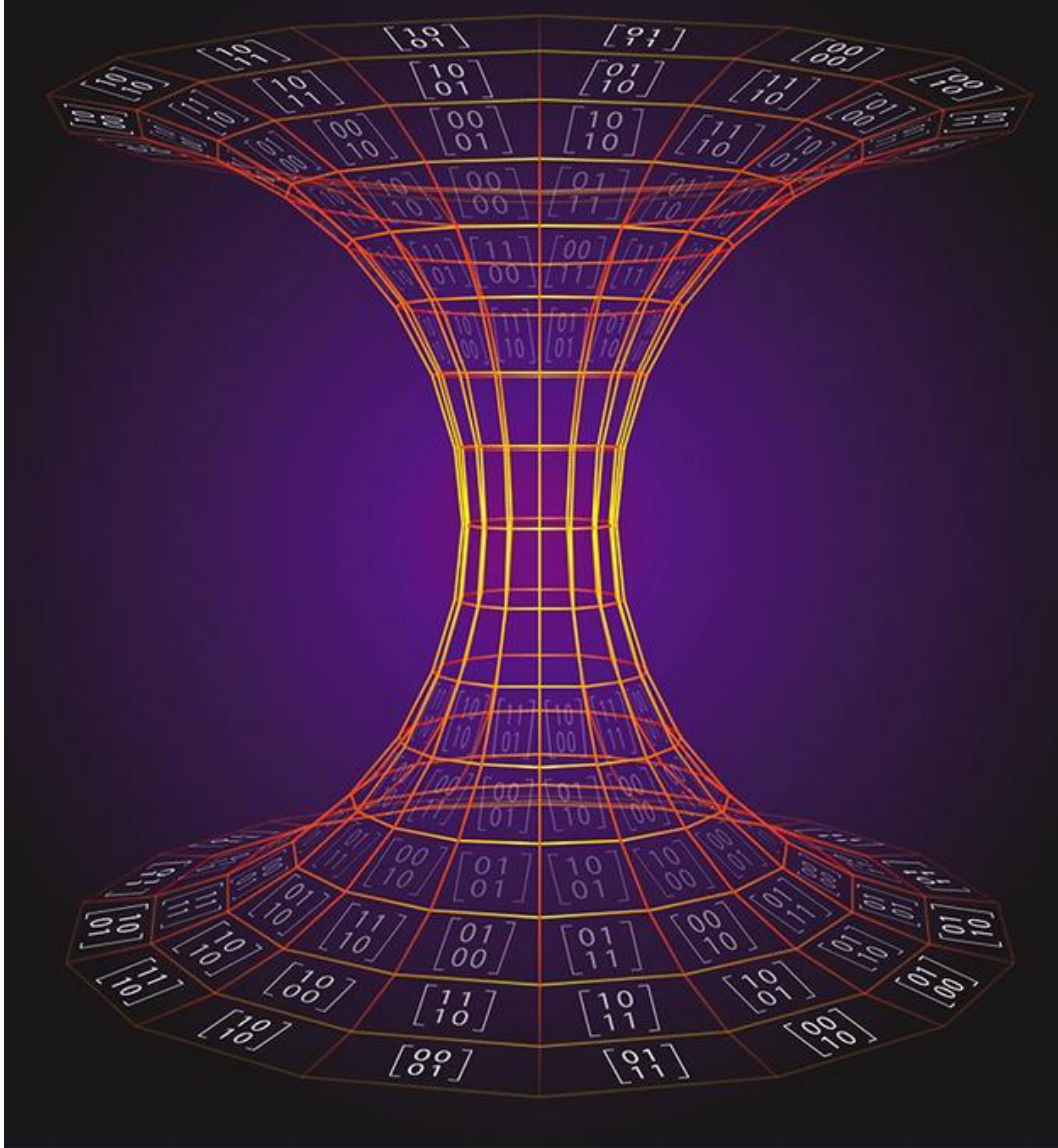
[Rinaldi et al., arxiv:2108.02942]

Outline

Matrix Models via QC, DL, and MC



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Matrix Models via QC, DL, and MC

- Introduction to Matrix Models

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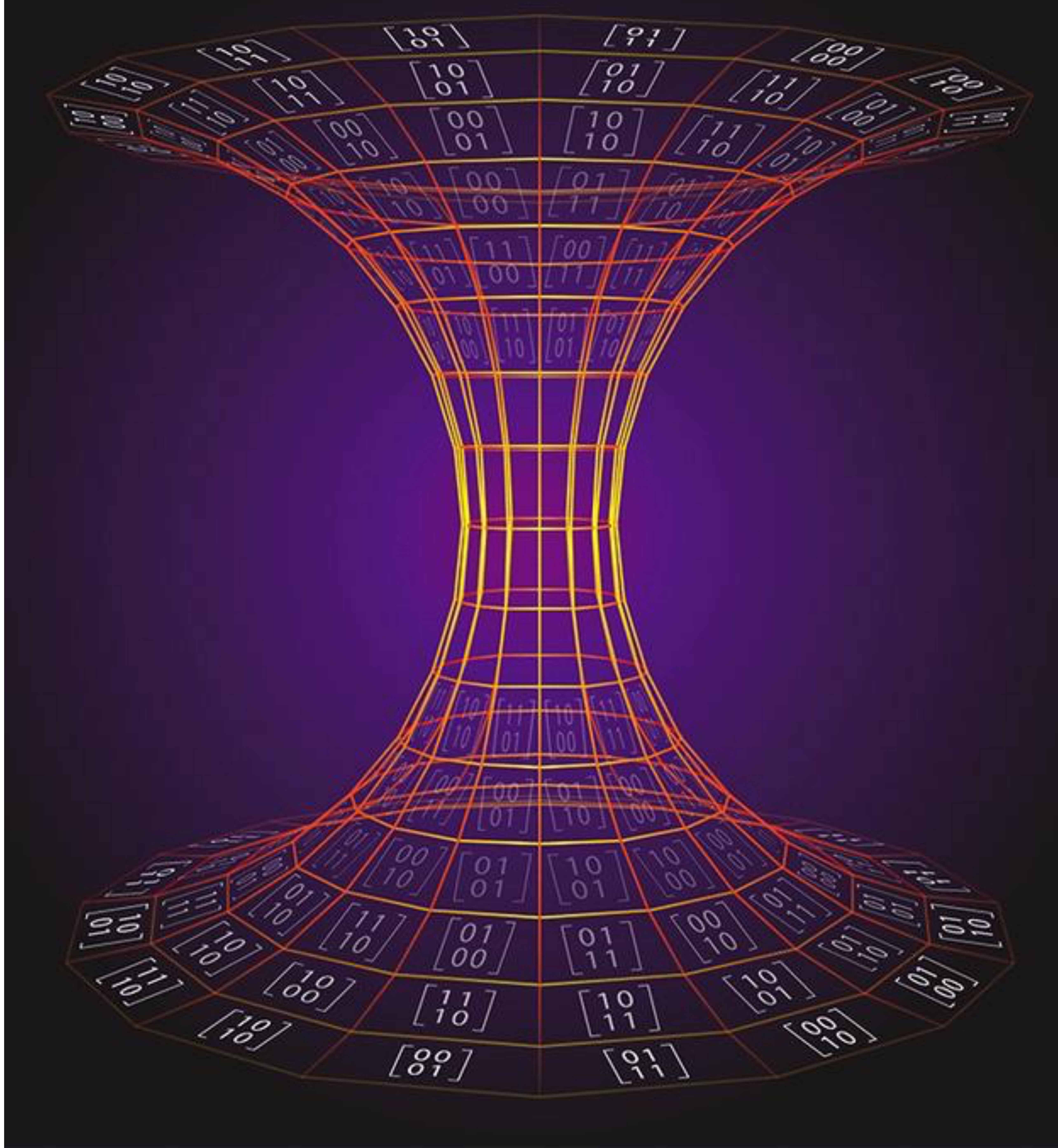

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Matrix Models via QC, DL, and MC

- Introduction to Matrix Models
- **Numerical techniques for matrix quantum mechanics**:
  - Truncated Hamiltonian
  - Quantum Computing
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  - Path integral Monte Carlo

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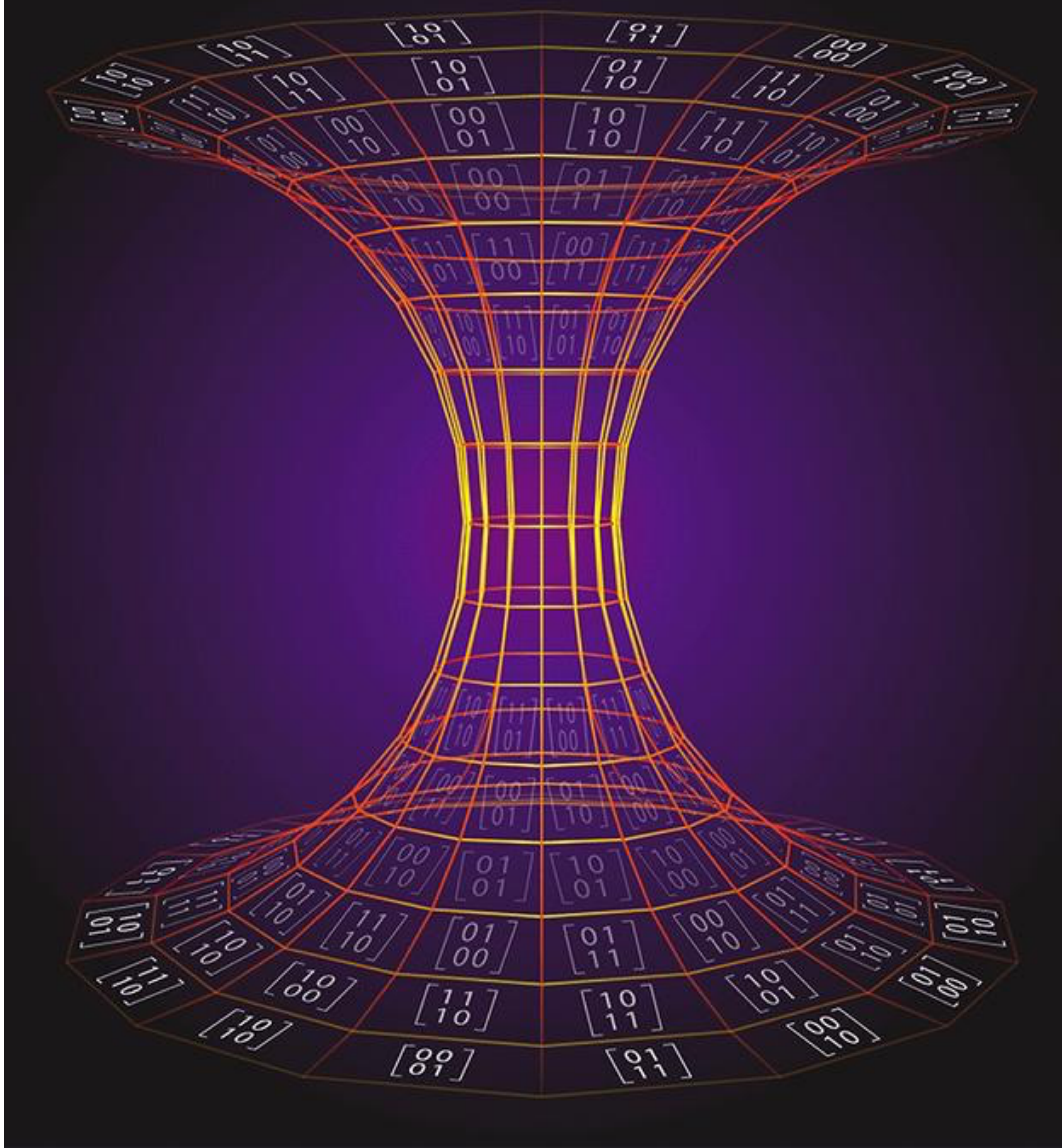

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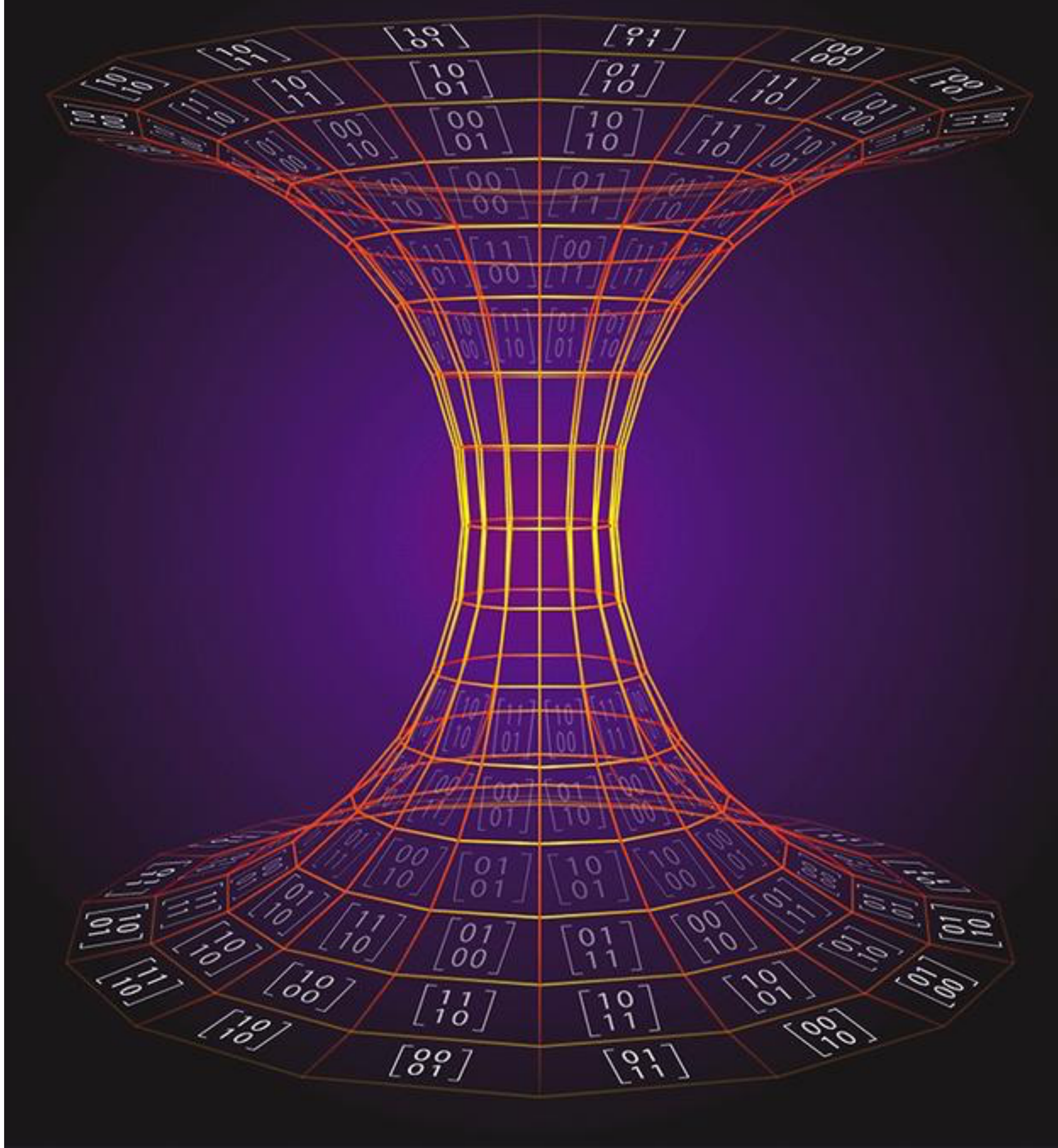
- Conclusions and challenges



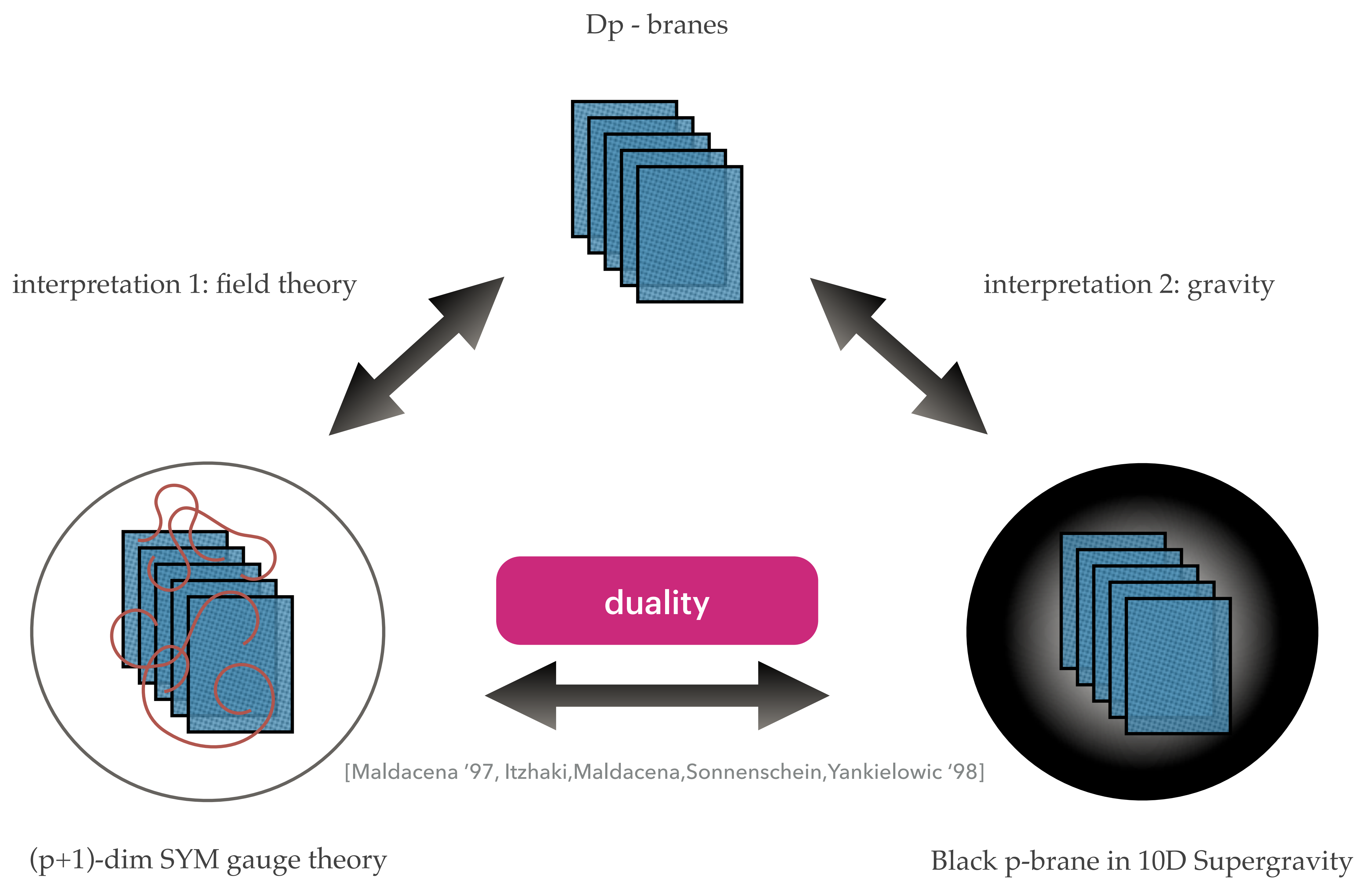
[Polchinski, arxiv:1010.6134]

Matrix Quantum Mechanics

Motivations

★Holographic duality → a quantum field theory **"is"** a gravitational theory

- D0-branes and open strings ⇔ Black hole in Type IIA superstring



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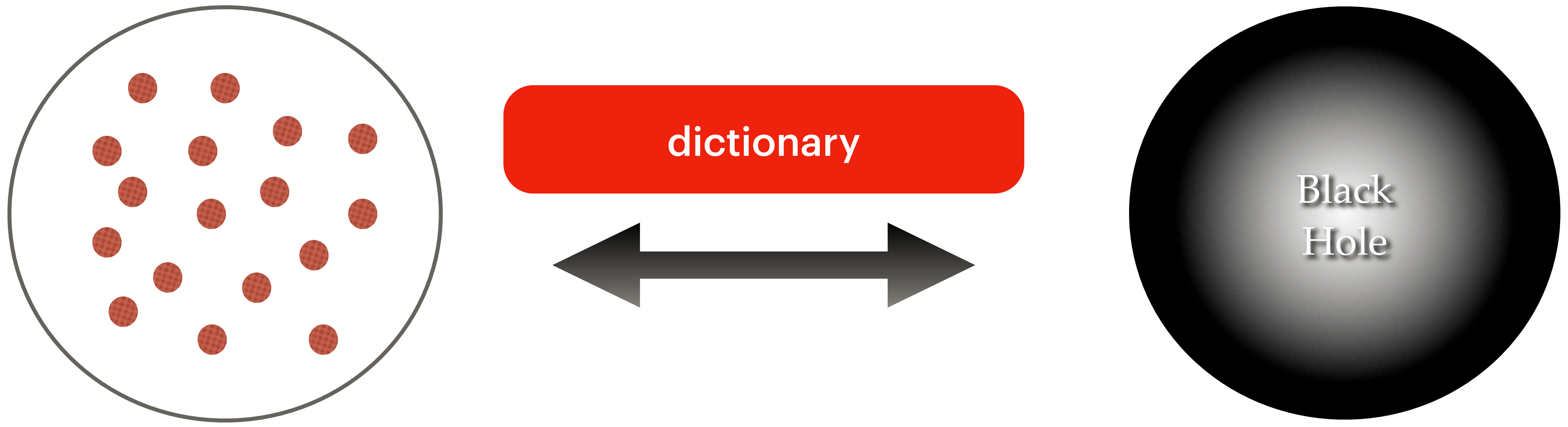
★Gauge/gravity duality → use QFT to study QG (*i.e.* emergent geometry)

- Supersymmetric QFT can be dimensionally reduced to matrix QM



(0+1)-dim maximally supersymmetric gauge theory

Type IIA superstring theory
in the near horizon limit of
a black 0-brane geometry

[BFSS, hep-th/9610043]

Matrix Quantum Mechanics

Interpretation

$$L = \frac{1}{2g_{YM}^2} \mathrm{Tr}\left\{ (D_t X_M)^2 + [X_M, X_{M'}]^2 + i\bar{\psi}^\alpha D_t \psi^\beta + \bar{\psi}^\alpha \gamma^M_{\alpha\beta} [X_M, \psi^\beta] \right\}$$

obtained from $\mathcal{N}$=1 U(N) SYM in (9+1)d via dimensional reduction to (0+1)d
or equivalently from $\mathcal{N}$=4 U(N) SYM in (3+1)d: it is maximally supersymmetric

$$S = \int_0^{1/T} dt\, L$$

$\lambda = g_{\mathrm{YM}}^2 N$ 't Hooft coupling

$X_M\,, M = 1, \ldots, 9\ (N \times N) \rightarrow$ hermitian scalars

$\psi^\alpha\,, \alpha = 1, \ldots, 16\ (N \times N) \rightarrow$ adjoint fermions

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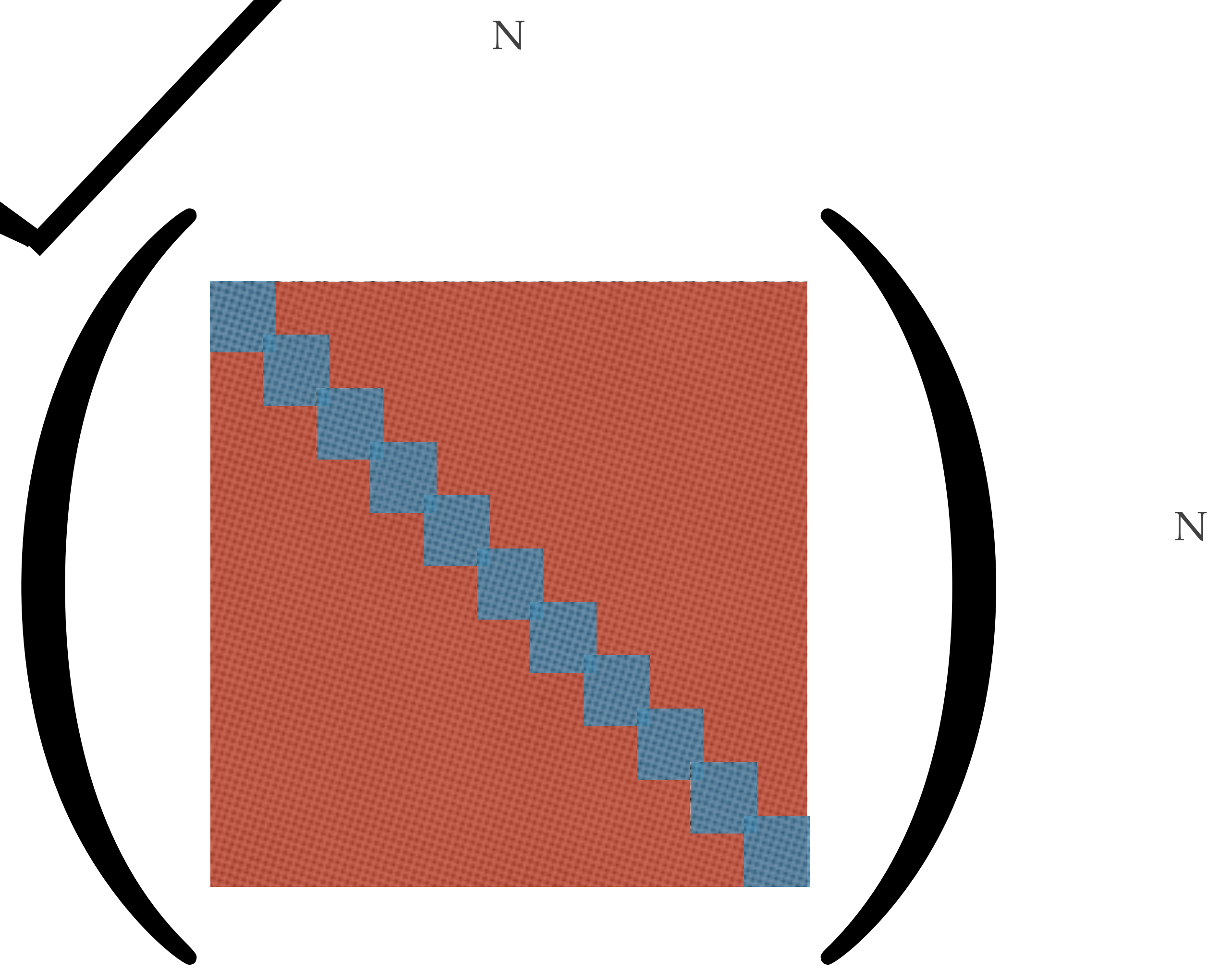

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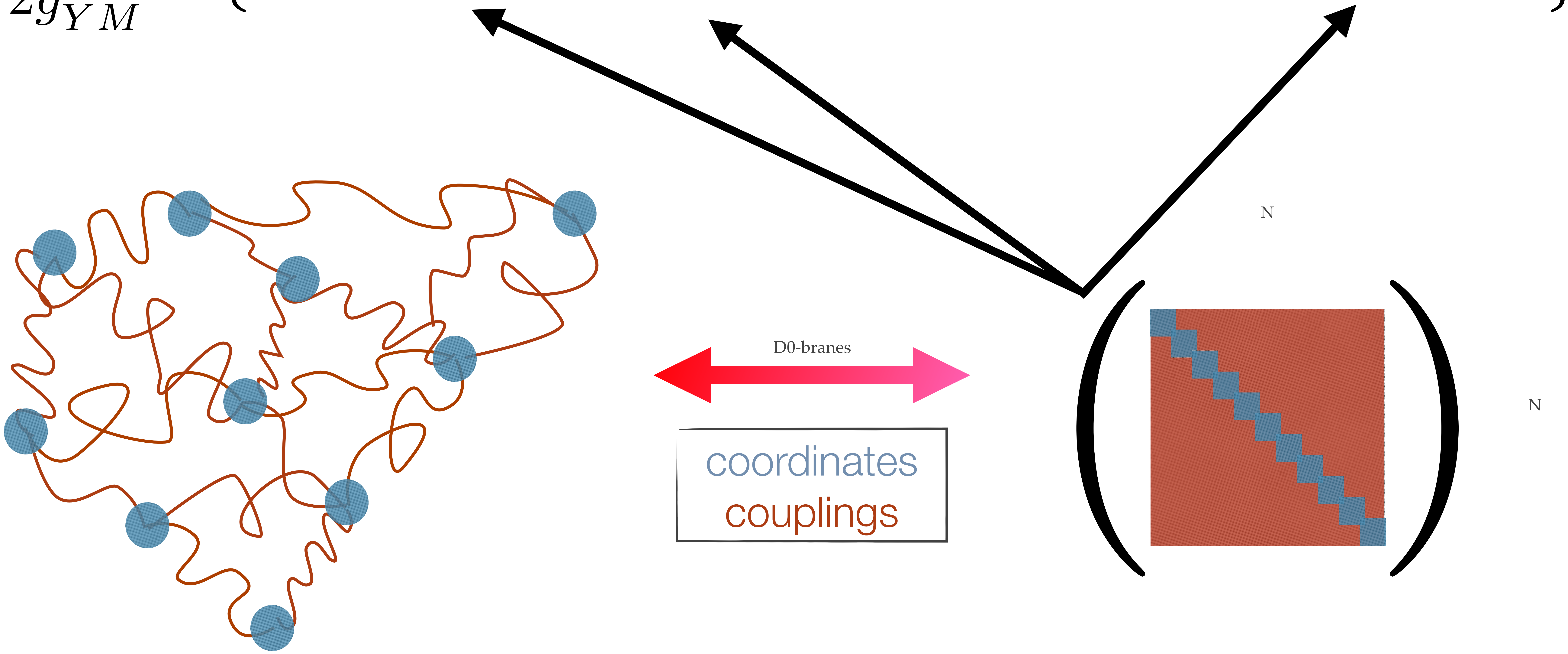

Numerical Methods

★HPC simulations using Path Integral-based methods on discrete grids: Monte Carlo sampling of quantum mechanical paths.

➡**Challenges**:

- Sign problem → paths are not weighted with a standard probability distribution (*i.e.* chem. pot., time evolution)
- Wave function → physics applications require knowledge of entanglement (*i.e.* **information paradox**)

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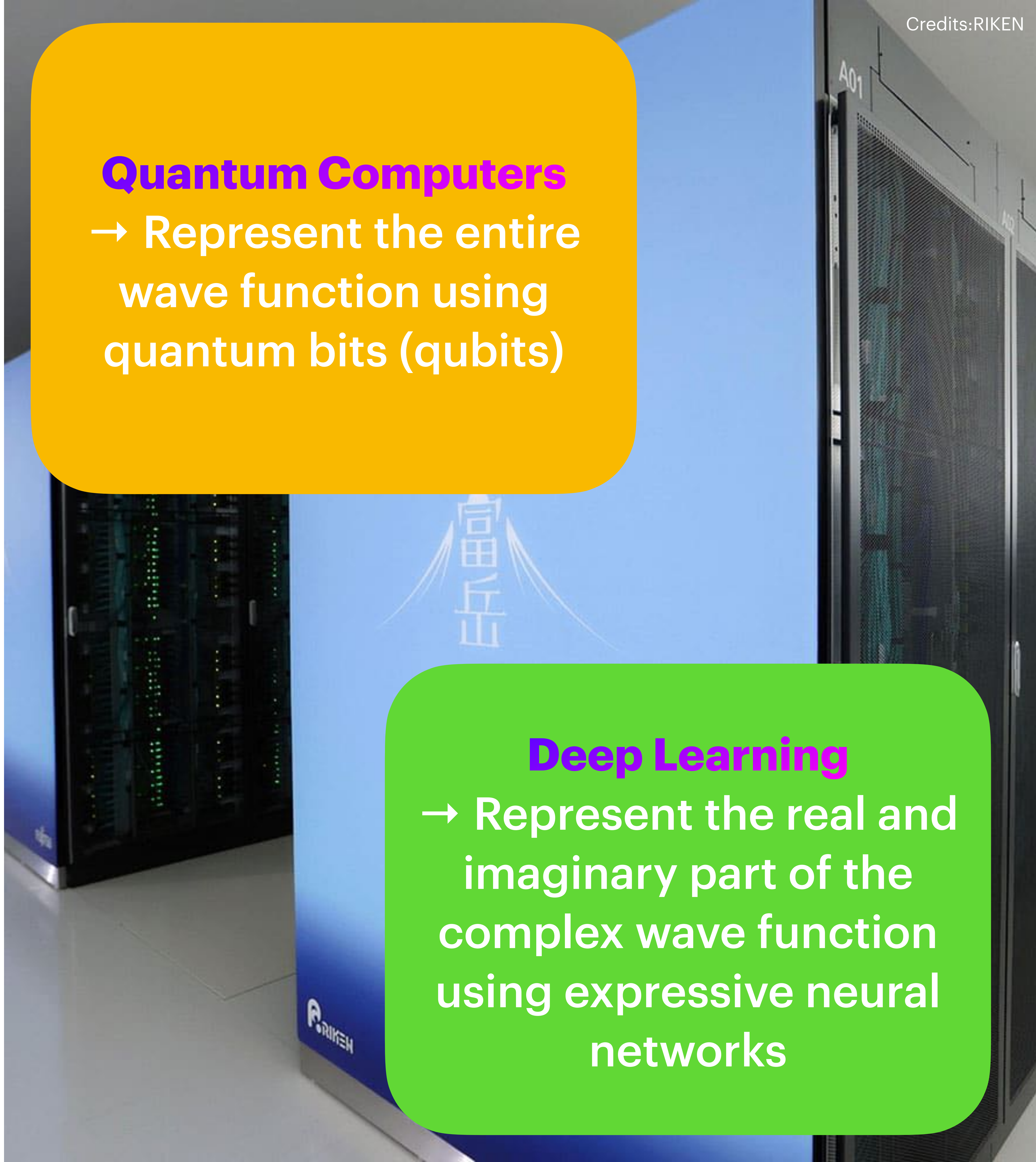
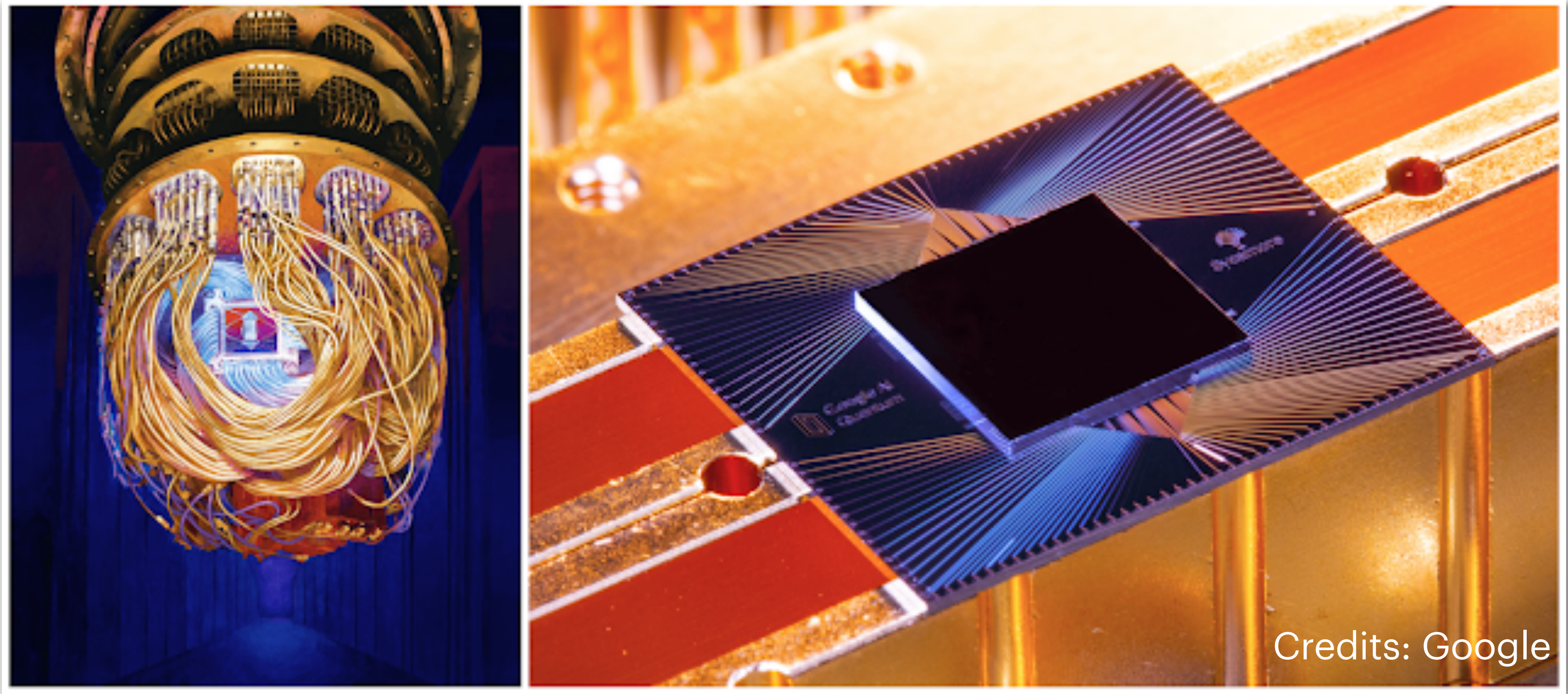


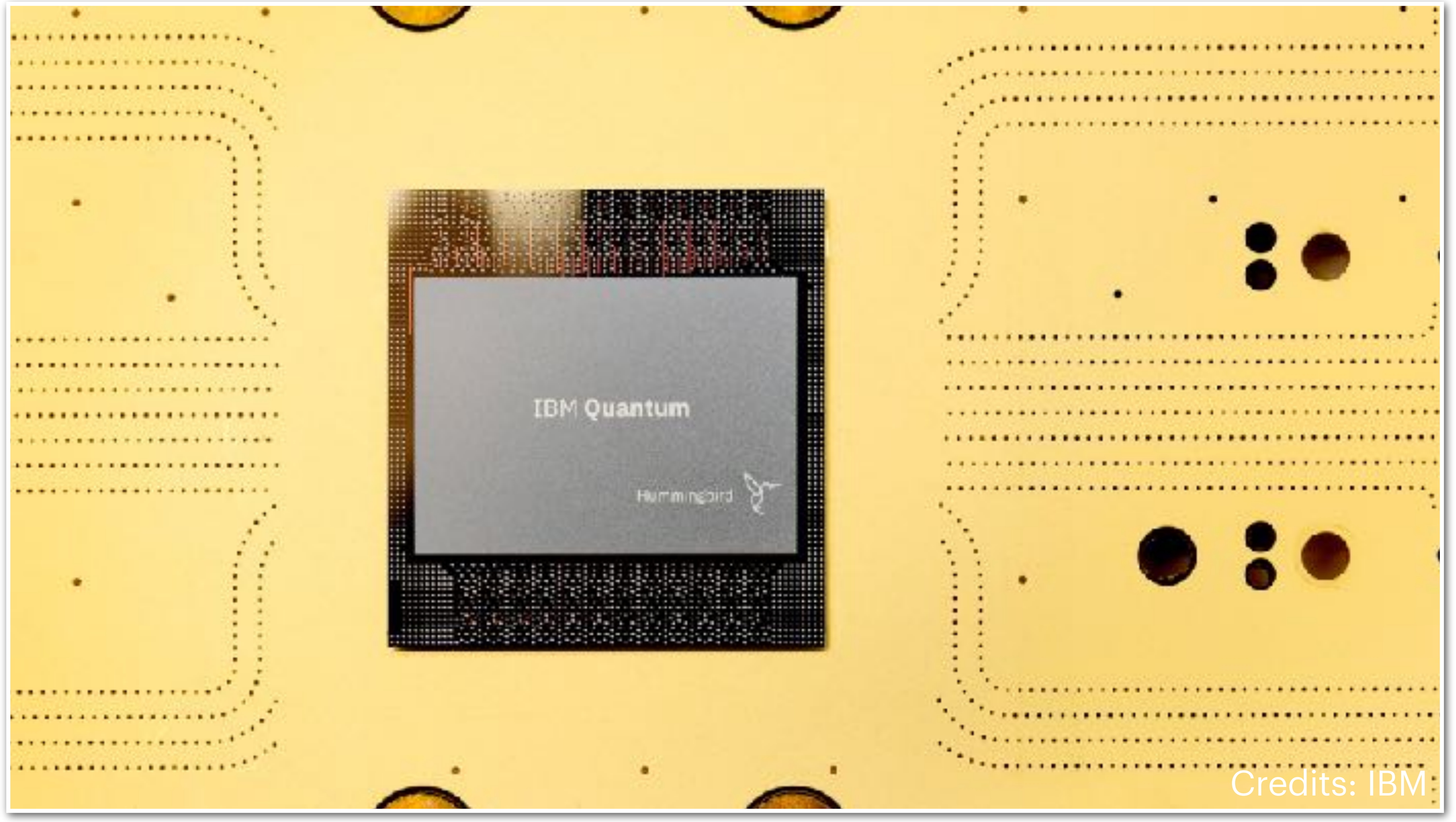
Quantum Computers
→ Represent the entire wave function using quantum bits (qubits)

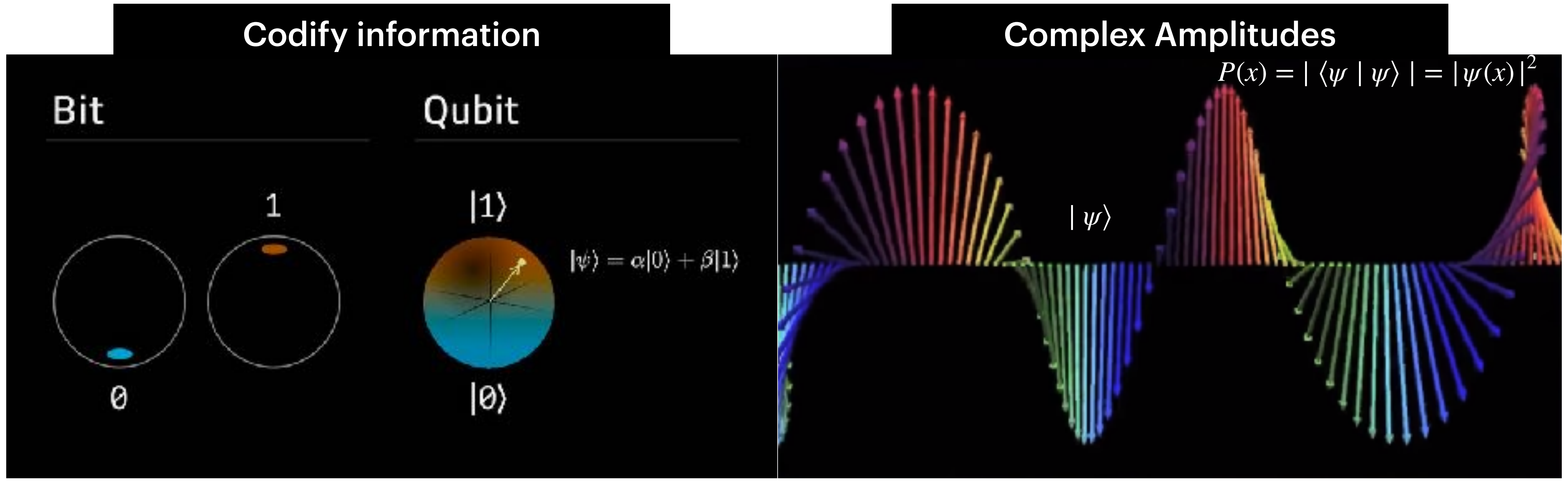
Deep Learning
→ Represent the real and imaginary part of the complex wave function using expressive neural networks

Quantum Technologies
the next computing revolution

- Feynman (1981): *"Nature isn't classical, dammit, and if you want to make a simulation of Nature, you'd better make it quantum mechanical, and by golly it's a wonderful problem because it doesn't look so easy."*
- Digital QC (~50 qubits → 1000 in 2 yr.) have opened new avenues for both scientific research and industrial applications







[Rinaldi et al., arxiv:2108.02942]

Numerical Methods for MQM

Prototypes

Bosonic Model

$$\hat{H}_{B2} = \mathrm{Tr}\left(\frac{1}{2}\hat{P}_I^2 + \frac{m^2}{2}\hat{X}_I^2 - \frac{g^2}{4}\left[\hat{X}_I, \hat{X}_J\right]^2\right)$$

Physical states are invariant under SU(N) Gauge Symmetry

Supersymmetric Model

$$\hat{H} = \hat{H}_{B2} + $$
$$+\mathrm{Tr}\left(\frac{g}{2}\hat{\xi}\left[-\hat{X}_1 - i\hat{X}_2, \hat{\xi}\right] + \frac{g}{2}\hat{\xi}^\dagger\left[-\hat{X}_1 + i\hat{X}_2, \hat{\xi}^\dagger\right] + \frac{3\mu}{2}\hat{\xi}^\dagger\hat{\xi}\right)$$
$$-\left(N^2 - 1\right)\mu$$

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FREE

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FREE — BOS. INTERACTION

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ZERO EN.

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<u>Challenge</u>: numerical methods on quantum computers have a limited number of qubits!

[Rinaldi et al., arxiv:2108.02942]

Numerical Methods for MQM

Prototype: small-scale system

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Bosonic Model

Example: N=2, D=2

FREE — BOS. INTERACTION

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1 2 3 $I = 1$

4 5 6 $I = 2$

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$I = 1,2 \qquad \alpha = 1,2,3$

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4 5 6 $\quad I = 2$

SYMMETRIES

$g^2 = 0$

$g^2 > 0$

[Rinaldi et al., arxiv:2108.02942]

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Bosonic Model

FREE | BOS. INTERACTION

$$\hat{H}_B = \mathrm{Tr}\left(\frac{1}{2}\hat{P}_I^2 + \frac{m^2}{2}\hat{X}_I^2 - \frac{g^2}{4}\left[\hat{X}_I, \hat{X}_J\right]^2\right)$$

Physical states are invariant under SU(N) Gauge Symmetry

$$\hat{X}_I = \sum_{\alpha=1}^{N^2-1} \hat{X}_I^{\alpha} \tau_{\alpha} \qquad I = 1,\ldots,D$$

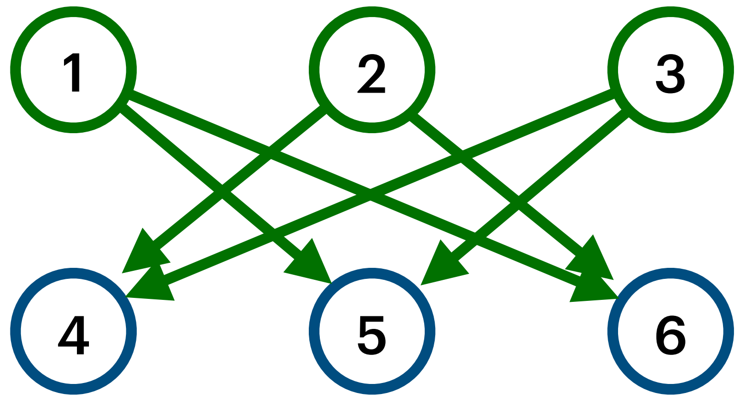
$\hat{X}_I^{\alpha}$ → bosonic degrees of freedom

τ_{α} → generators of SU(N) group

Example: N=2, D=2

$$\hat{H}_B = \sum_{\alpha,I}\left(\frac{1}{2}\hat{P}_{I\alpha}^2 + \frac{m^2}{2}\hat{X}_{I\alpha}^2\right) + \frac{g^2}{4}\sum_{\gamma,I,J}\left(\sum_{\alpha,\beta} f_{\alpha\beta\gamma}\hat{X}_I^{\alpha}\hat{X}_J^{\beta}\right)^2$$

$I = 1,2 \qquad \alpha = 1,2,3$



$I = 1$

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SYMMETRIES

$g^2 = 0$

$g^2 > 0$

Numerical Methods for MQM

Prototype: small-scale system

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FREE | BOS. INTERACTION

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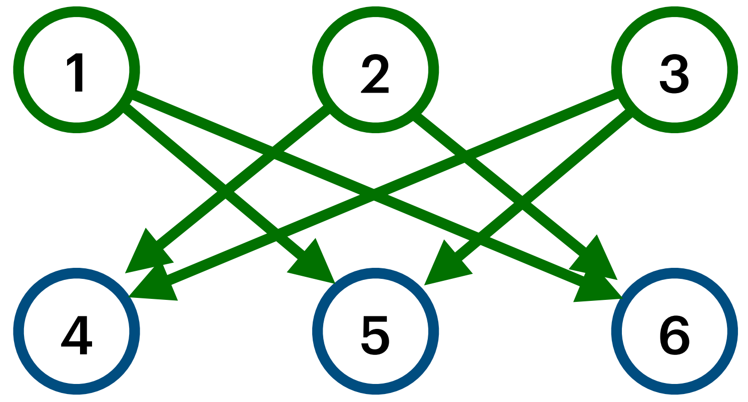

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[Rinaldi et al., arxiv:2108.02942]

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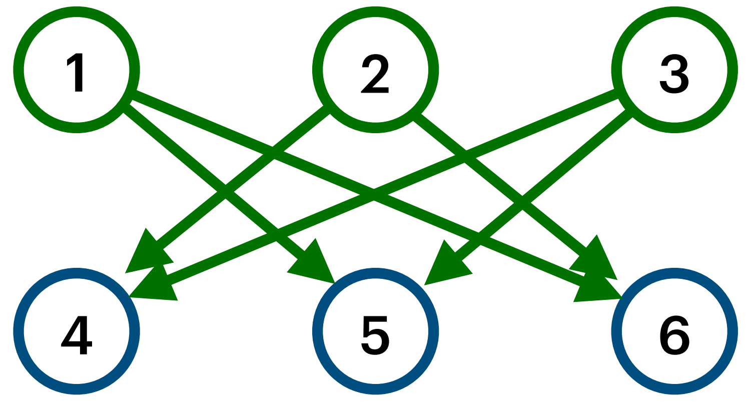

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[Gharibyan et al., arxiv:2011.06573]

Hilbert space regularization

Truncation

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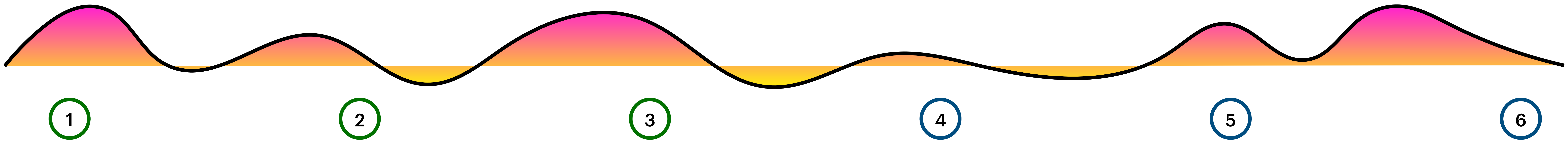

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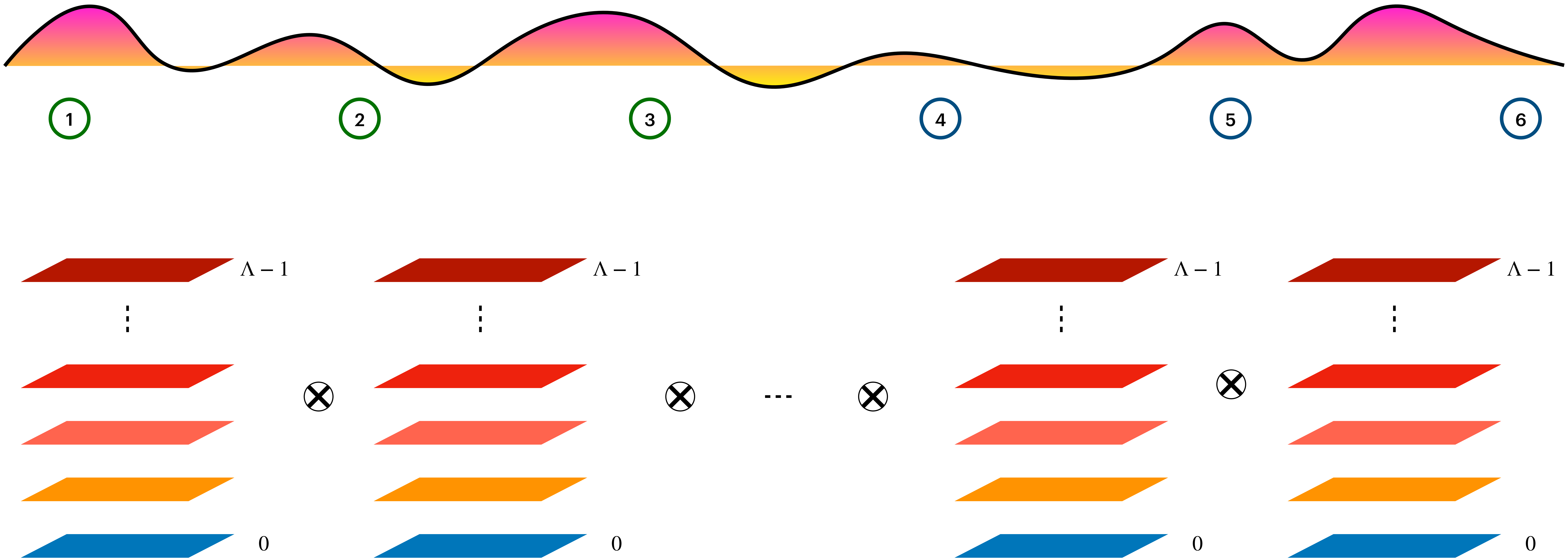

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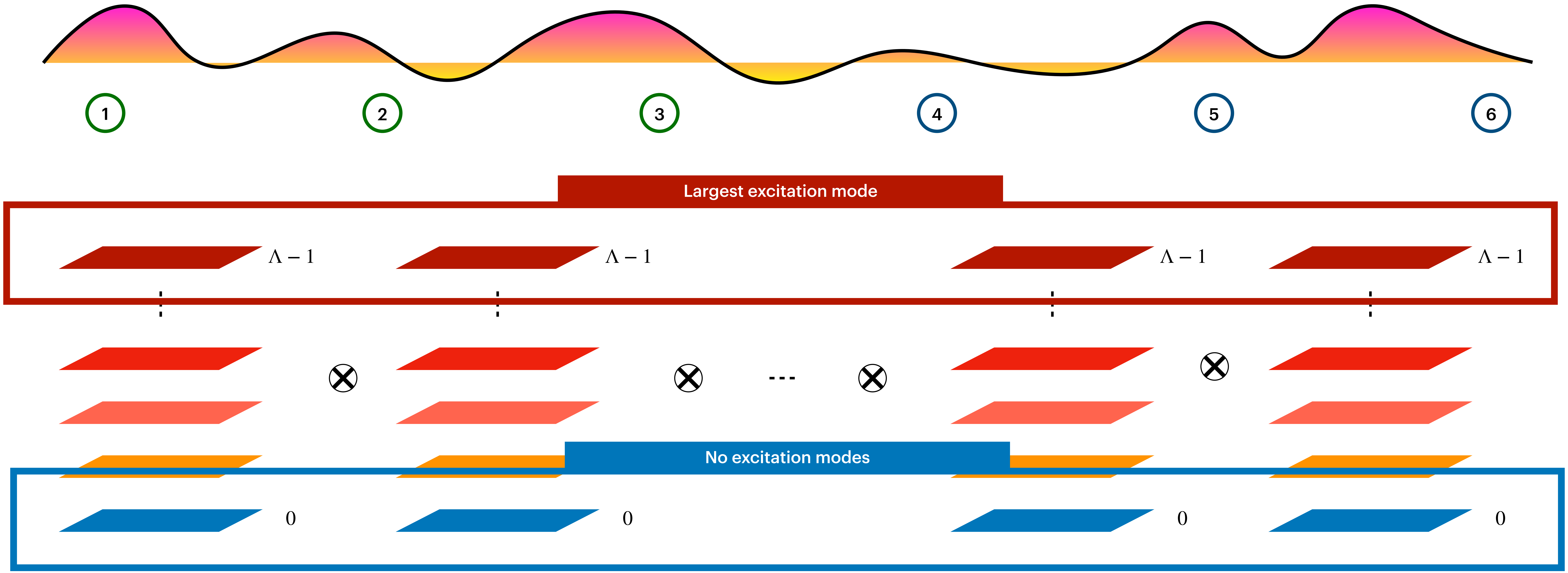

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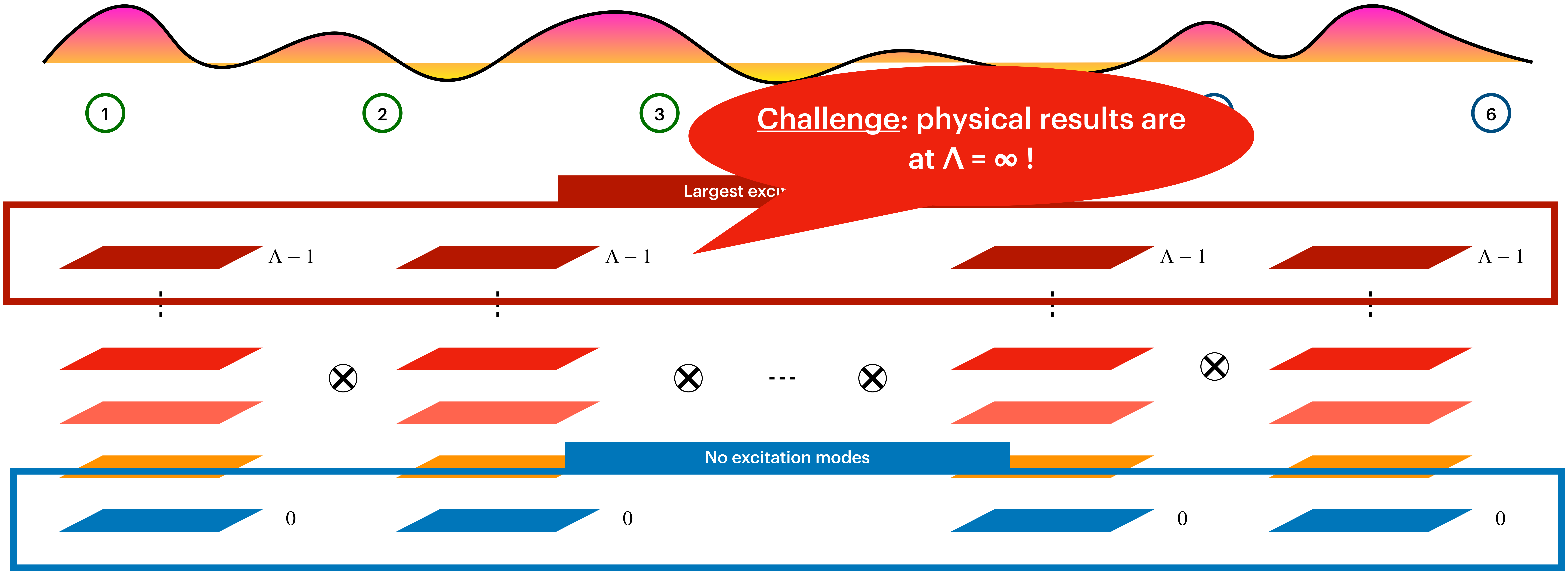

https://github.com/erinaldi/bmn2-qutip

[Rinaldi et al., arxiv:2108.02942]

QuTiP

Quantum Toolbox in Python

Results

Small-scale: N=2, D=2, $\Lambda \to \infty$

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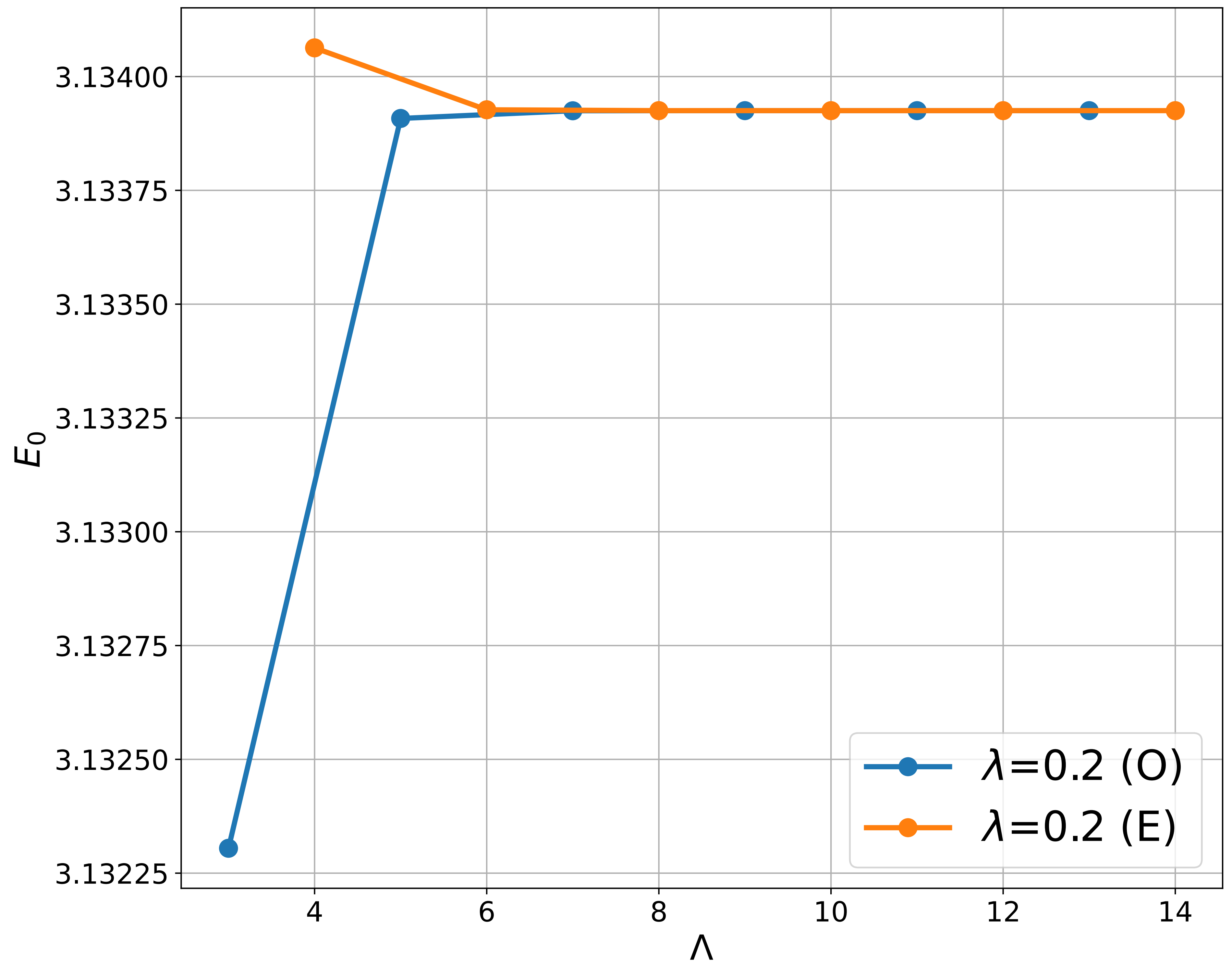

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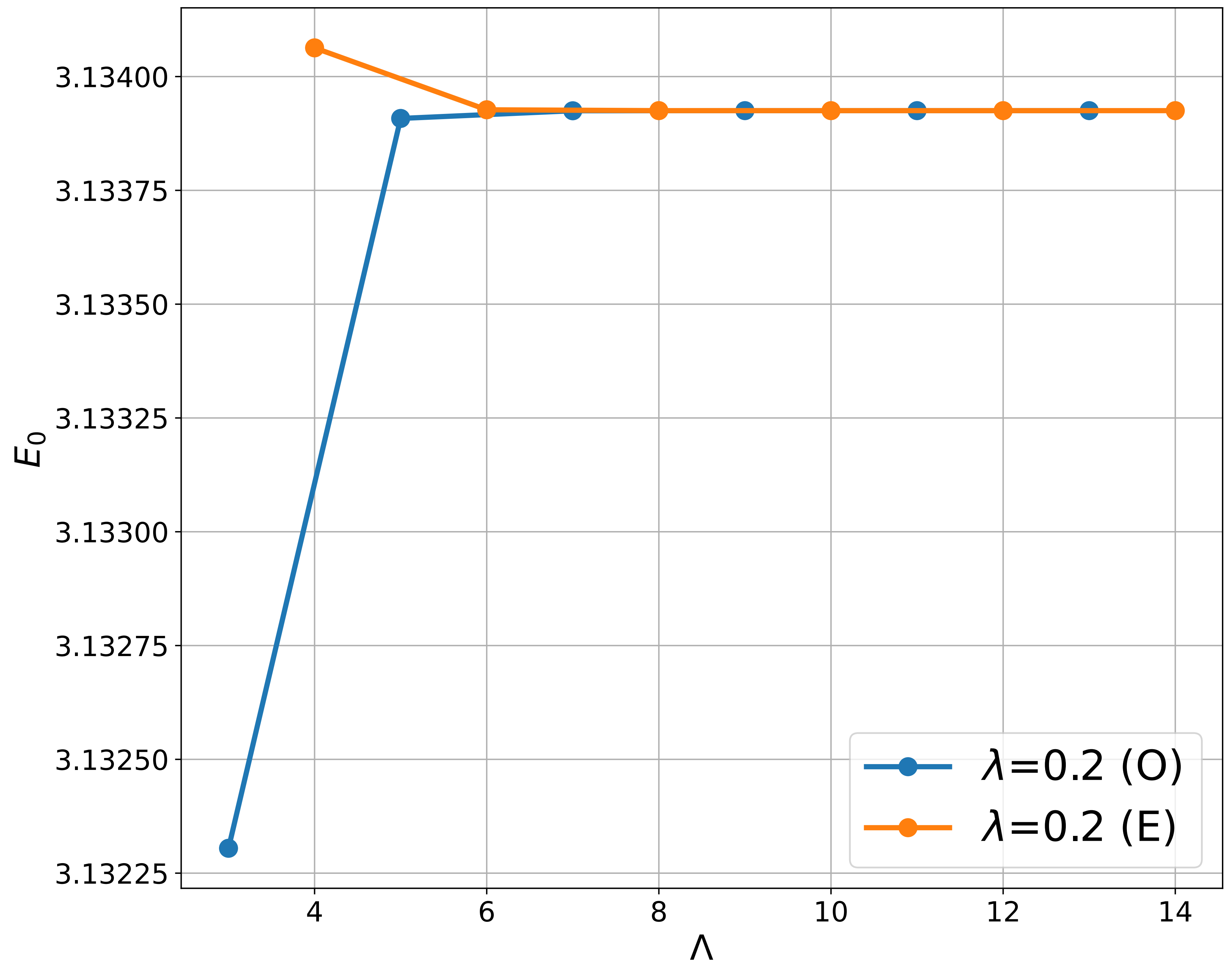

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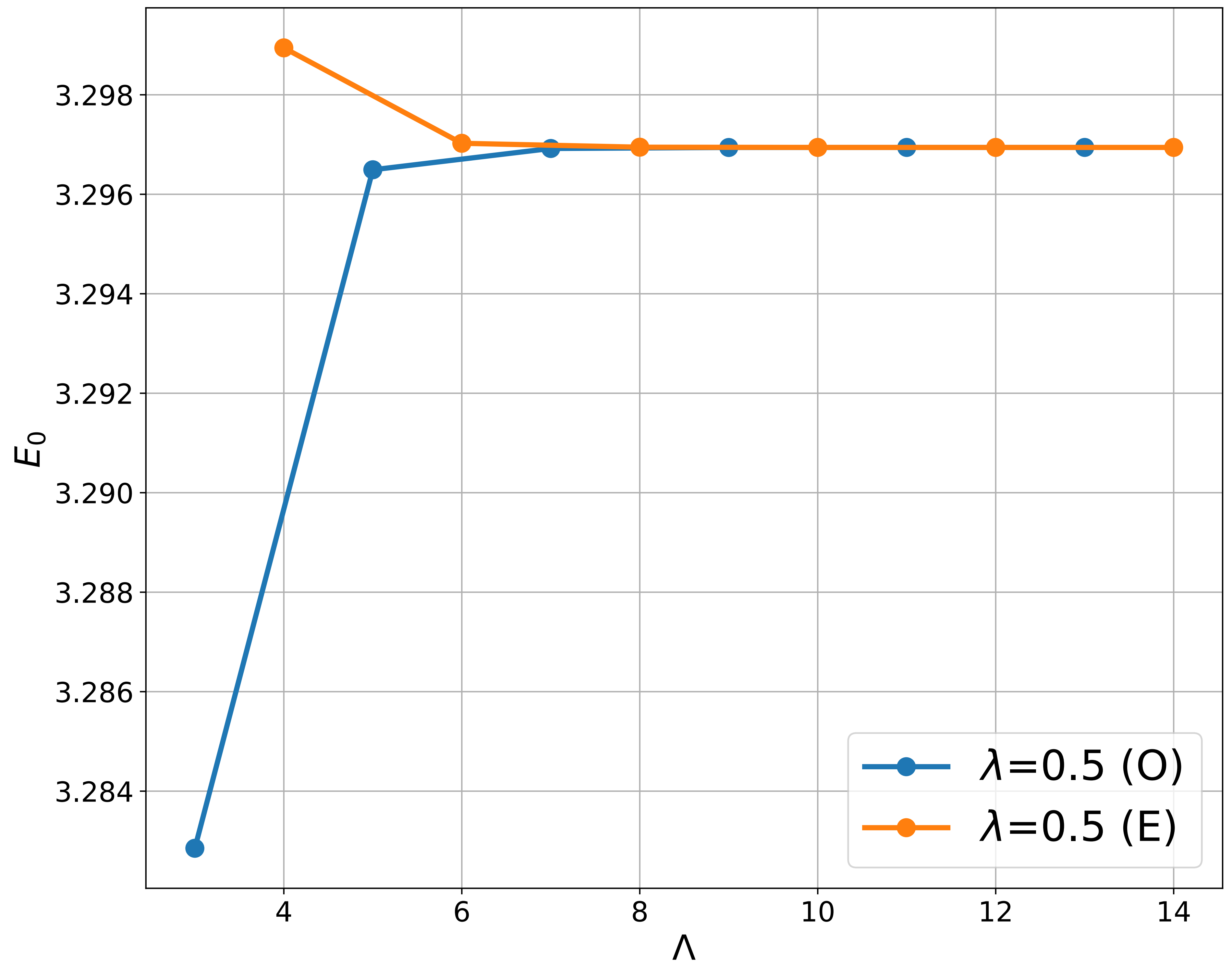

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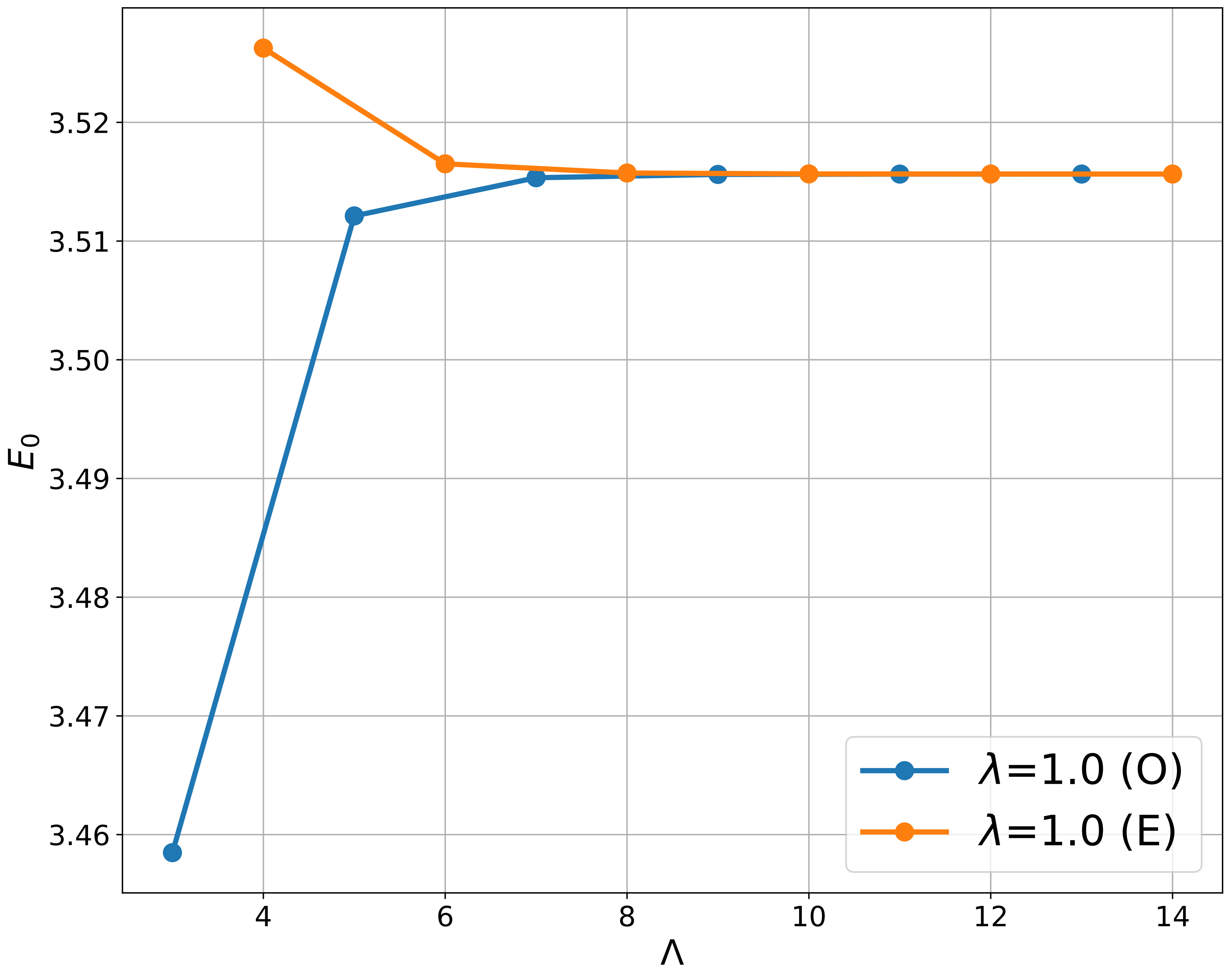

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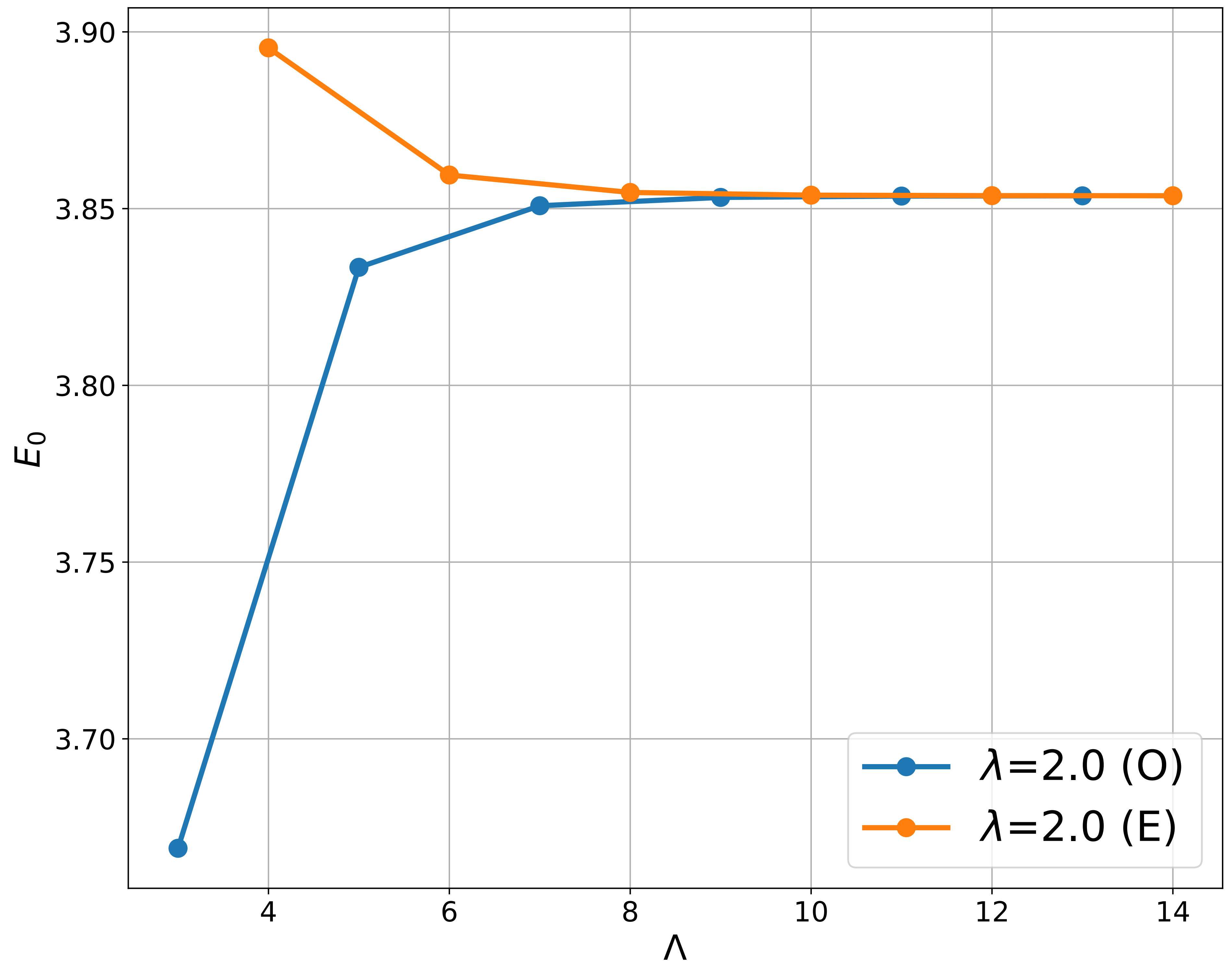

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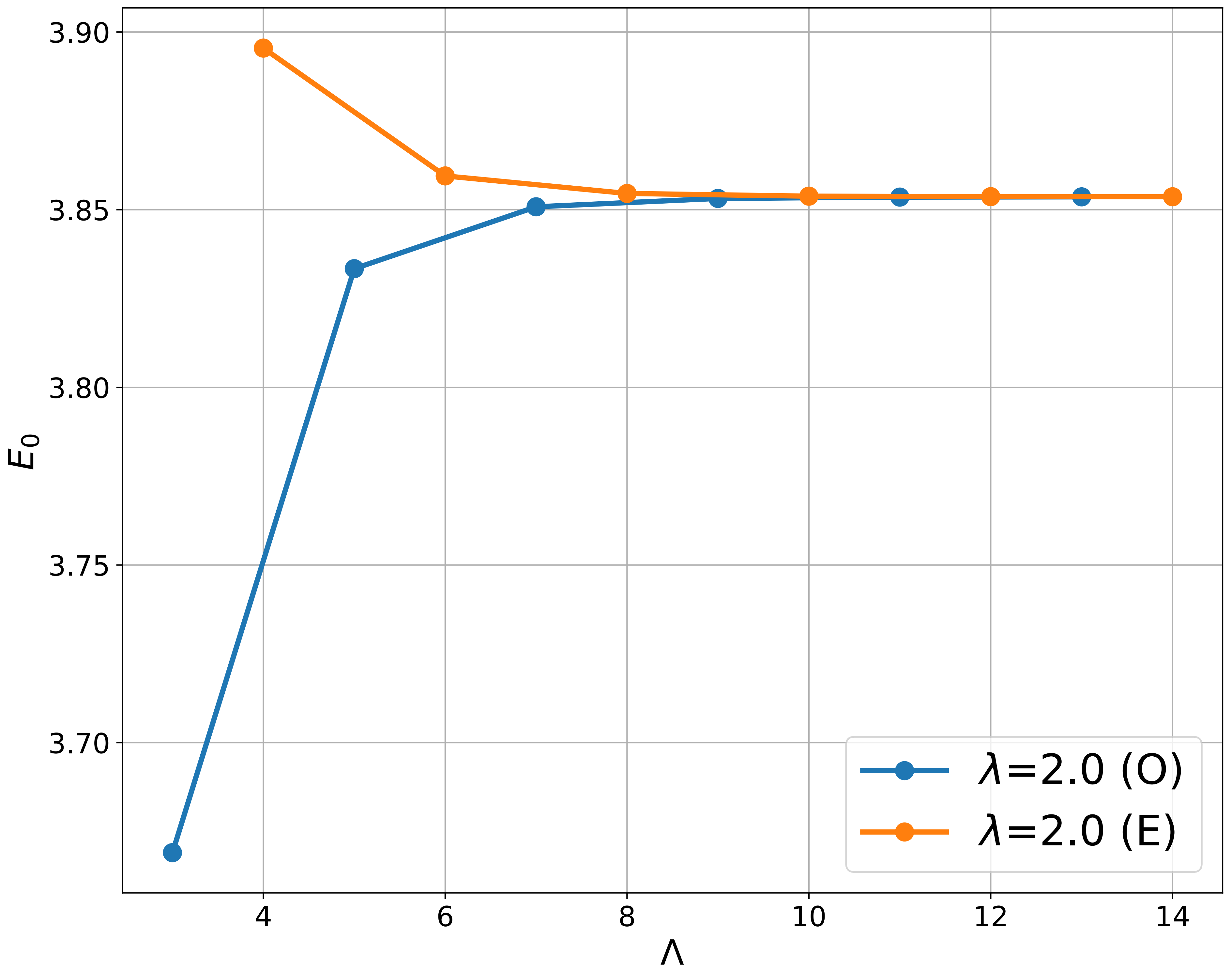

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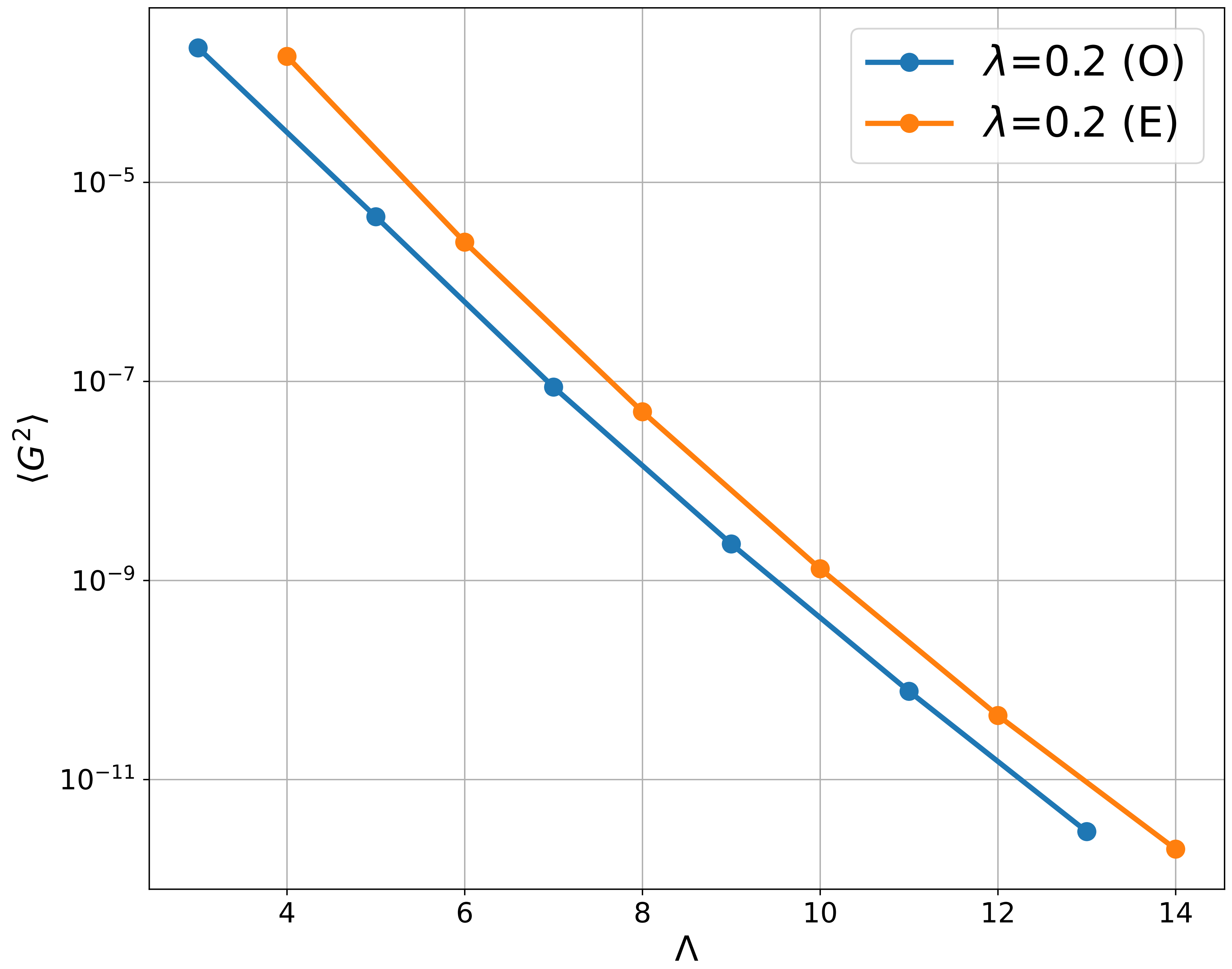

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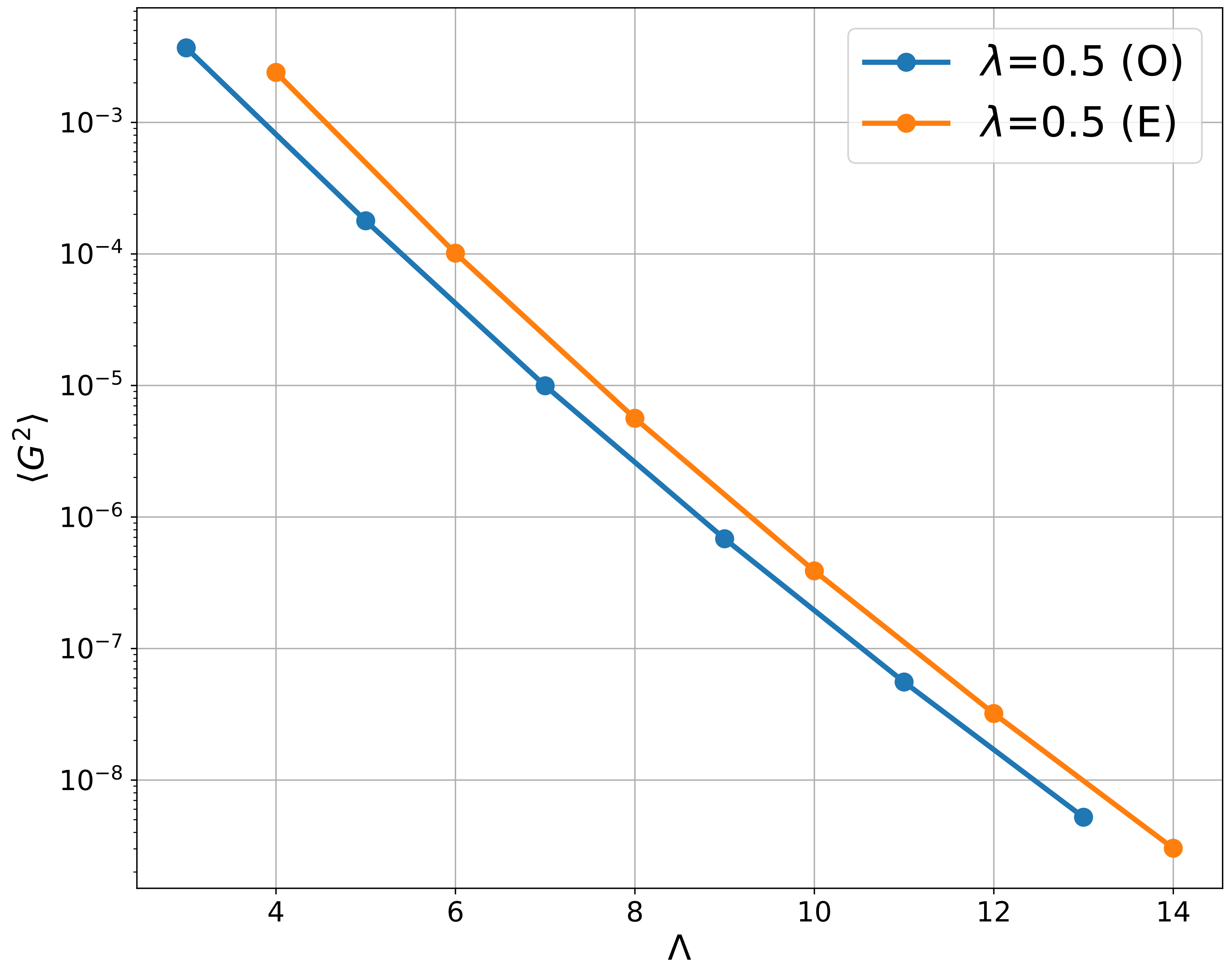

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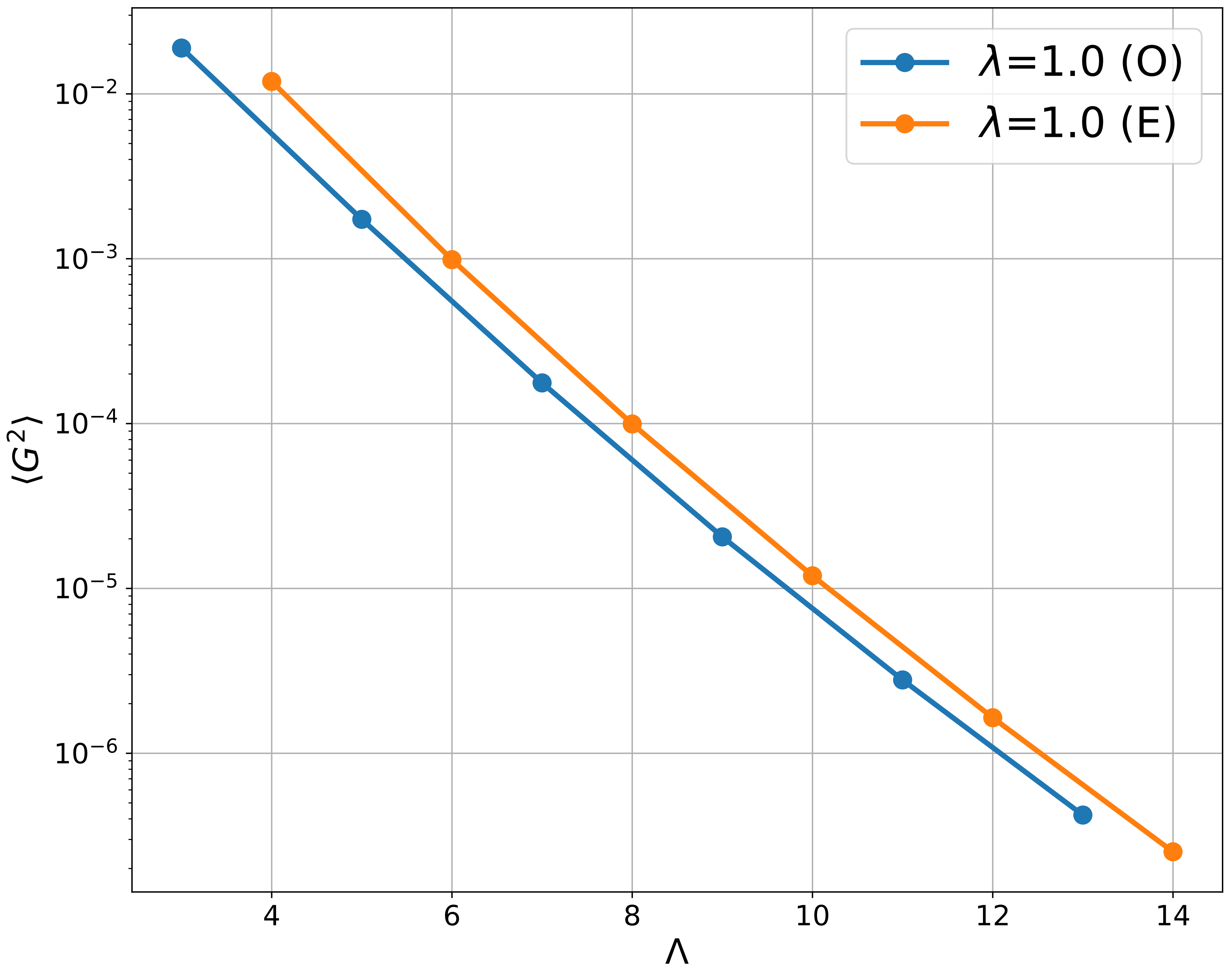

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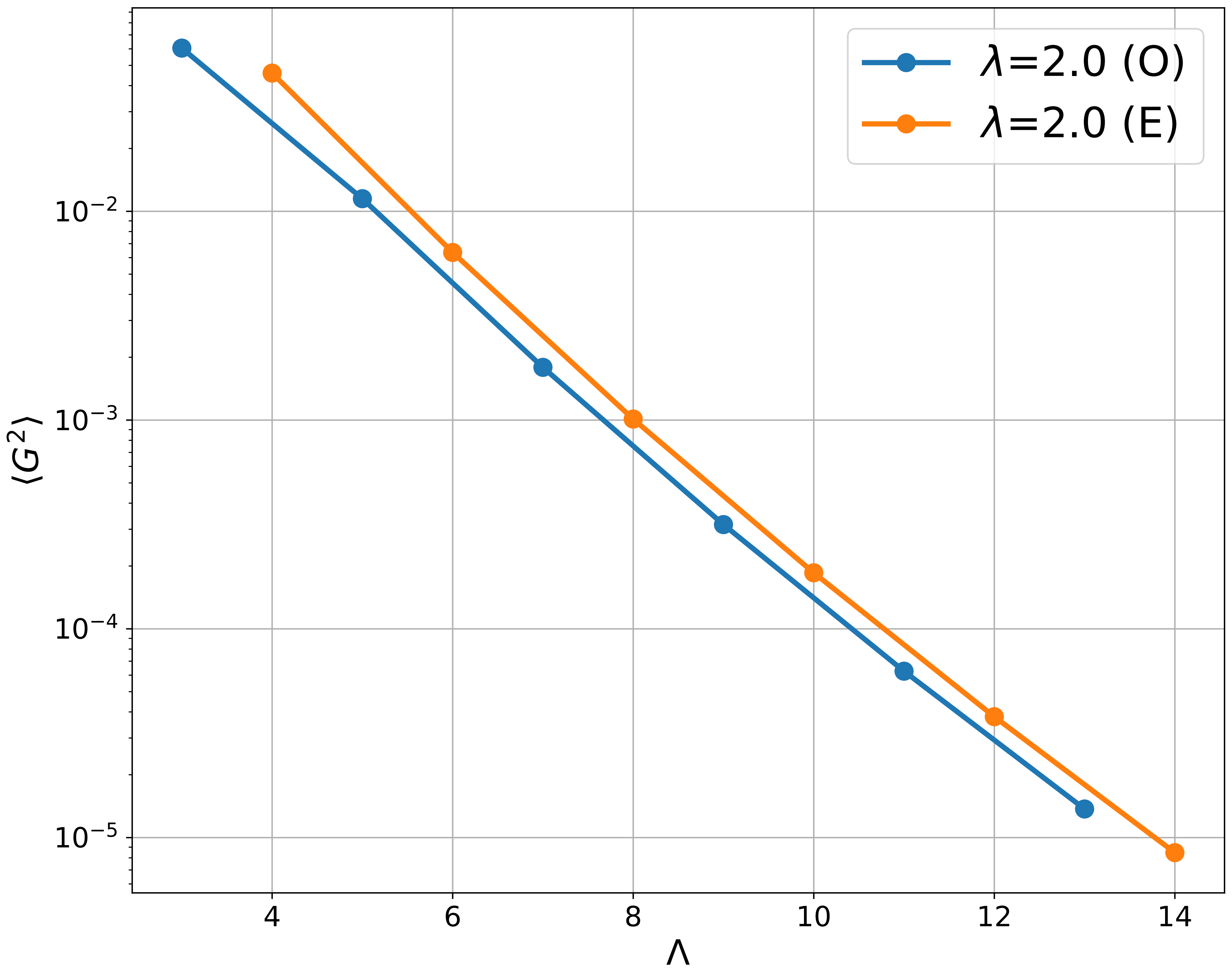

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[Rinaldi et al., arxiv:2108.02942]

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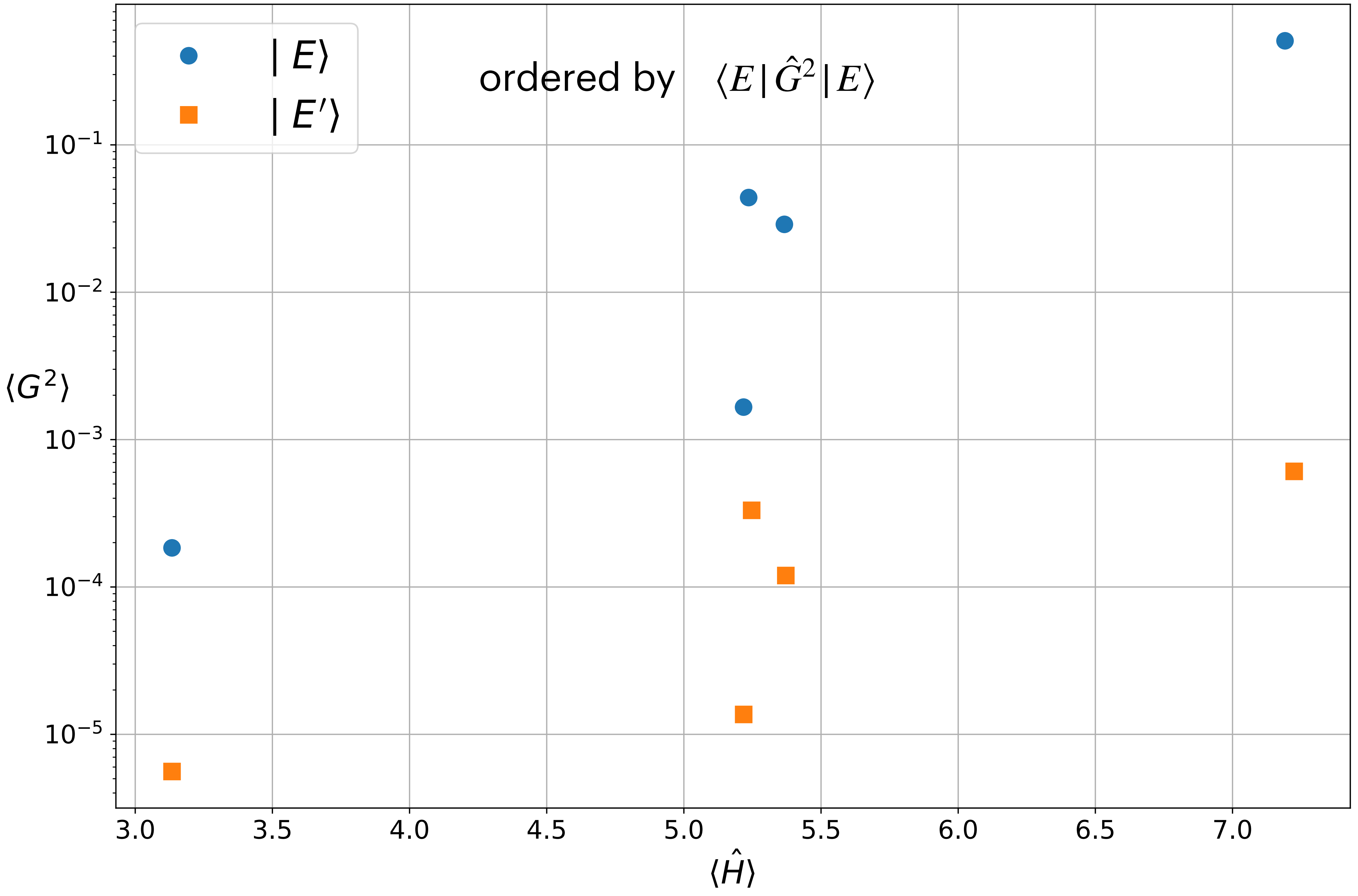

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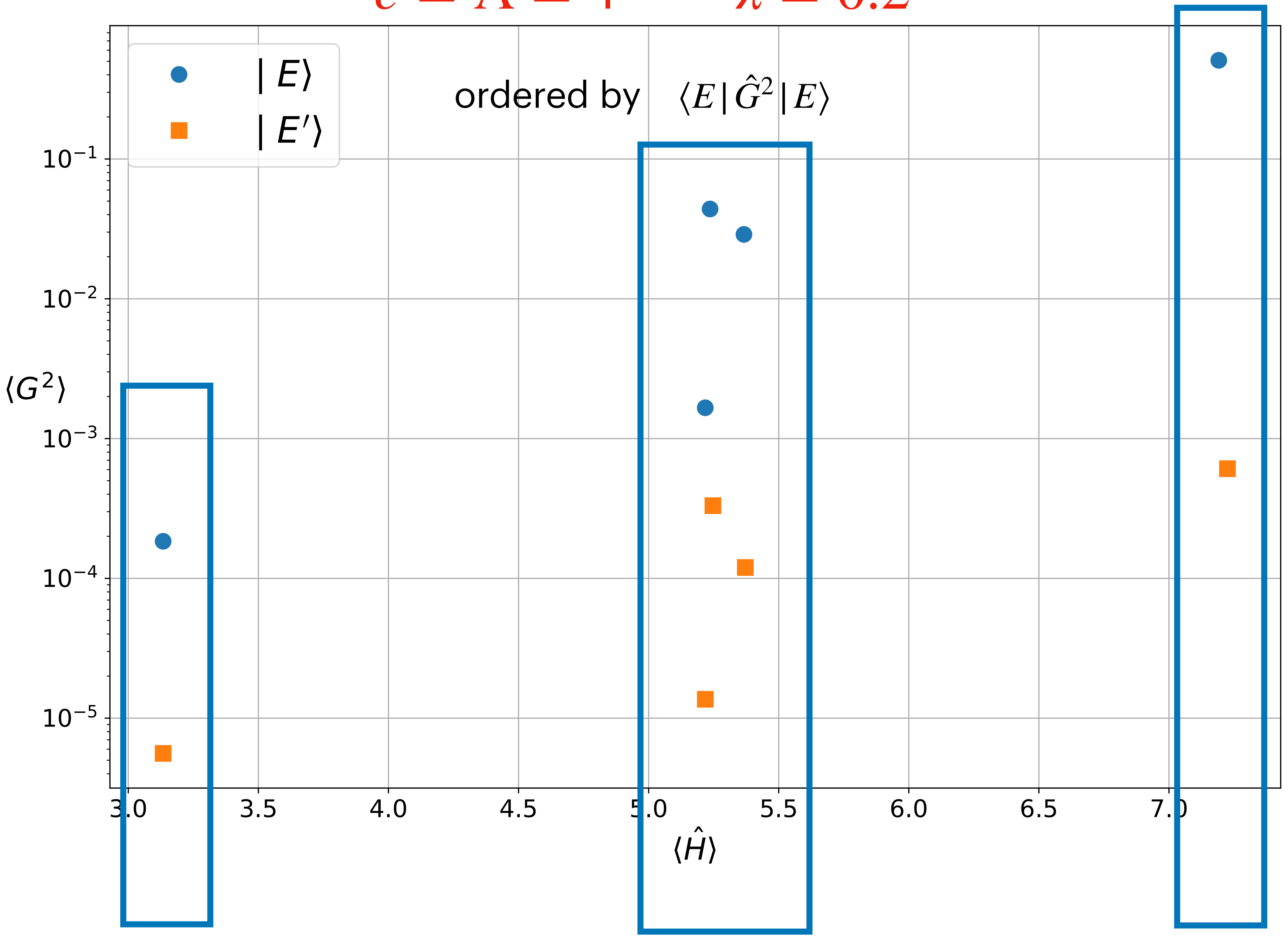

https://github.com/erinaldi/bmn2-qiskit

[Rinaldi et al., arxiv:2108.02942]

Qiskit

Qubitization of MQM

Small-scale: N=2, D=2, $\Lambda \to \infty$

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Qiskit

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Small-scale: N=2, D=2, Λ → ∞

Truncation Level

Λ = 2

Λ = 4

Λ = 8

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Each boson is 1 qubit

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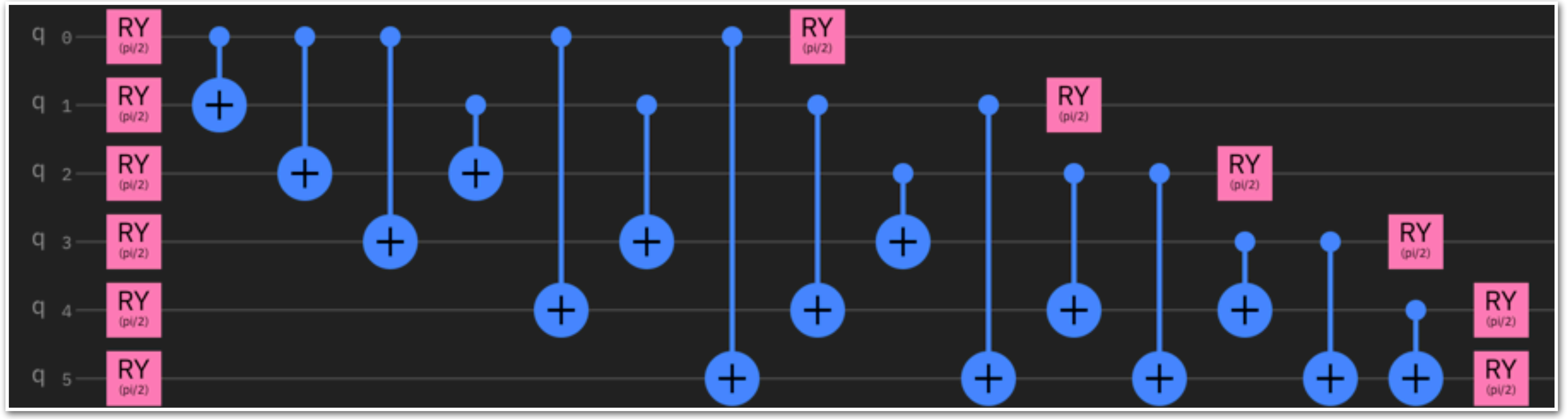

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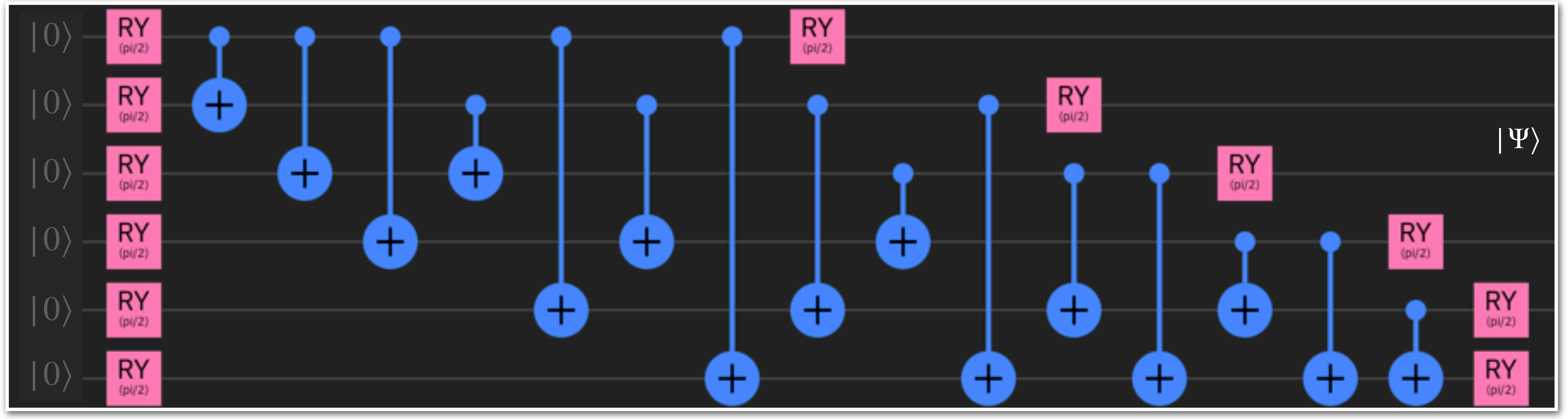

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Truncation Level

$\Lambda = 2$	Each boson is 1 qubit	$\log_2 \Lambda^6 = 6$ qubits
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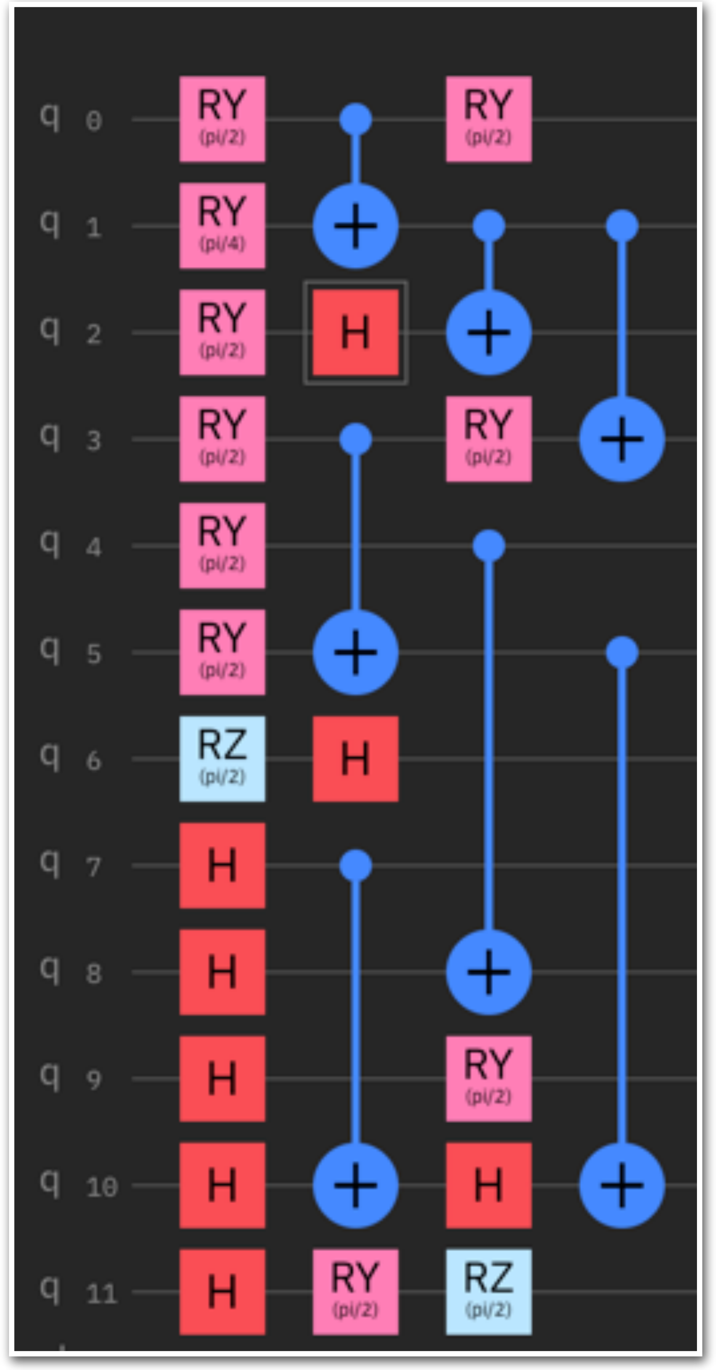

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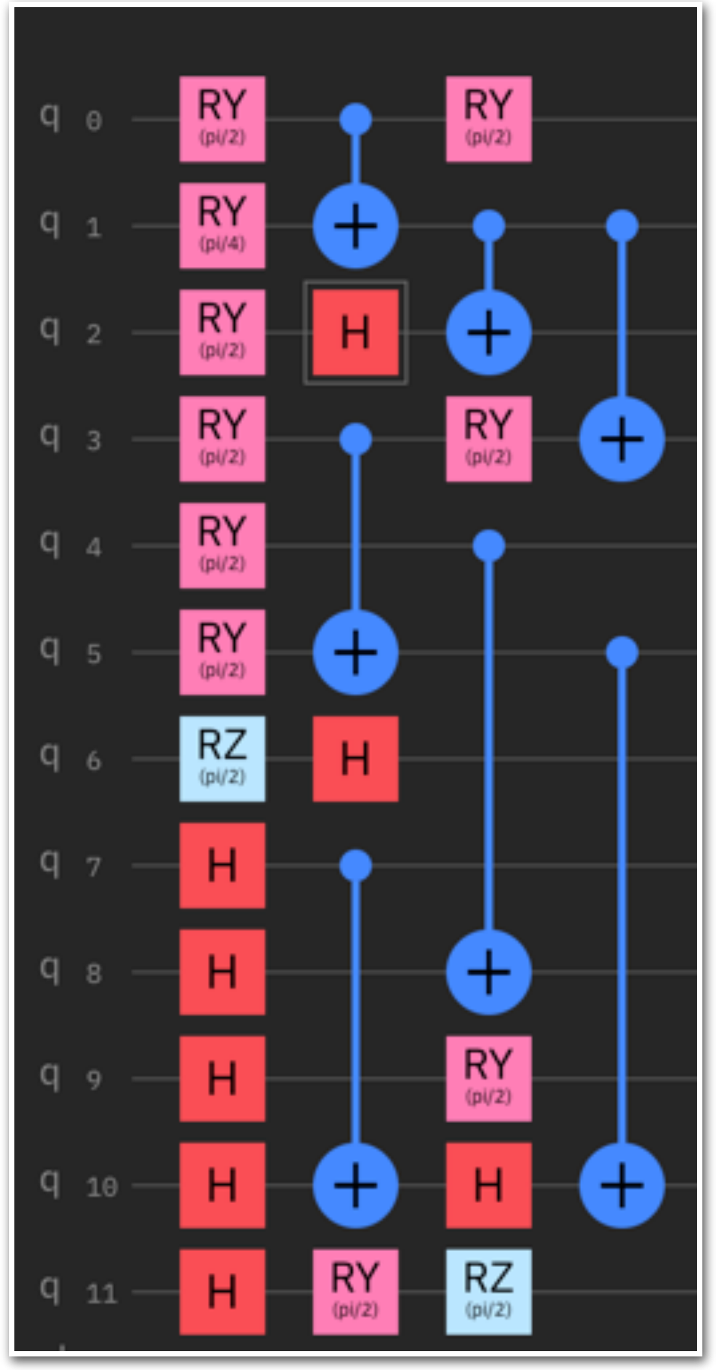

Qiskit

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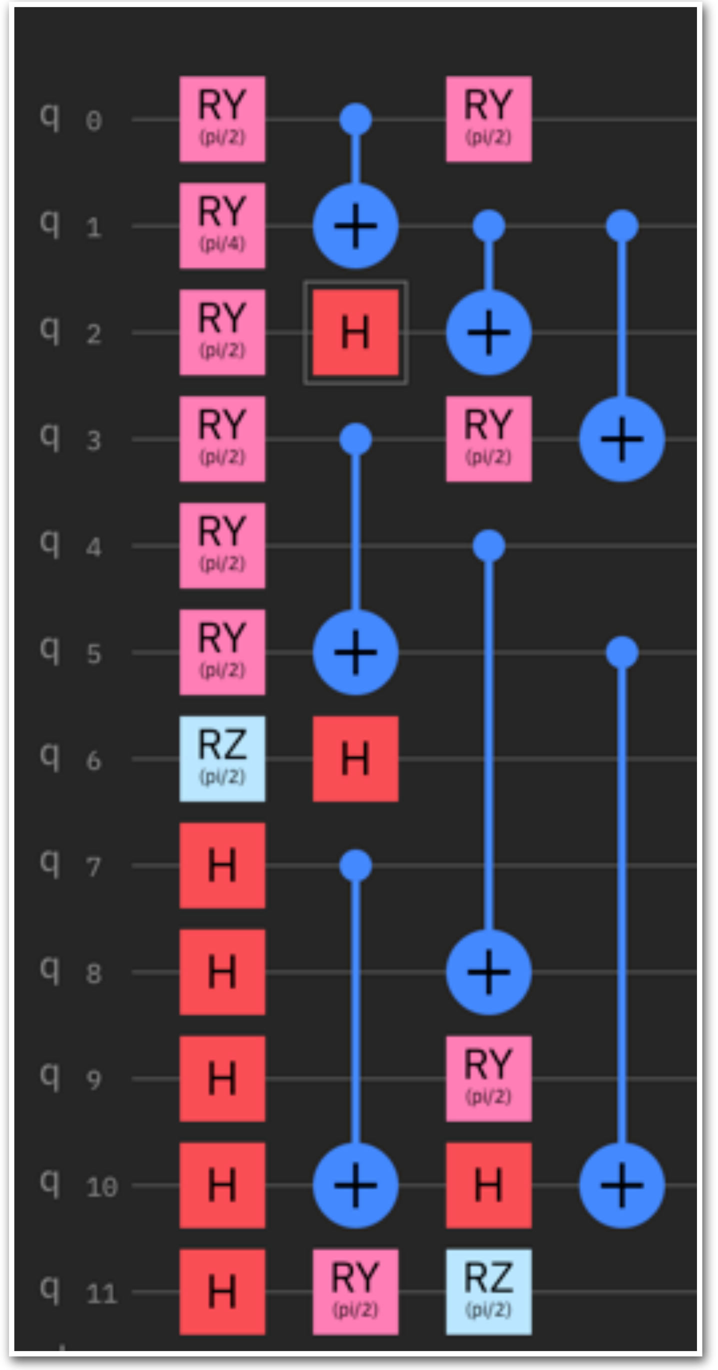
limited resources



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Qubitization of MQM

Small-scale: N=2, D=2, Λ → ∞

Truncation Level

Λ = 2	Each boson is 1 qubit	$\log_2 \Lambda^6$ = 6 qubits
Λ = 4	Each boson is 2 qubits	$\log_2 \Lambda^6$ = 12 qubits
Λ = 8	Each b[illegible]on [illegible] qubits	$\log_2 \Lambda^6$ = 18 qubits

limited resources

$$\hat{H}_B = \sum_{\alpha,I}\left(\frac{1}{2}\hat{P}^2_{I\alpha} + \frac{m^2}{2}\hat{X}^2_{I\alpha}\right) + \frac{g^2}{4}\sum_{\gamma,I,J}\left(\sum_{\alpha,\beta} f_{\alpha\beta\gamma}\hat{X}^\alpha_I \hat{X}^\beta_J\right)^2 \qquad I = 1,2 \qquad \alpha = 1,2,3$$

Build matrix Hamiltonian which gets mapped to qubits

Qiskit

Qubitization of MQM

Small-scale: N=2, D=2, $\Lambda \to \infty$

Truncation Level

$\Lambda = 2$	Each boson is 1 qubit	$\log_2 \Lambda^6 = 6$ qubits
$\Lambda = 4$	Each boson is 2 qubits	$\log_2 \Lambda^6 = 12$ qubits
$\Lambda = 8$	Each b… on i… qubits	$\log_2 \Lambda^6 = 18$ qubits

limited resources

Rewrite $X_i \to a_i$
Annihilation operator for site "i"

$$\hat{a}_i = \hat{I}_1 \otimes \ldots \otimes \hat{I}_{i-1} \otimes \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \otimes \hat{I}_{i+1} \otimes \ldots \otimes \hat{I}_6$$

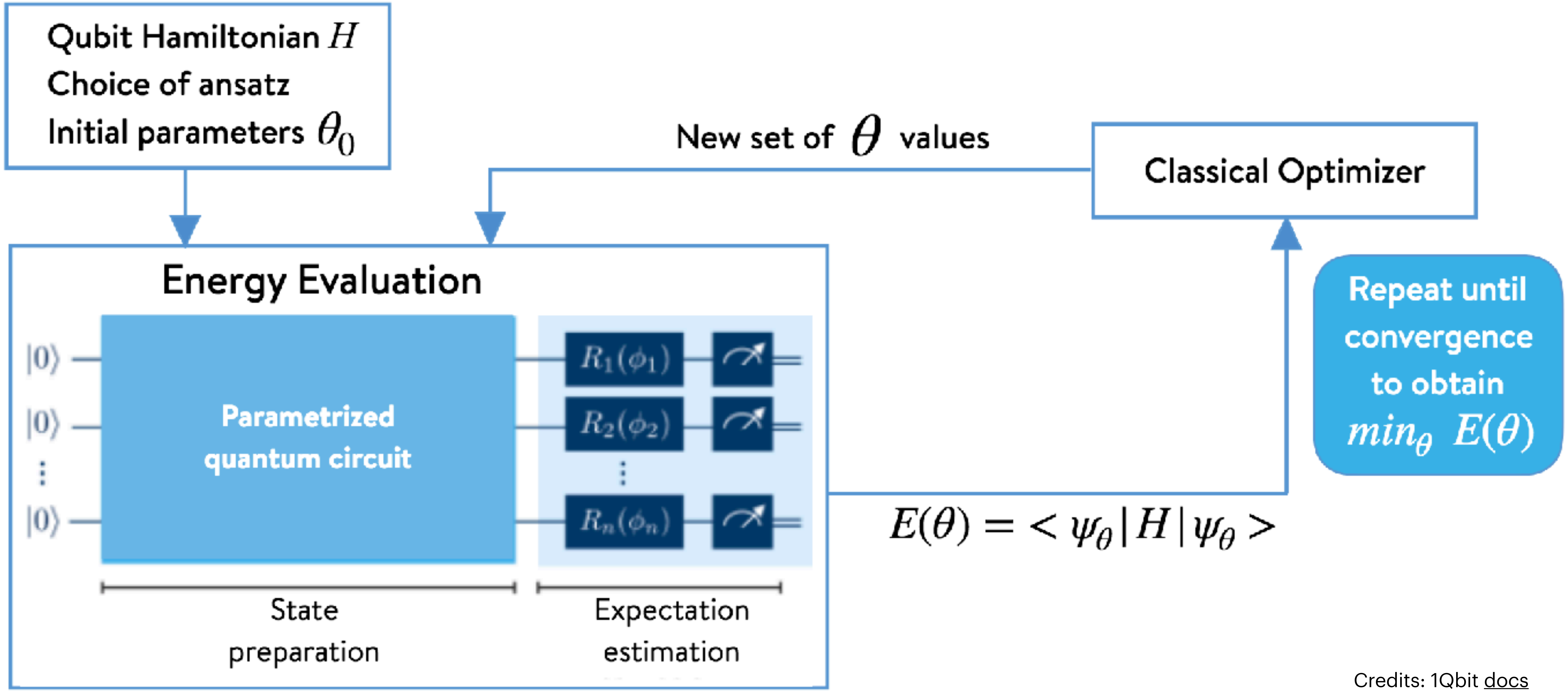
$$\hat{a}_i = \hat{I}_1 \otimes \ldots \otimes \hat{I}_{i-1} \otimes \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & \sqrt{2} & 0 \\ 0 & 0 & 0 & \sqrt{3} \\ 0 & 0 & 0 & 0 \end{pmatrix} \otimes \hat{I}_{i+1} \otimes \ldots \otimes \hat{I}_6$$

$$\hat{H}_B = \sum_{\alpha,I} \left(\frac{1}{2}\hat{P}^2_{I\alpha} + \frac{m^2}{2}\hat{X}^2_{I\alpha} \right) + \frac{g^2}{4} \sum_{\gamma,I,J} \left(\sum_{\alpha,\beta} f_{\alpha\beta\gamma} \hat{X}^\alpha_I \hat{X}^\beta_J \right)^2 \qquad I = 1,2 \qquad \alpha = 1,2,3$$

Build matrix Hamiltonian which gets mapped to qubits

Quantum Computing

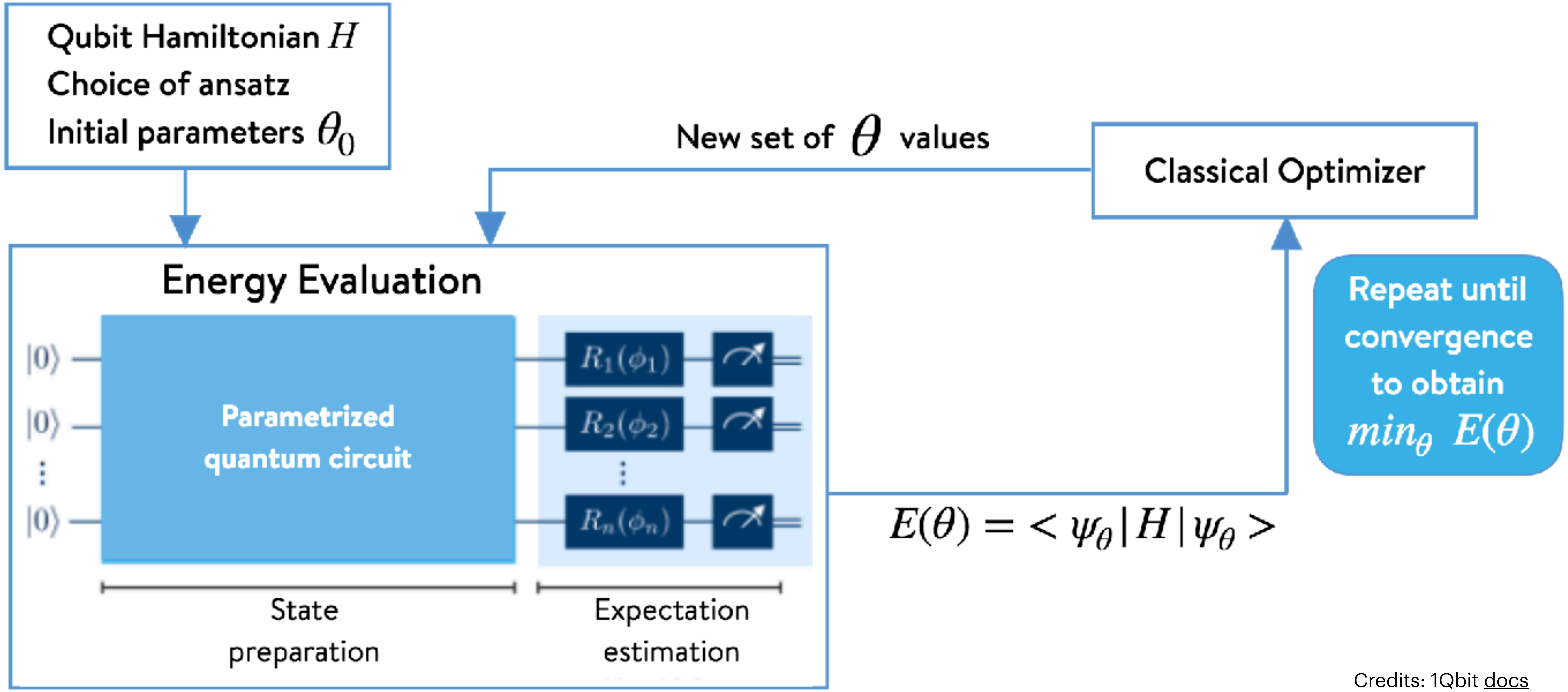
Variational Quantum Eigensolver - VQE



Credits: 1Qbit docs

Quantum Computing

Variational Quantum Eigensolver - VQE



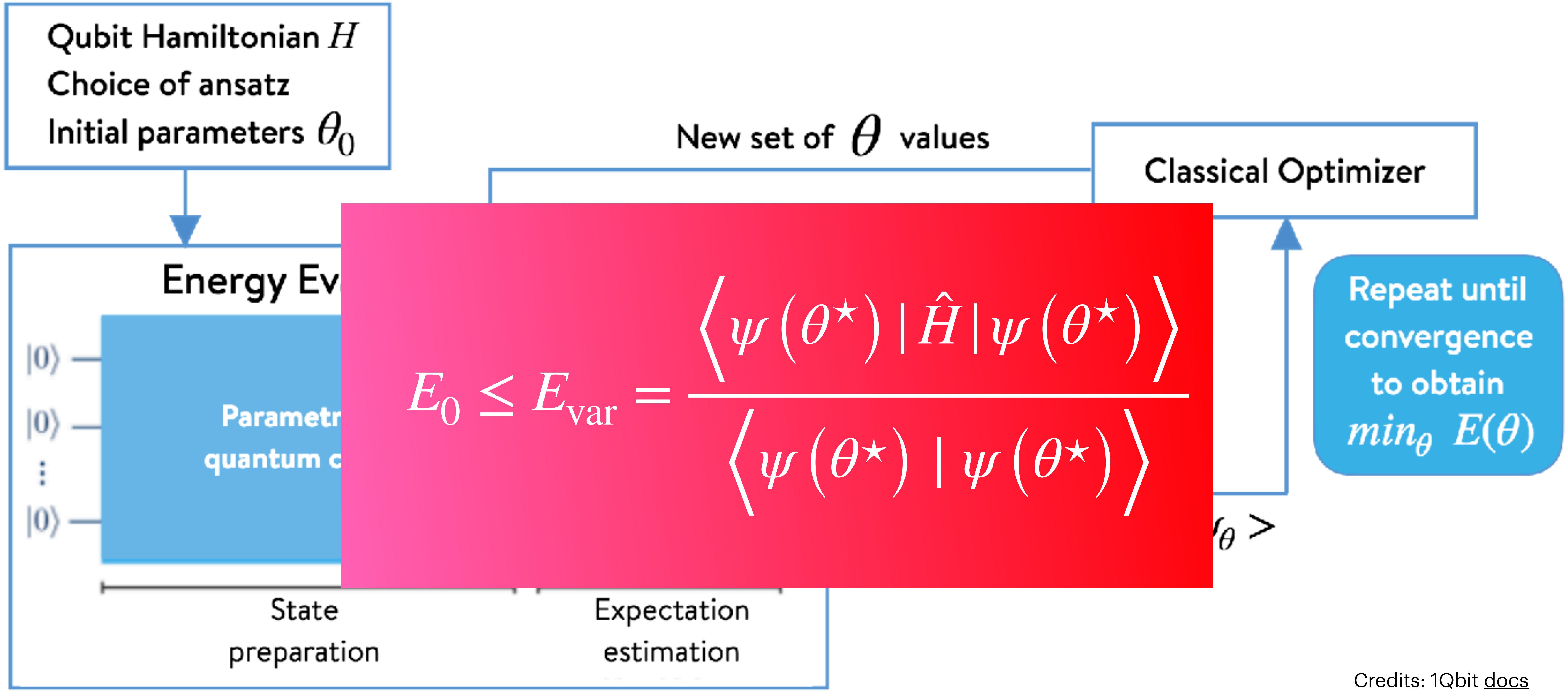
Credits: 1Qbit docs

PQC → Variational Ansatz for |Φ〉 → Evaluation of cost function → E(θ) → Optimize parameters → θ*

[Peruzzo et al., arxiv:1304.3061]

Quantum Computing

Variational Quantum Eigensolver - VQE



$$E_0 \leq E_{\text{var}} = \frac{\left\langle \psi\left(\theta^\star\right) | \hat{H} | \psi\left(\theta^\star\right) \right\rangle}{\left\langle \psi\left(\theta^\star\right) | \psi\left(\theta^\star\right) \right\rangle}$$

Credits: 1Qbit docs

PQC → Variational Ansatz for |Φ⟩ → Evaluation of cost function → E(θ) → Optimize parameters → θ*

https://github.com/erinaldi/bmn2-qiskit

[Rinaldi et al., arxiv:2108.02942]

Qiskit

VQE details

Small-scale: N=2, D=2, Λ → ∞

$\Lambda = 2$

$\log_2 \Lambda^6 = 6$ qubits

https://github.com/erinaldi/bmn2-qiskit

[Rinaldi et al., arxiv:2108.02942]

Qiskit

VQE details

Small-scale: N=2, D=2, $\Lambda \to \infty$

$\Lambda = 2$ | $\log_2 \Lambda^6 = 6$ qubits → Choose Variational Ansatz

https://github.com/erinaldi/bmn2-qiskit

[Rinaldi et al., arxiv:2108.02942]

Qiskit

VQE details

Small-scale: N=2, D=2, Λ → ∞

$\Lambda = 2$

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Choose Variational Ansatz

PQC → Variational Ansatz for $|\Phi\rangle$

https://github.com/erinaldi/bmn2-qiskit

Qiskit

[Rinaldi et al., arxiv:2108.02942]

VQE details

Small-scale: N=2, D=2, $\Lambda \rightarrow \infty$

$\Lambda = 2$

$\log_2 \Lambda^6 = 6$ qubits

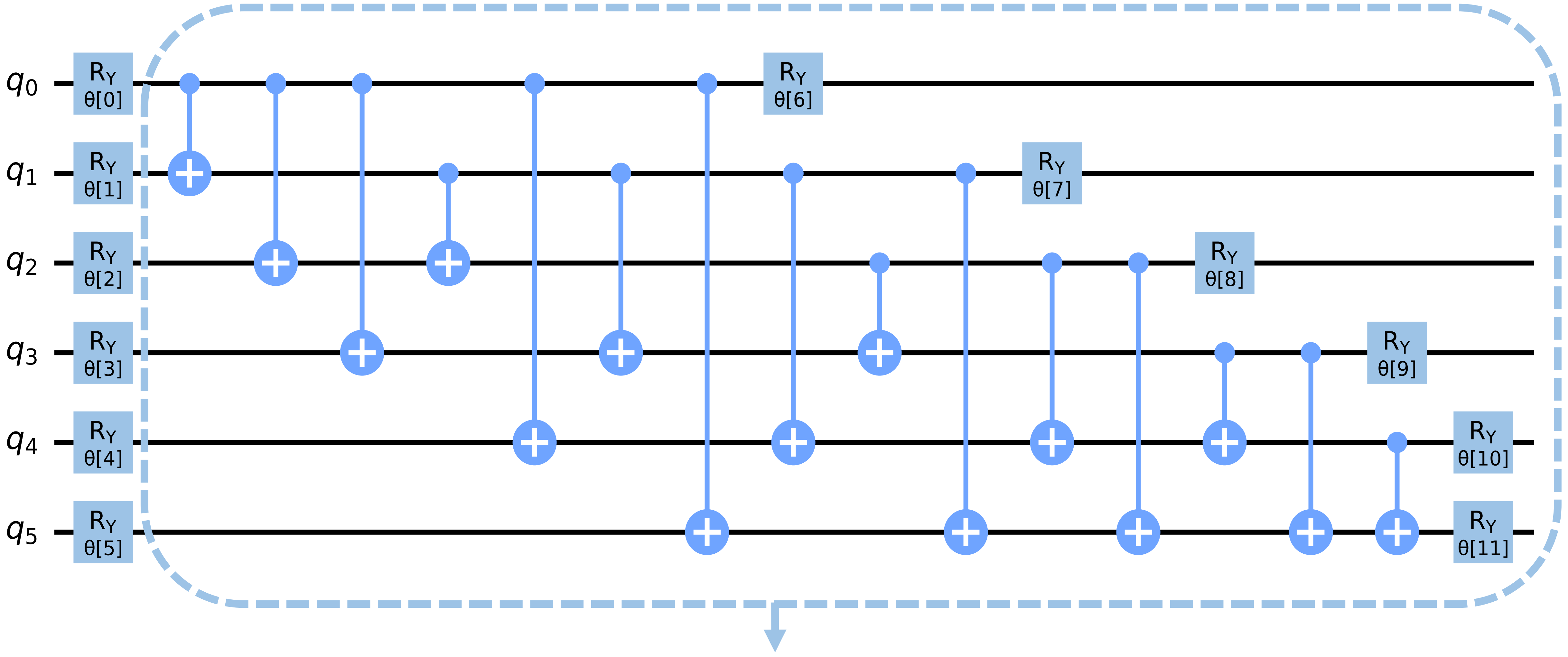
→

Choose Variational Ansatz

PQC → Variational Ansatz for $|\Phi\rangle$

depth = 1

parameters = 12



R_y Variational Form

https://github.com/erinaldi/bmn2-qiskit

Qiskit

[Rinaldi et al., arxiv:2108.02942]

VQE details

Small-scale: N=2, D=2, Λ → ∞

$\Lambda = 2$

$\log_2 \Lambda^6 = 6$ qubits

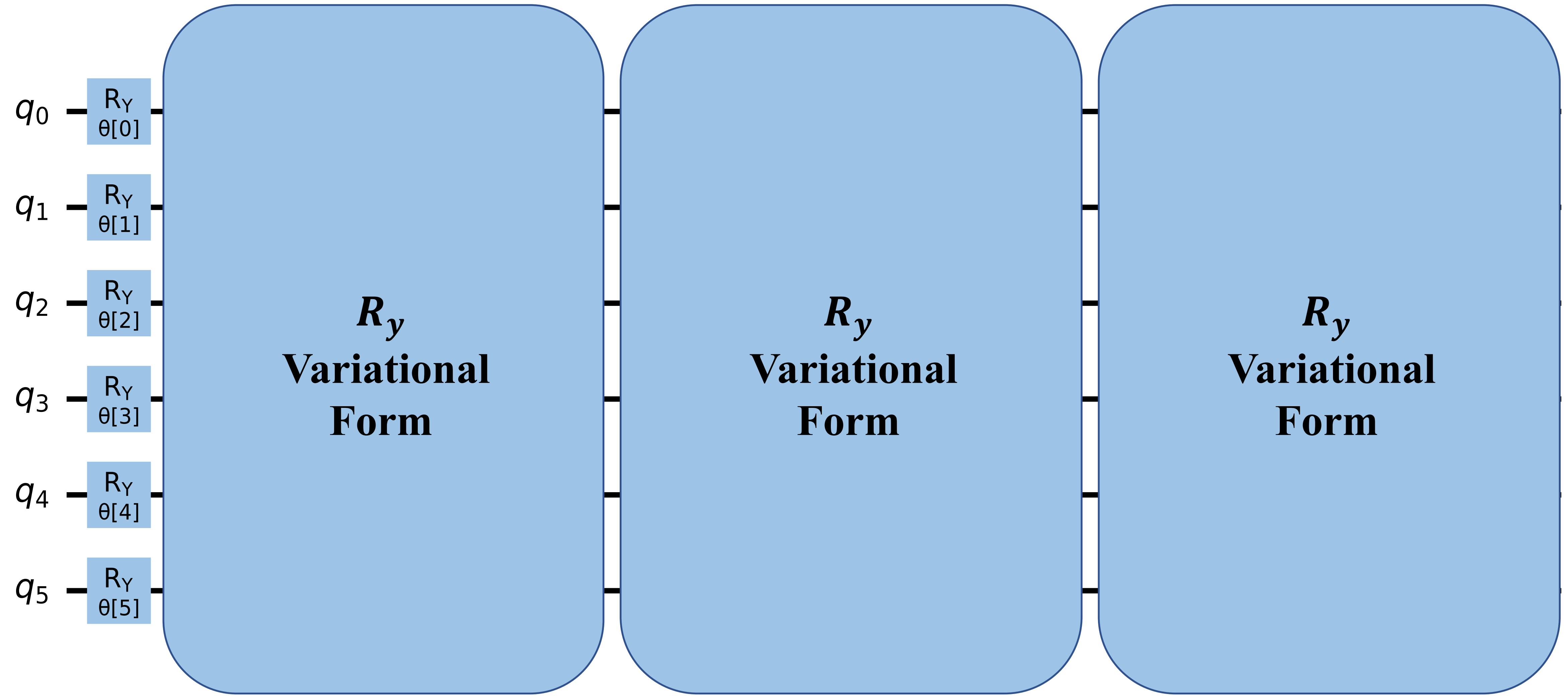
→

Choose Variational Ansatz

PQC → Variational Ansatz for $|\Phi\rangle$

depth = 3

parameters = 24



https://github.com/erinaldi/bmn2-qiskit

Qiskit

[Rinaldi et al., arxiv:2108.02942]

VQE details

Small-scale: N=2, D=2, Λ → ∞

Λ = 2

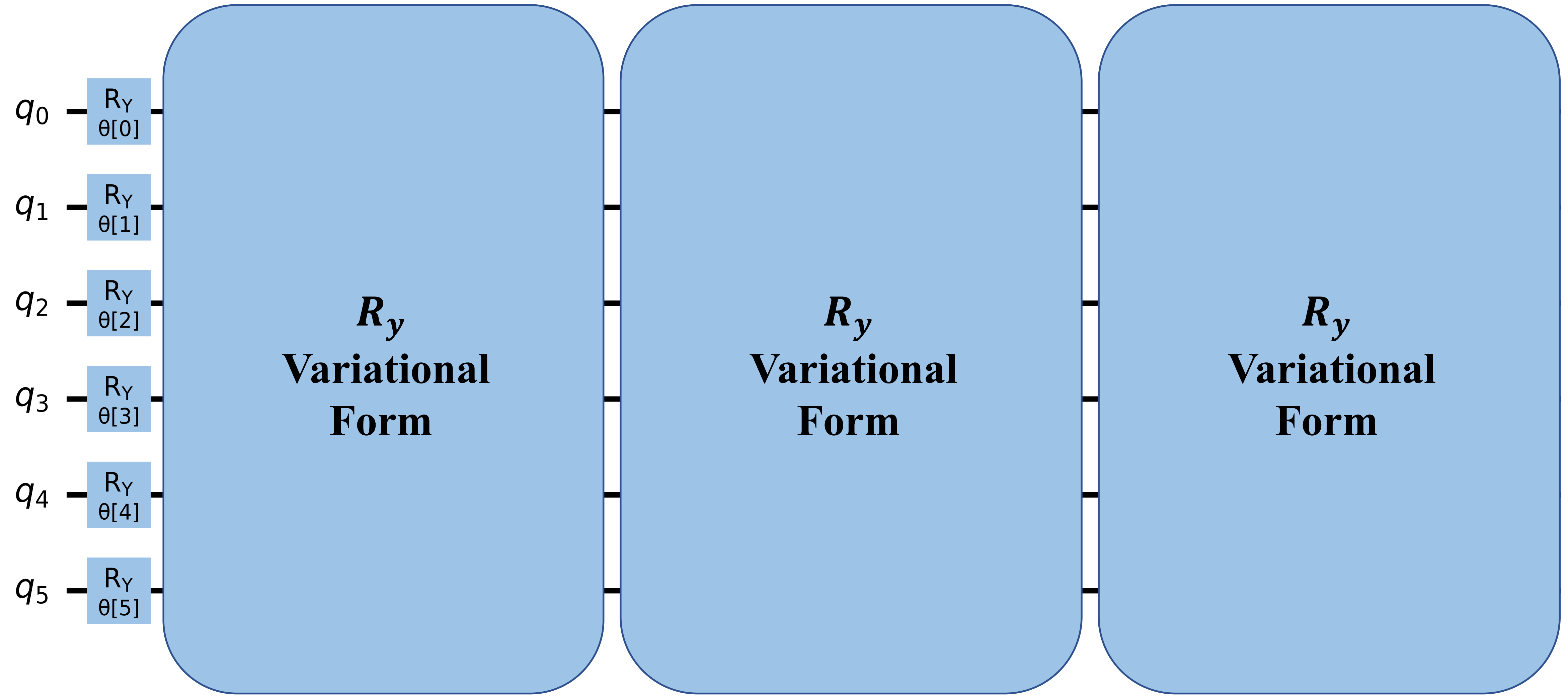
$\log_2 \Lambda^6$ = 6 qubits

→ Choose Variational Ansatz

PQC → Variational Ansatz for |Φ〉

depth = 3

parameters = 24



Run each multiple instances of PQC from different initial points

https://github.com/erinaldi/bmn2-qiskit

[Rinaldi et al., arxiv:2108.02942]

Qiskit

VQE details

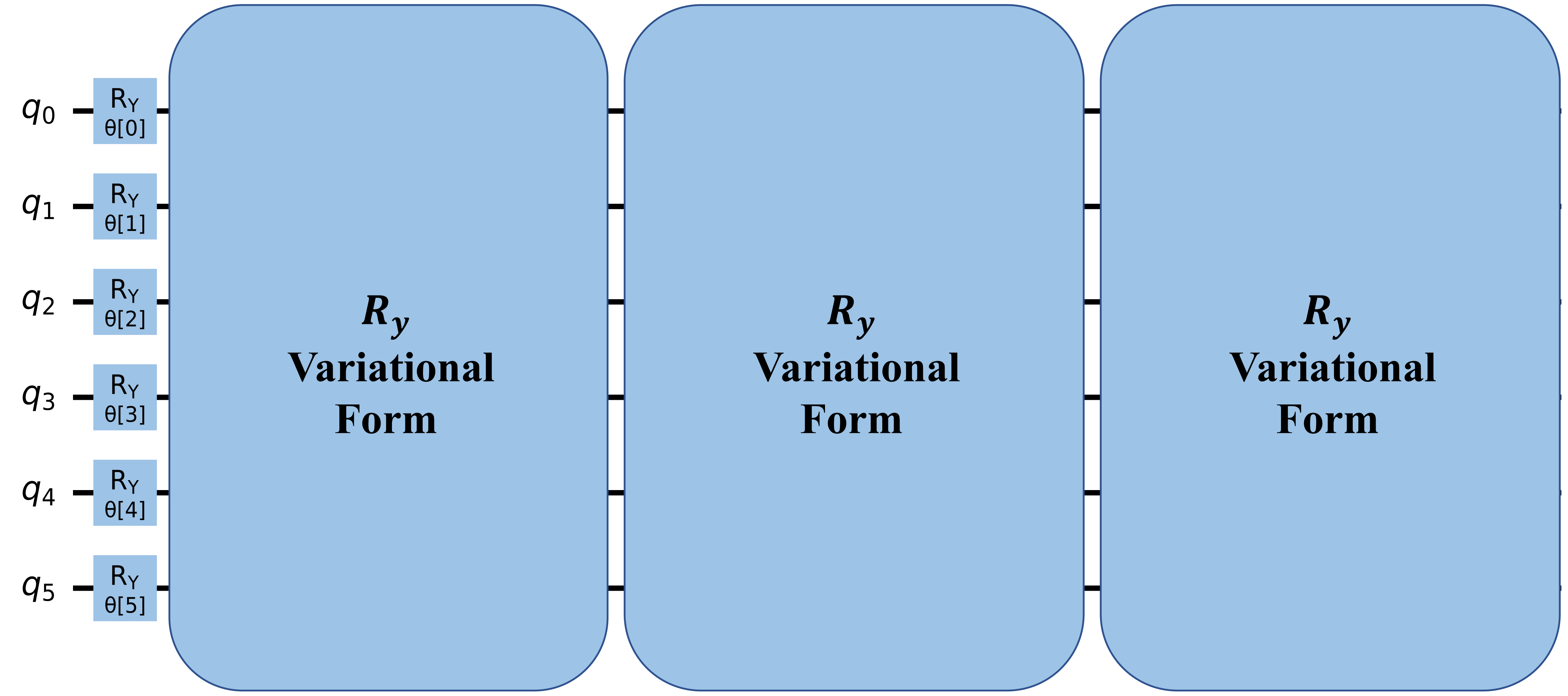
Small-scale: N=2, D=2, $\Lambda \to \infty$

$\Lambda = 2$ | $\log_2 \Lambda^6 = 6$ qubits → Choose Variational Ansatz → Choose Quantum Simulator

PQC → Variational Ansatz for $|\Phi\rangle$

depth = 3

parameters = 24



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Qiskit

[Rinaldi et al., arxiv:2108.02942]

VQE details

Small-scale: N=2, D=2, $\Lambda \to \infty$

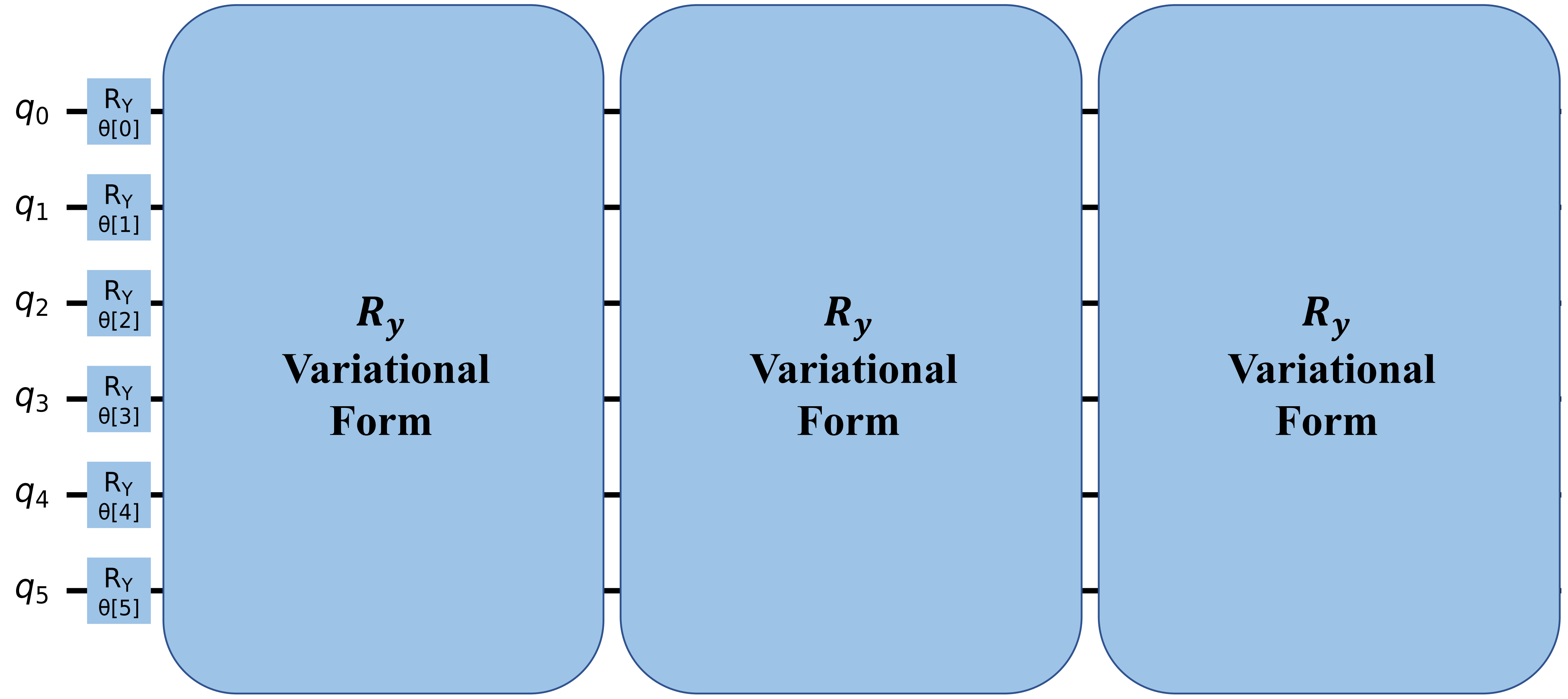
$\Lambda = 2$ | $\log_2 \Lambda^6 = 6$ qubits → Choose Variational Ansatz → Choose Quantum Simulator

PQC → Variational Ansatz for $|\Phi\rangle$

depth = 3

parameters = 24

Evaluation of cost function → $E(\theta)$



- Statevector simulator

Run each multiple instances of PQC from different initial points

https://github.com/erinaldi/bmn2-qiskit

[Rinaldi et al., arxiv:2108.02942]

Qiskit

VQE details

Small-scale: N=2, D=2, Λ → ∞

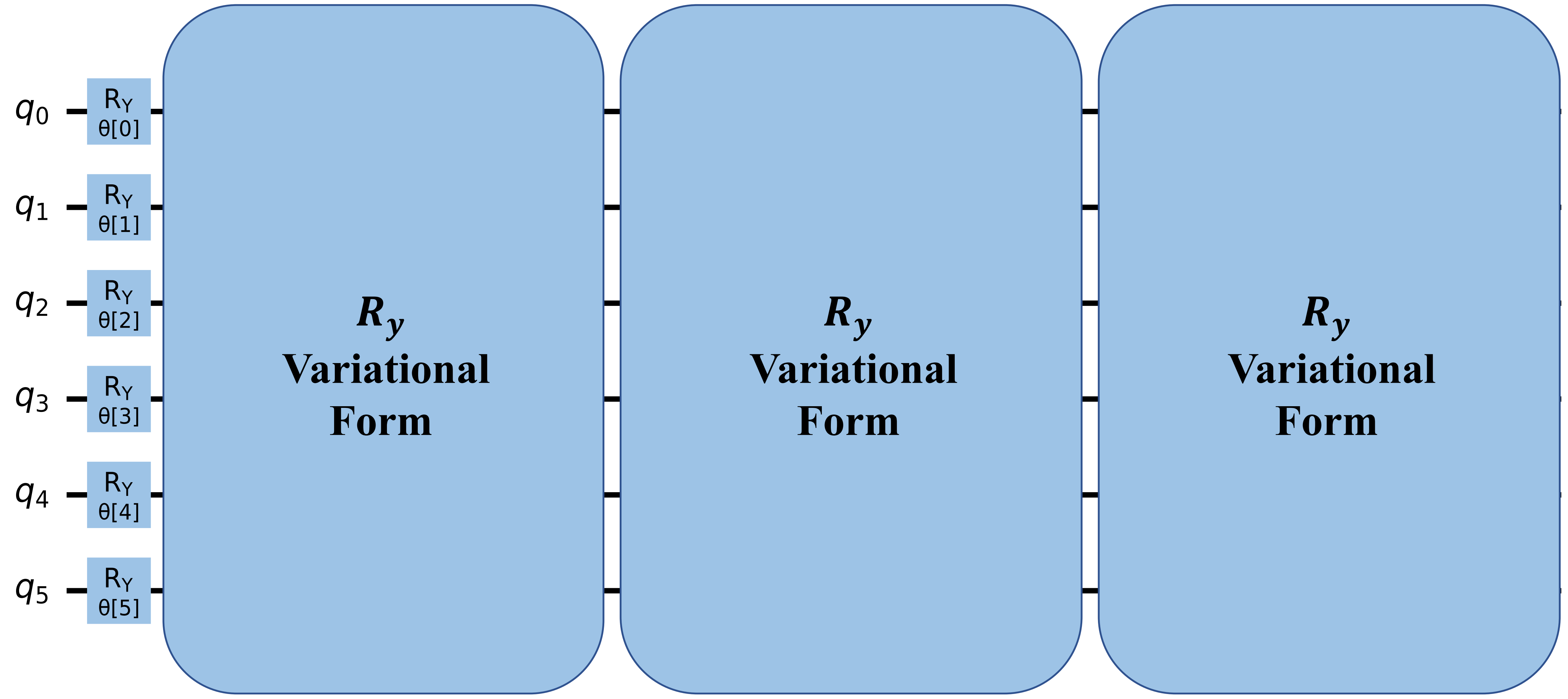
Λ = 2 | $\log_2 \Lambda^6$ = 6 qubits → Choose Variational Ansatz → Choose Quantum Simulator → Choose Classical Optimizer

PQC → Variational Ansatz for |Φ〉

depth = 3

parameters = 24

Evaluation of cost function → E(θ)



- Statevector simulator

Run each multiple instances of PQC from different initial points

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[Rinaldi et al., arxiv:2108.02942]

Qiskit

VQE details

Small-scale: N=2, D=2, Λ → ∞

$\Lambda = 2$ | $\log_2 \Lambda^6 = 6$ qubits → Choose Variational Ansatz → Choose Quantum Simulator → Choose Classical Optimizer

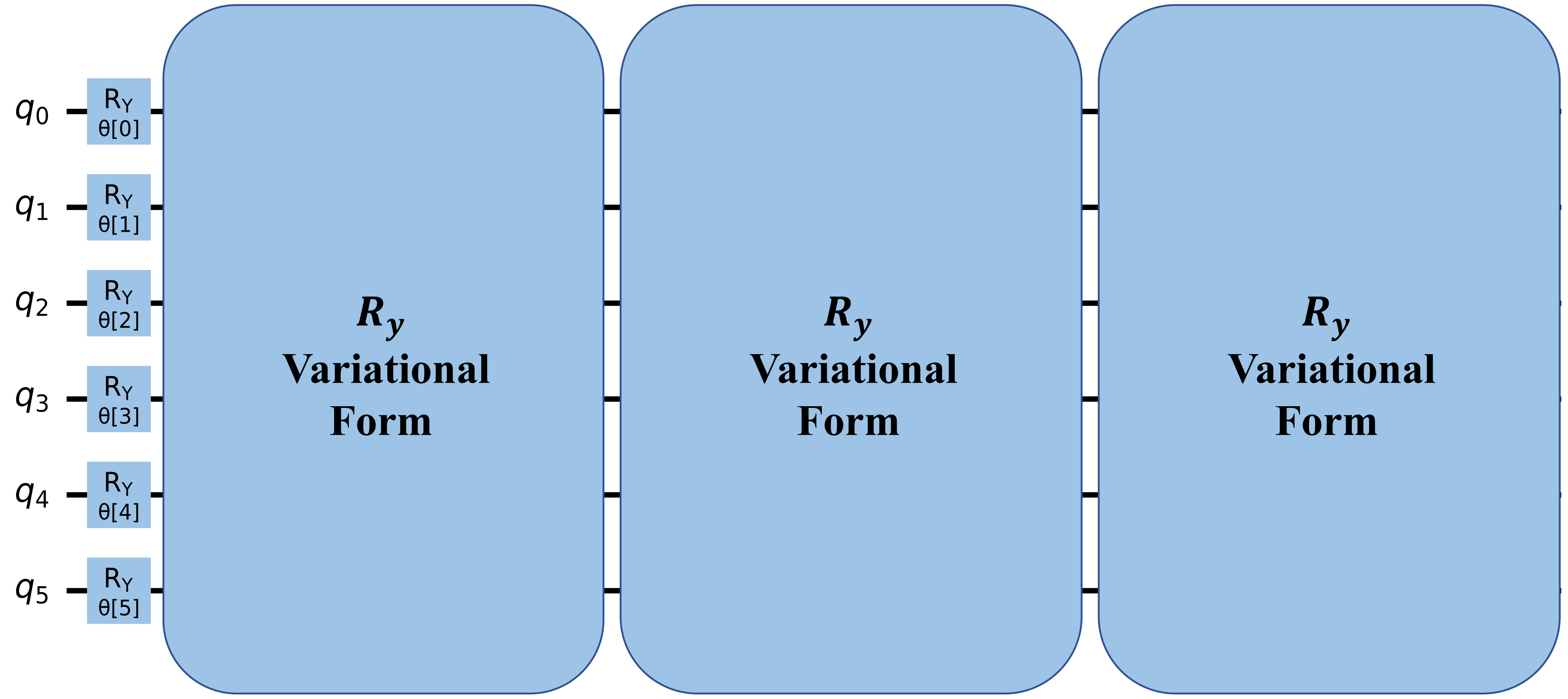
PQC → Variational Ansatz for |Φ 〉

depth = 3

parameters = 24

Evaluation of cost function → E(θ)

Optimize parameters → θ*



- Statevector simulator

- Least SQuares Programming optimizer (SLSQP)
- Constrained Optimization By Linear Approximation optimizer (COBYLA)
- Limited-memory BFGS Bound optimizer (L-BFGS-B)
- Nelder-Mead

Run each optimizer with a max. number of iterations

Run each multiple instances of PQC from different initial points

https://github.com/erinaldi/bmn2-qiskit

[Rinaldi et al., arxiv:2108.02942]

Qiskit

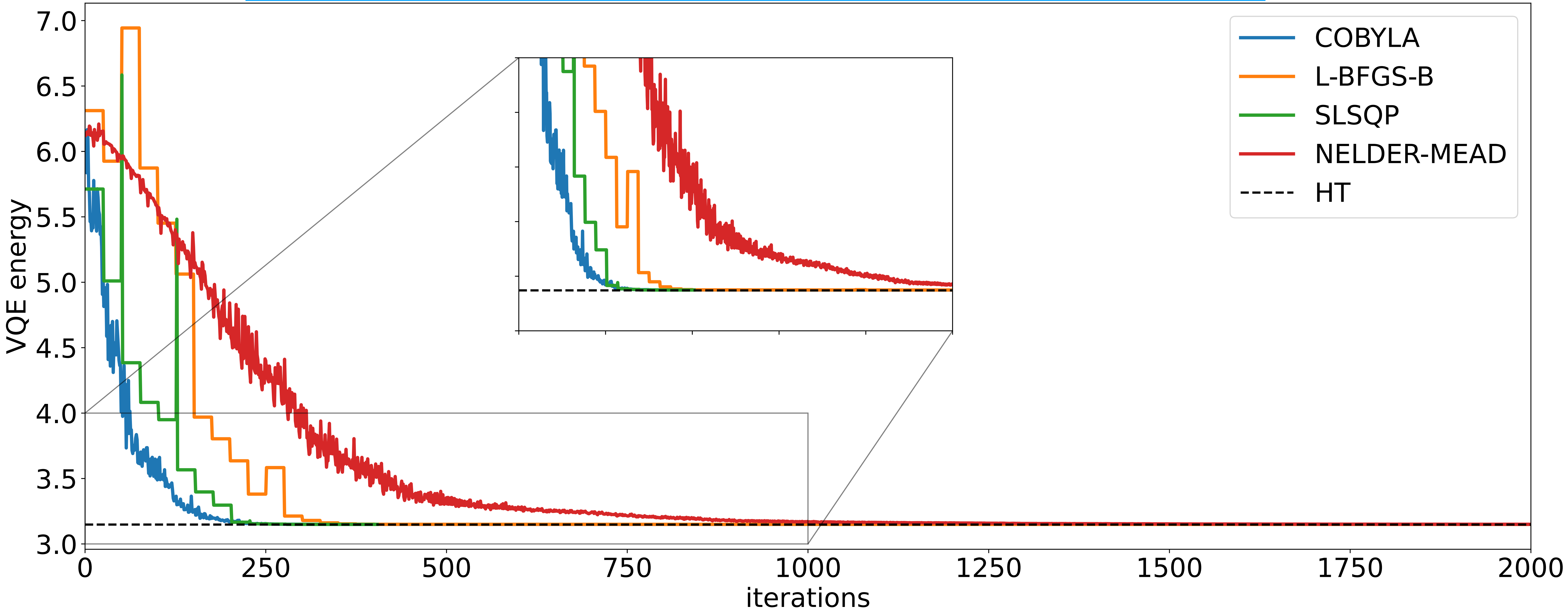
Results

Small-scale: N=2, D=2, $\Lambda \rightarrow \infty$

$\Lambda = 2$ | $\log_2 \Lambda^6 = 6$ qubits

Optimizer	Var. form: R_y				Var. form: $R_y R_z$			
	Min.	Max.	Mean	Std.	Min.	Max.	Mean	Std.
COBYLA	3.149370	4.147156	3.159740	0.099739	**3.149157**	3.150034	3.149862	0.000202
L-BFGS-B	3.149268	4.150000	3.159886	0.100012	3.149375	4.148751	3.159925	0.099882
SLSQP	3.149397	4.150000	3.164968	0.111340	3.149377	4.149946	3.164980	0.111349
NELDER-MEAD	**3.148972**	3.195922	3.150774	0.005065	3.149516	4.149891	3.171468	0.140469

PQC with y rotation gates: depth = 3 → 24 parameters | Best out of 100 runs



Best VQE (100 runs): $E_0 = 3.148972$

Exact Diagonalization: $E_0 = 3.14808$

https://github.com/erinaldi/bmn2-qiskit

[Rinaldi et al., arxiv:2108.02942]

Qiskit

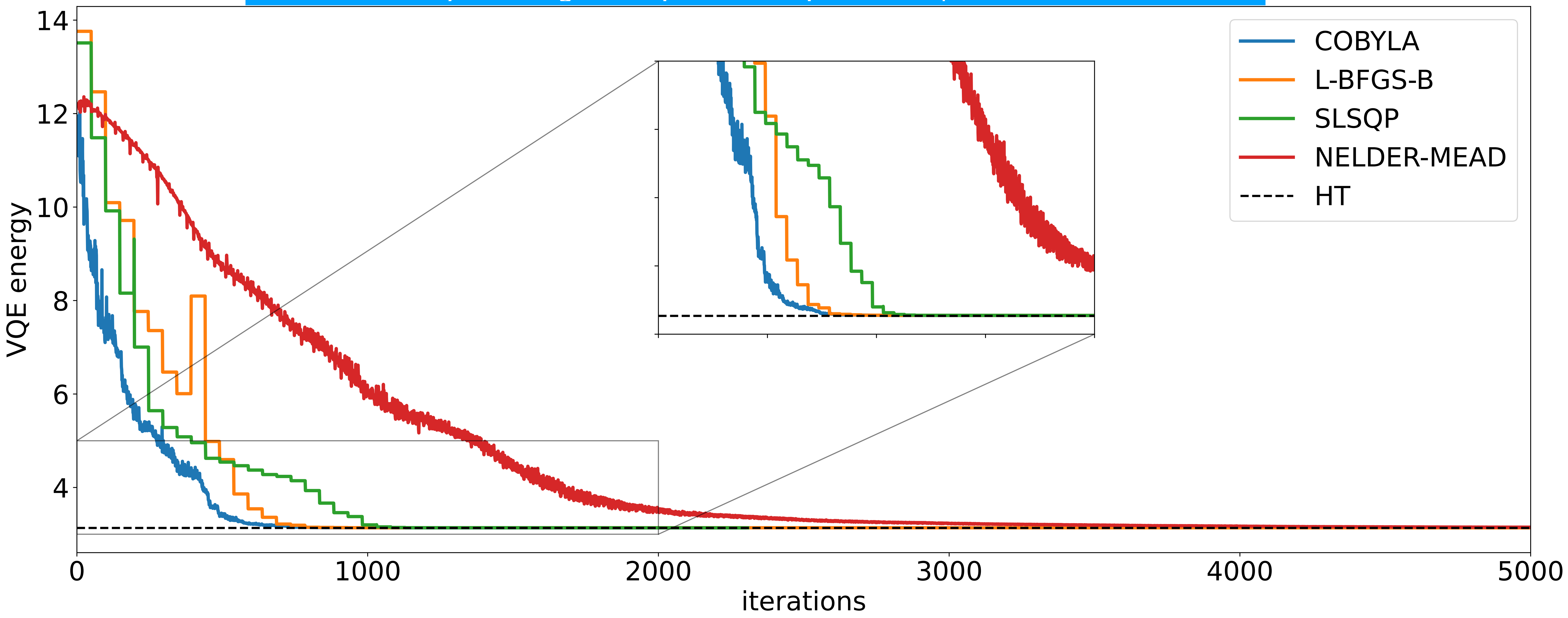
Results

Small-scale: N=2, D=2, $\Lambda \to \infty$

$\Lambda = 4$ | $\log_2 \Lambda^6$ = 12 qubits

Optimizer	Var. form: R_y				Var. form: $R_y R_z$			
	Min.	Max.	Mean	Std.	Min.	Max.	Mean	Std.
COBYLA	**3.137059**	4.769101	3.251414	0.347646	3.137237	4.782013	3.378628	0.472015
L-BFGS-B	**3.137059**	5.769553	3.283462	0.434162	**3.137050**	4.286367	3.243110	0.307549
SLSQP	3.137060	5.769554	3.327706	0.471957	3.137059	4.232419	3.236925	0.290855
NELDER-MEAD	3.137471	5.713976	3.492673	0.478810	3.273614	6.443055	4.428032	0.758732

PQC with y rotation gates: depth = 3 → 24 parameters | Best out of 100 runs



Best VQE (100 runs): $E_0 = 3.137$

Exact Diagonalization: $E_0 = 3.13406$

https://github.com/erinaldi/bmn2-qiskit

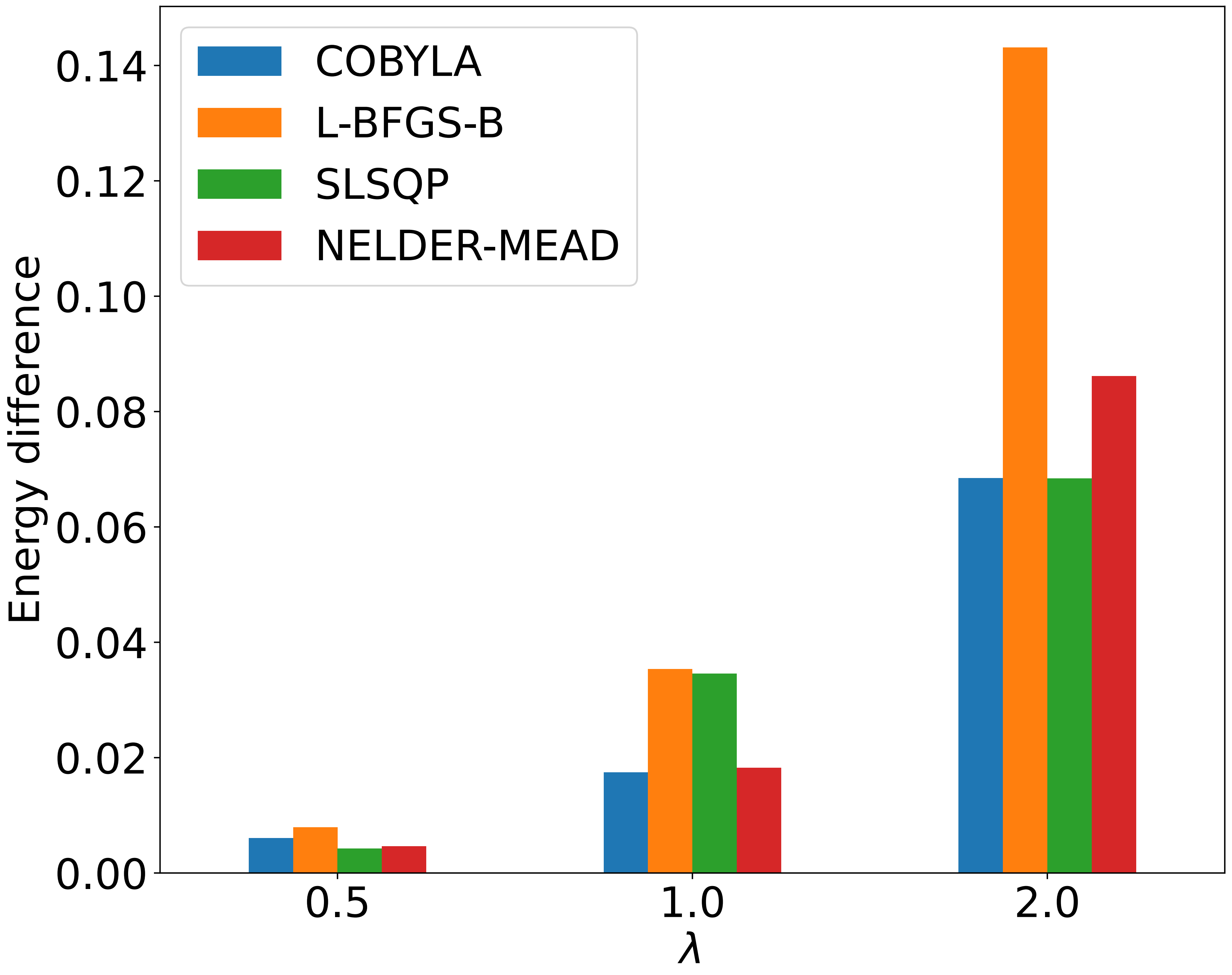
[Rinaldi et al., arxiv:2108.02942]

Qiskit

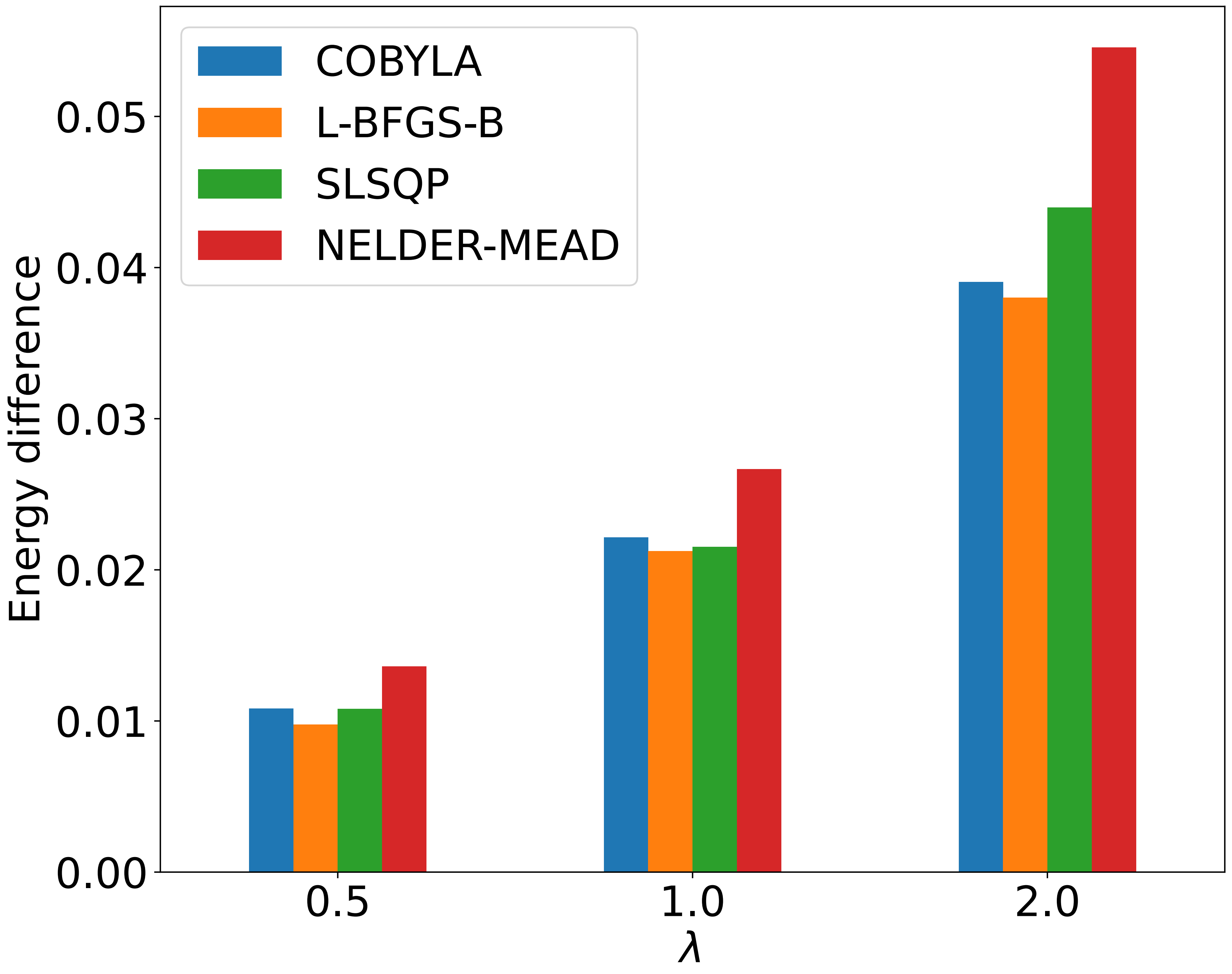
Results

Small-scale: N=2, D=2, $\Lambda \to \infty$

$\Lambda = 2$ | $\log_2 \Lambda^6 = 6$ qubits



$\Lambda = 4$ | $\log_2 \Lambda^6 = 12$ qubits



PQC with y rotation gates: depth = 3 → 24 parameters | Best out of 100 runs

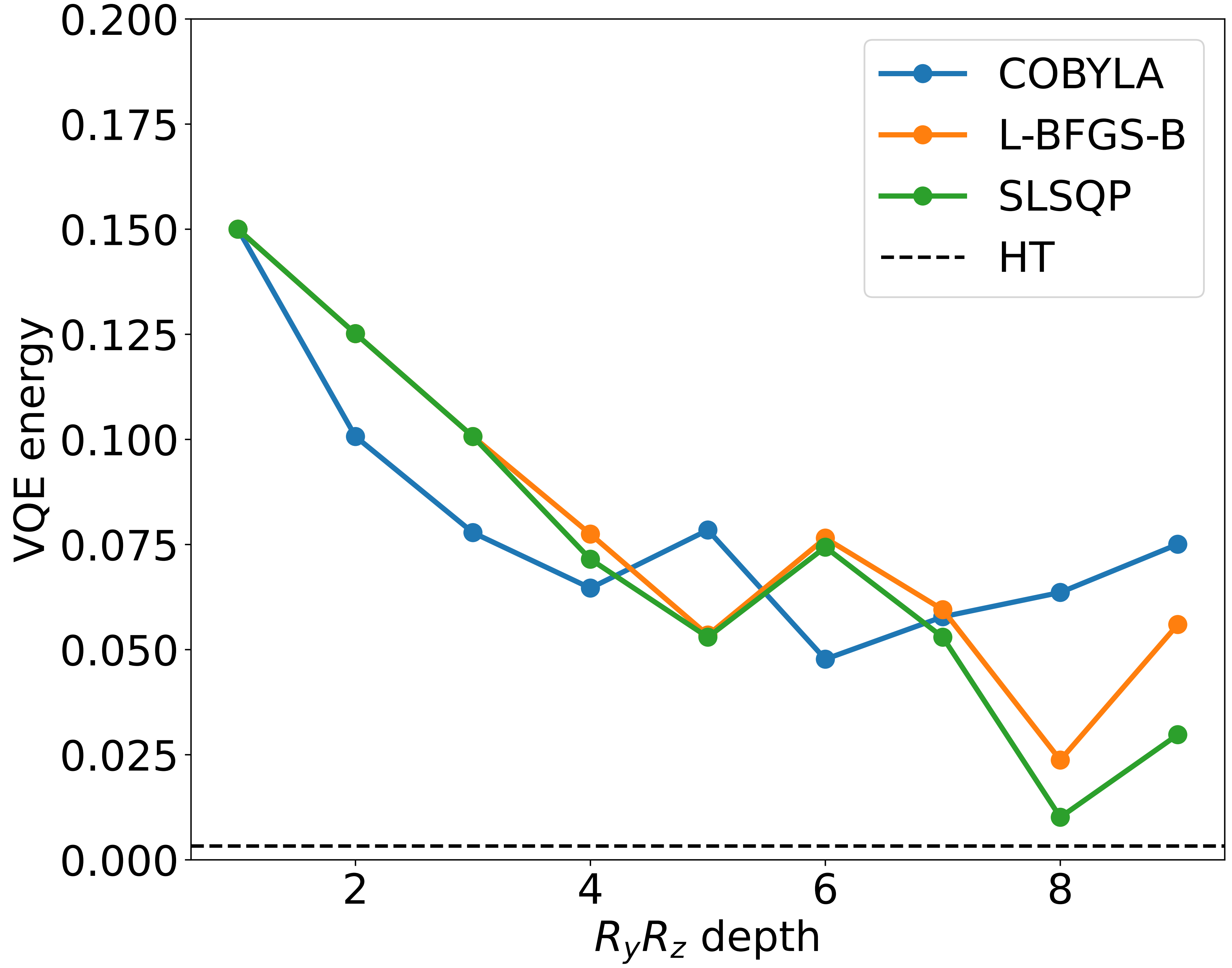
Qiskit

Results

Supersymmetric N=2 D=2 at large coupling

$\Lambda = 2$

$\log_2 \Lambda^6 = 9$ qubits

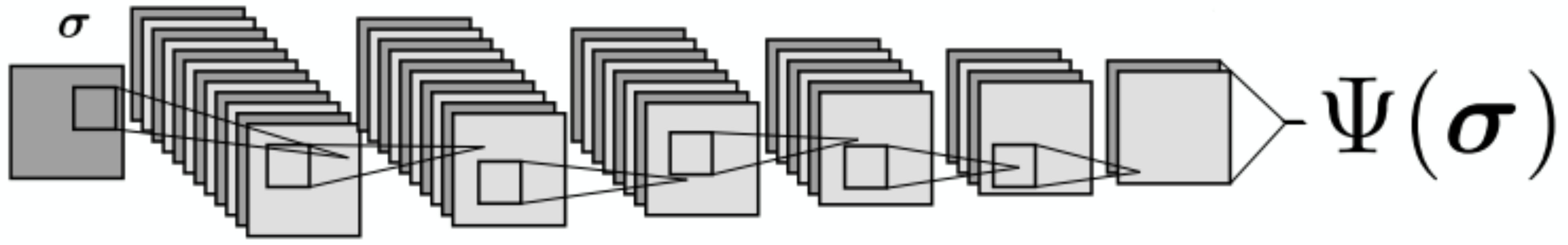


	depth = 5				depth = 9	
λ	COBYLA	L-BFGS-B	SLSQP	NELDER-MEAD	Best	HT (exact)
0.5	0.088492	0.139702	0.134517	0.406003	0.02744	0.01690
1.0	0.135800	0.219268	0.308781	0.752459	0.07900	0.04829
2.0	0.387977	0.622704	0.522396	1.271939	0.17688	0.08385

[Carleo and Troyer, Science 2017]

Deep Learning

Variational Quantum Monte Carlo with Neural Quantum States



$$\psi_\theta(X) = \langle X \mid \psi_\theta \rangle$$

$$E_\theta \equiv \left\langle \psi_\theta \middle| \hat{H} \middle| \psi_\theta \right\rangle = \int dX \left| \psi_\theta(X) \right|^2 \cdot \frac{\left\langle X \middle| \hat{H} \middle| \psi_\theta \right\rangle}{\psi_\theta(X)} = \mathbf{E}_{X \sim |\psi_\theta|^2} \left[\epsilon_\theta(X) \right]$$

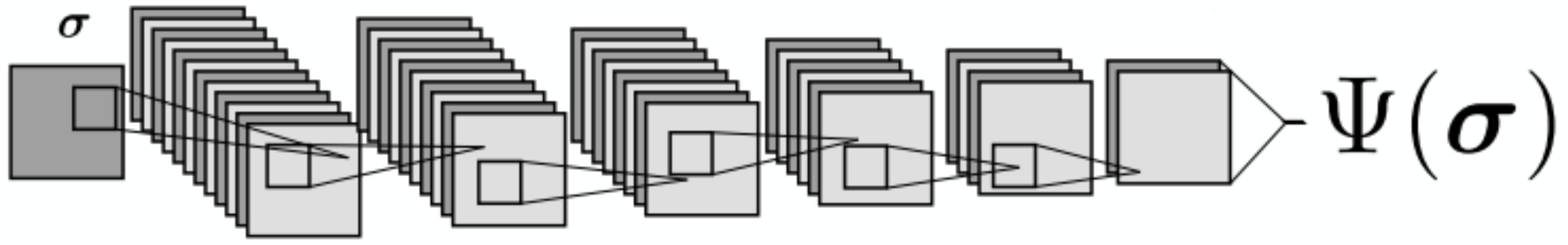
$$\nabla_\theta E_\theta = \mathbf{E}_{X \sim |\psi_\theta|^2} \left[\nabla_\theta \epsilon_\theta(X) \right] + \mathbf{E}_{X \sim |\psi_\theta|^2} \left[\epsilon_\theta(X) \, \nabla_\theta \ln \left| \psi_\theta \right|^2 \right]$$

$$\theta' = \theta - \beta \nabla_\theta E_\theta$$

[Carleo and Troyer, Science 2017]

Deep Learning

Variational Quantum Monte Carlo with Neural Quantum States



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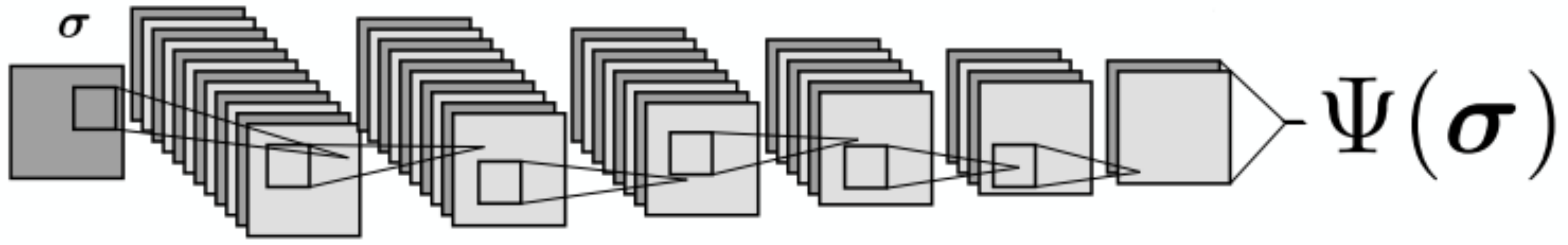
$$\theta' = \theta - \beta \nabla_\theta E_\theta$$

NQS → Variational Ansatz for |Φ 〉 → Evaluation of cost function → E(θ) → Optimize parameters → θ*

[Carleo and Troyer, Science 2017]

Deep Learning

Variational Quantum Monte Carlo with Neural Quantum States



$$\psi_\theta(X) = \langle X \mid \psi_\theta \rangle$$

Choice of Neural Network Architecture

$$E_\theta \equiv \langle \psi_\theta | \hat{H} | \psi_\theta \rangle = \int dX \left| \psi_\theta(X) \right|^2 \cdot \frac{\langle X | \hat{H} | \psi_\theta \rangle}{\psi_\theta(X)} = \mathbf{E}_{X \sim |\psi_\theta|^2} \left[\epsilon_\theta(X) \right]$$

$$\nabla_\theta E_\theta = \mathbf{E}_{X \sim |\psi_\theta|^2} \left[\nabla_\theta \epsilon_\theta(X) \right] + \mathbf{E}_{X \sim |\psi_\theta|^2} \left[\epsilon_\theta(X) \nabla_\theta \ln \left| \psi_\theta \right|^2 \right]$$

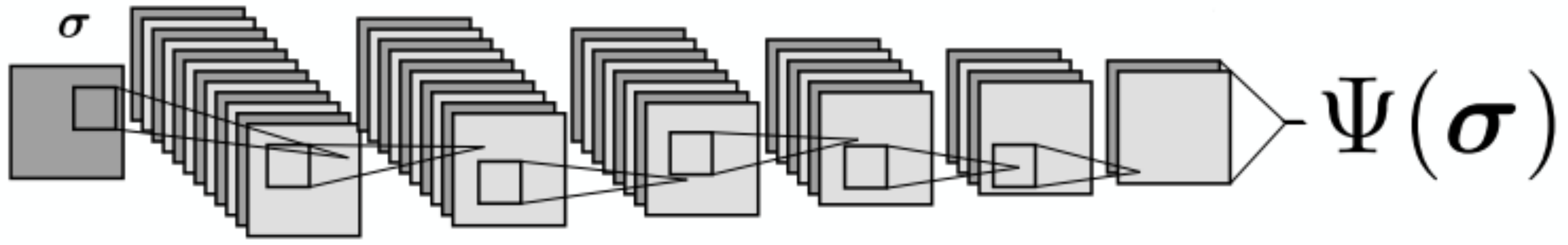
$$\theta' = \theta - \beta \nabla_\theta E_\theta$$

NQS → Variational Ansatz for |Φ〉 → Evaluation of cost function → E(θ) → Optimize parameters → θ*

[Carleo and Troyer, Science 2017]

Deep Learning

Variational Quantum Monte Carlo with Neural Quantum States



$$\psi_\theta(X) = \langle X \mid \psi_\theta \rangle$$

Choice of Neural Network Architecture

$$E_\theta \equiv \langle \psi_\theta | \hat{H} | \psi_\theta \rangle = \int dX \left| \psi_\theta(X) \right|^2 \cdot \frac{\langle X | \hat{H} | \psi_\theta \rangle}{\psi_\theta(X)} = \mathbf{E}_{X \sim |\psi_\theta|^2} \left[\epsilon_\theta(X) \right]$$

Choice of Monte Carlo Sampling

$$\nabla_\theta E_\theta = \mathbf{E}_{X \sim |\psi_\theta|^2} \left[\nabla_\theta \epsilon_\theta(X) \right] + \mathbf{E}_{X \sim |\psi_\theta|^2} \left[\epsilon_\theta(X) \, \nabla_\theta \ln \left| \psi_\theta \right|^2 \right]$$

$$\theta' = \theta - \beta \nabla_\theta E_\theta$$

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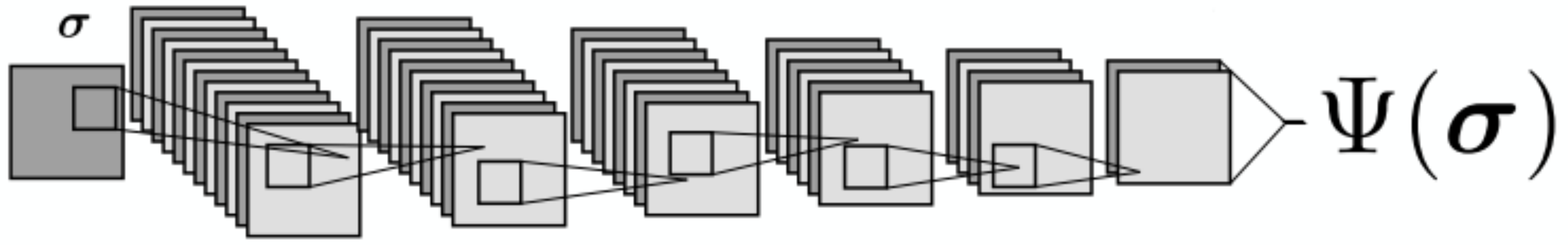
[Carleo and Troyer, Science 2017]

Deep Learning

Variational Quantum Monte Carlo with Neural Quantum States



$$\psi_\theta(X) = \langle X \mid \psi_\theta \rangle$$

Choice of Neural Network Architecture

$$E_\theta \equiv \langle \psi_\theta | \hat{H} | \psi_\theta \rangle = \int dX \left| \psi_\theta(X) \right|^2 \cdot \frac{\langle X | \hat{H} | \psi_\theta \rangle}{\psi_\theta(X)} = \mathbf{E}_{X \sim |\psi_\theta|^2} \left[\epsilon_\theta(X) \right]$$

Choice of Monte Carlo Sampling

$$\nabla_\theta E_\theta = \mathbf{E}_{X \sim |\psi_\theta|^2} \left[\nabla_\theta \epsilon_\theta(X) \right] + \mathbf{E}_{X \sim |\psi_\theta|^2} \left[\epsilon_\theta(X) \, \nabla_\theta \ln \left| \psi_\theta \right|^2 \right]$$

$$\theta' = \theta - \beta \nabla_\theta E_\theta$$

Choice of learning algorithm

NQS → Variational Ansatz for |Φ⟩ → Evaluation of cost function → E(θ) → Optimize parameters → θ*

Neural quantum state

Small-scale: N=2, D=2

Wave function

$$\psi(X) = |\psi(X)| e^{i\theta(X)}$$

Neural quantum state

Small-scale: N=2, D=2

Wave function

$$\psi(X) = |\psi(X)| e^{i\theta(X)} \longrightarrow |\psi(X)| = \sqrt{p_\theta(X)}$$

Neural quantum state

Small-scale: N=2, D=2

Wave function

$$\psi(X) = |\psi(X)|\, e^{i\theta(X)} \longrightarrow |\psi(X)| = \sqrt{p_\theta(X)}$$

$$p_\theta(X) = p\left(x_1; F_\theta^0\right) p\left(x_2; F_\theta^1\left(x_1\right)\right) p\left(x_3; F_\theta^2\left(x_1, x_2\right)\right)\ldots$$

Autoregressive Flow

https://github.com/hanxzh94/minimal_BMN_MQM

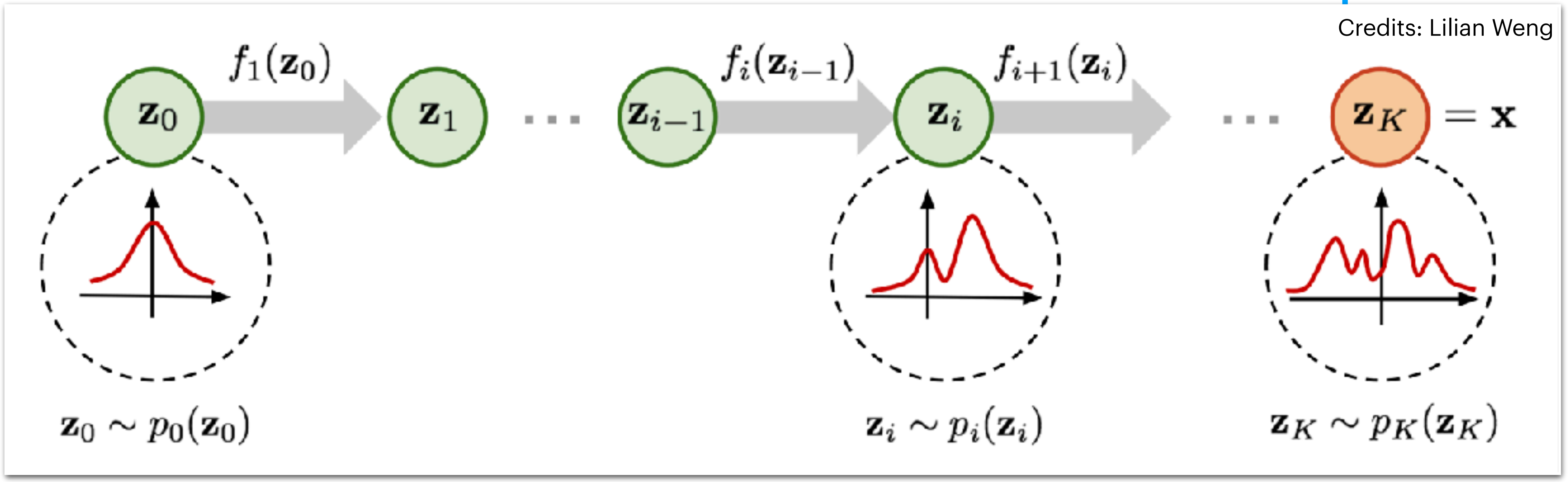
[Rinaldi et al., arxiv:2108.02942]

Neural quantum state

Small-scale: N=2, D=2

Wave function

$$\psi(X) = |\psi(X)| e^{i\theta(X)} \longrightarrow |\psi(X)| = \sqrt{p_\theta(X)}$$



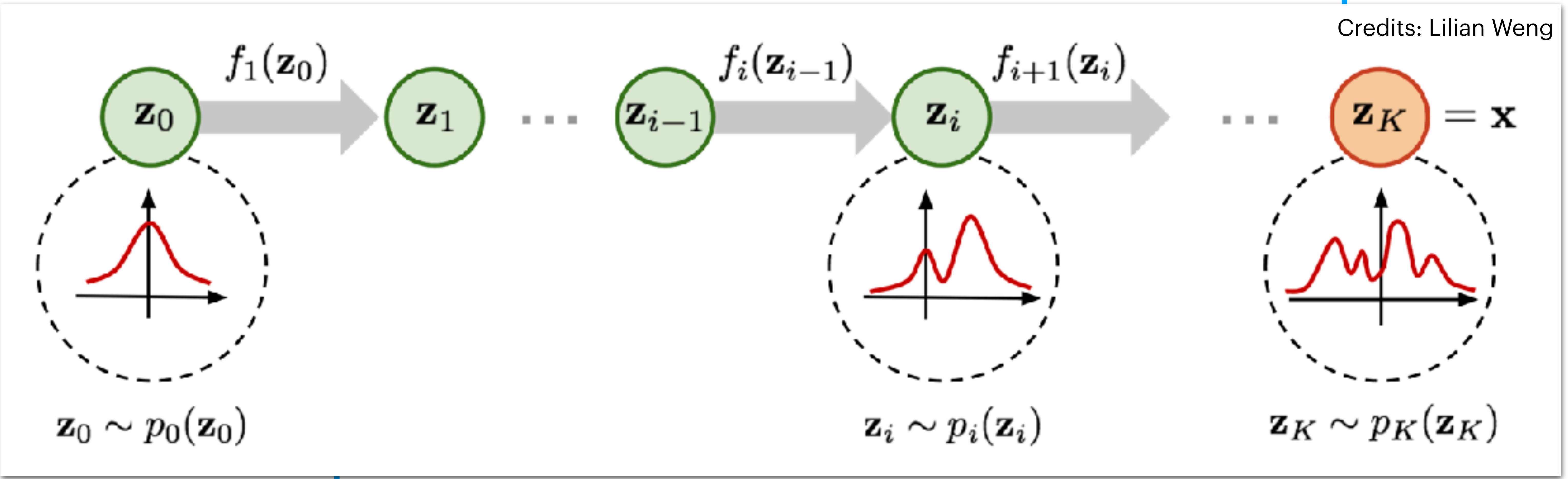
Autoregressive Flow

Neural quantum state

Small-scale: N=2, D=2

Wave function

$$\psi(X) = |\psi(X)| e^{i\theta(X)} \longrightarrow |\psi(X)| = \sqrt{p_\theta(X)}$$



Autoregressive Flow

Prob. distribution

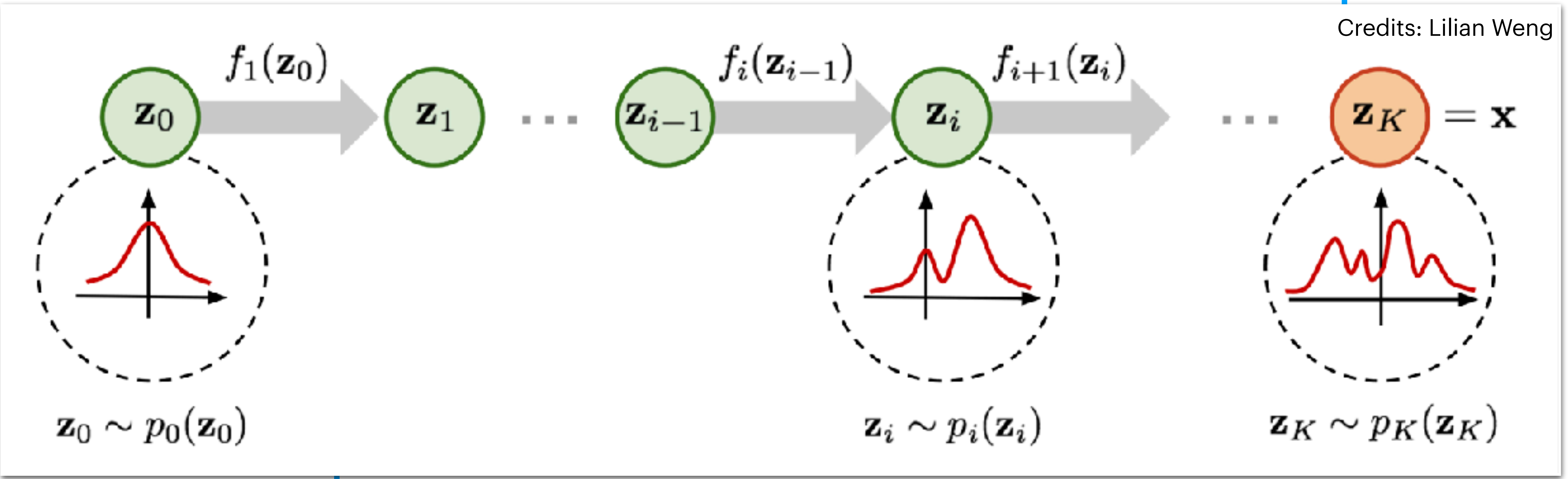
$$p\left(x_1; F_\theta^0\right)$$

Neural quantum state

Small-scale: N=2, D=2

Wave function

$$\psi(X) = |\psi(X)| e^{i\theta(X)} \longrightarrow |\psi(X)| = \sqrt{p_\theta(X)}$$



Autoregressive Flow

Parametrization

Prob. distribution

$$p\left(x_1; F_\theta^0\right) \longrightarrow F_\theta^i = A_\theta^{i,m} \circ \tanh \circ A_\theta^{i,m-1} \circ \tanh \circ \cdots \circ A_\theta^{i,2} \circ \tanh \circ A_\theta^{i,1}$$

$$A_\theta^{i,a}(\vec{x}) = M_\theta^{i,a} \cdot \vec{x} + \vec{b}_\theta^{i,a}$$

https://github.com/hanxzh94/minimal_BMN_MQM

[Rinaldi et al., arxiv:2108.02942]

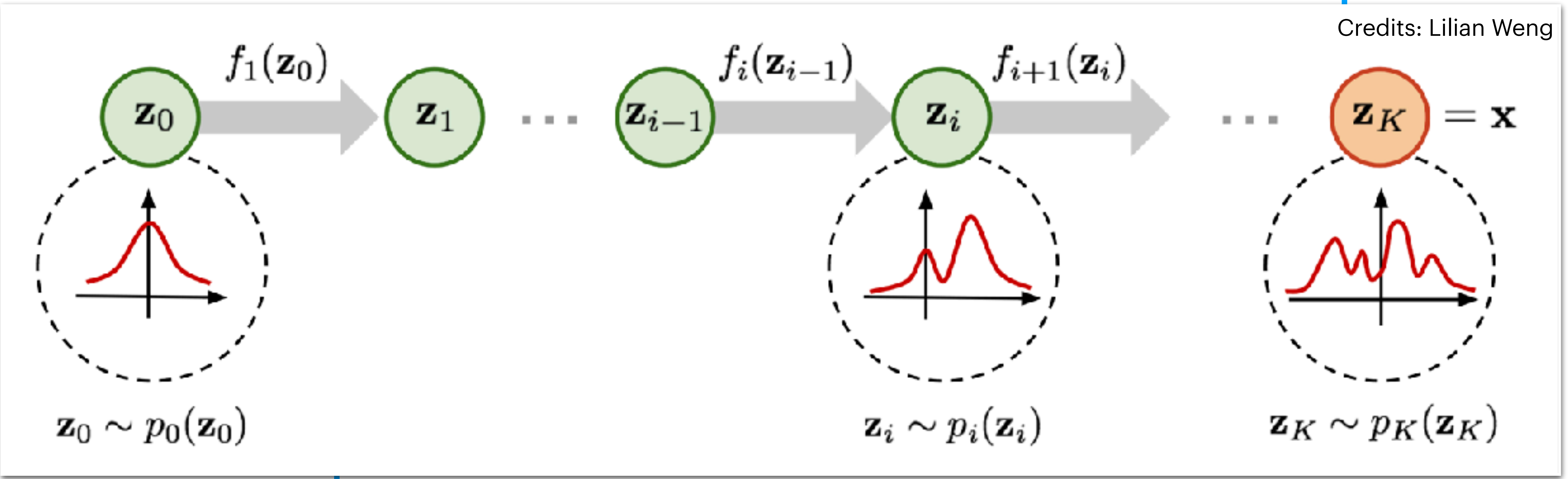
Neural quantum state

Small-scale: N=2, D=2

No truncation Λ

Wave function

$$\psi(X) = |\psi(X)|\, e^{i\theta(X)} \longrightarrow |\psi(X)| = \sqrt{p_\theta(X)}$$



Autoregressive Flow

Parametrization

Prob. distribution $p\left(x_1; F^0_\theta\right) \longrightarrow$

$$F^i_\theta = A^{i,m}_\theta \circ \tanh \circ A^{i,m-1}_\theta \circ \tanh \circ \cdots \circ A^{i,2}_\theta \circ \tanh \circ A^{i,1}_\theta$$

$$A^{i,a}_\theta(\vec{x}) = M^{i,a}_\theta \cdot \vec{x} + \vec{b}^{\,i,a}_\theta$$

https://github.com/hanxzh94/minimal_BMN_MQM

[Rinaldi et al., arxiv:2108.02942]

Neural quantum state

Small-scale: N=2, D=2

No truncation Λ

Direct Sampling

Wave function

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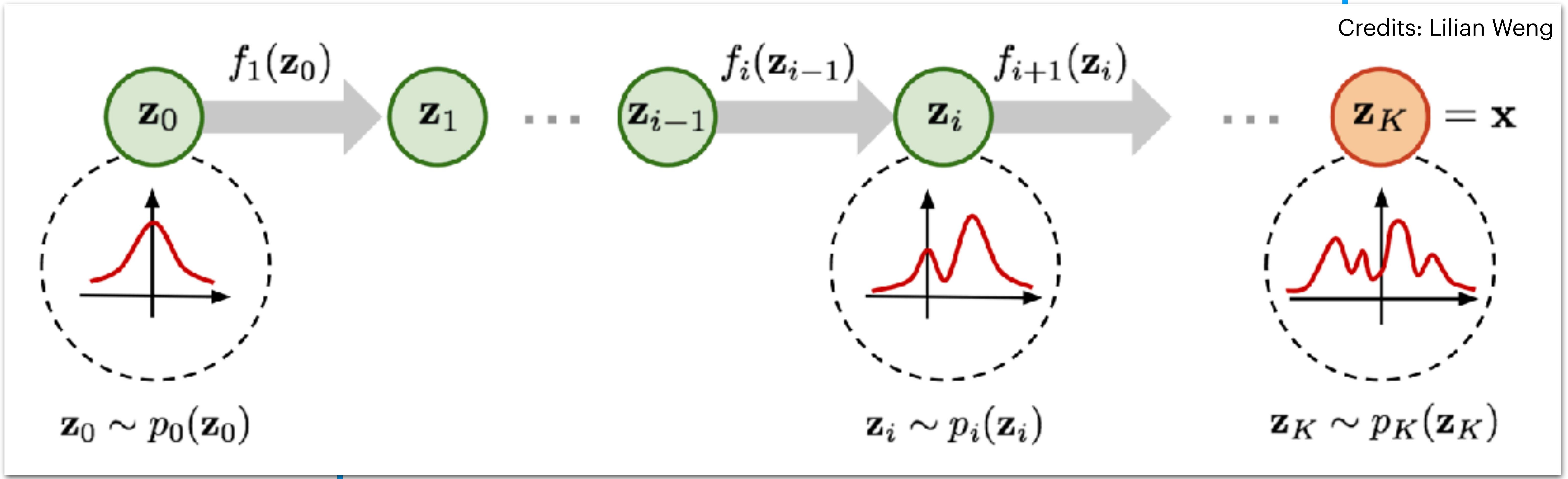

Credits: Lilian Weng

Autoregressive Flow

Parametrization

Prob. distribution

$$p\left(x_1; F_\theta^0\right) \longrightarrow F_\theta^i = A_\theta^{i,m} \circ \tanh \circ A_\theta^{i,m-1} \circ \tanh \circ \cdots \circ A_\theta^{i,2} \circ \tanh \circ A_\theta^{i,1}$$

$$A_\theta^{i,a}(\vec{x}) = M_\theta^{i,a} \cdot \vec{x} + \vec{b}_\theta^{i,a}$$

Results

Small-scale: N=2, D=2

θ

$$|\psi(X)| = \sqrt{p_\theta(X)}$$

$$X = [x_1, \ldots, x_6]$$

NQS → Variational Ansatz for |Φ〉

https://github.com/hanxzh94/minimal_BMN_MQM

[Rinaldi et al., arxiv:2108.02942]

Results

Small-scale: N=2, D=2

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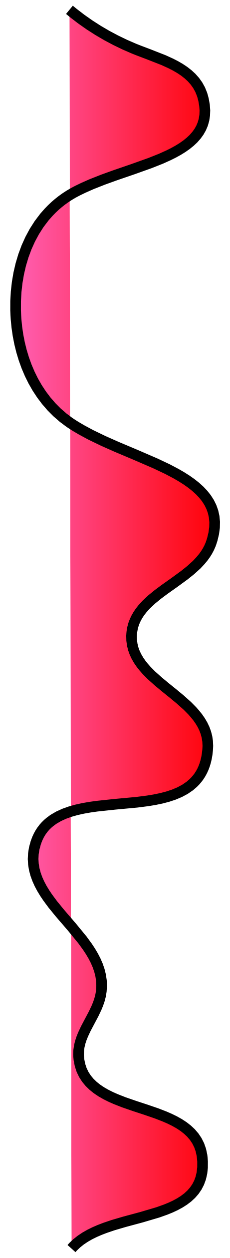
$$|\psi(X)| = \sqrt{p_\theta(X)}$$

$X = [x_1, \ldots, x_6]$

$$E_{\theta'} = \mathbf{E}_{X \sim |\psi_{\theta'}|^2} \left[\epsilon_{\theta'}(X) \right]$$

θ'



NQS → Variational Ansatz for |Φ〉

Evaluation of cost function → E(θ)

https://github.com/hanxzh94/minimal_BMN_MQM

[Rinaldi et al., arxiv:2108.02942]

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Small-scale: N=2, D=2

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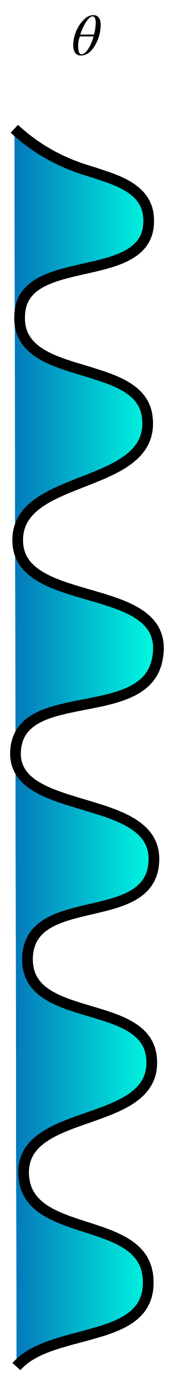

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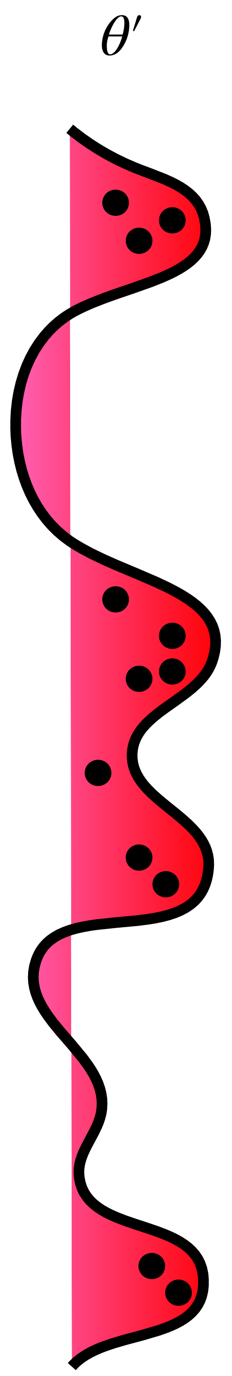

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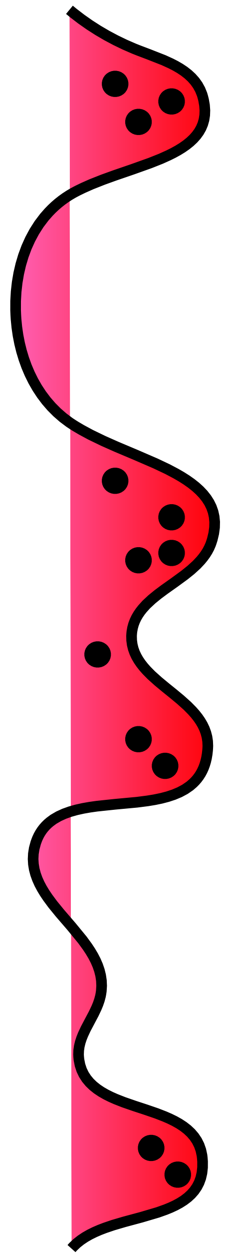
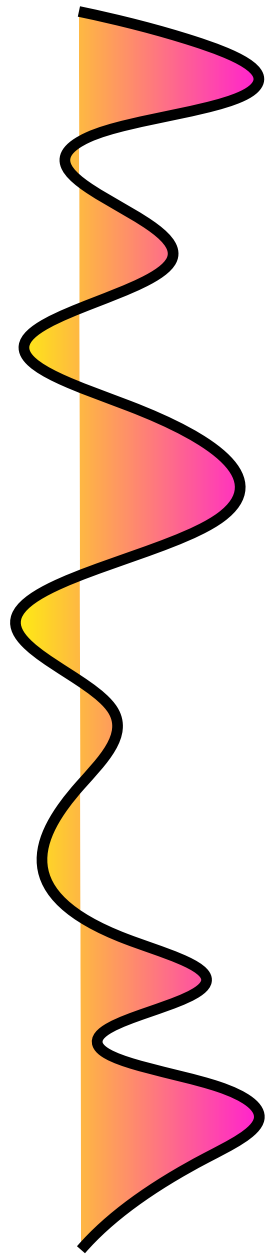


$\theta^\star$



$$|\mathrm{Ground\ State}\rangle = \psi_{\theta^\star}(X)$$

$$E_{\theta^\star} = \mathbf{E}_{X \sim |\psi_{\theta^\star}|^2} \left[\epsilon_{\theta^\star}(X)\right]$$

NQS → Variational Ansatz for |Φ 〉

Evaluation of cost function → E(θ)

Optimize parameters → θ*

Results

Small-scale: N=2, D=2

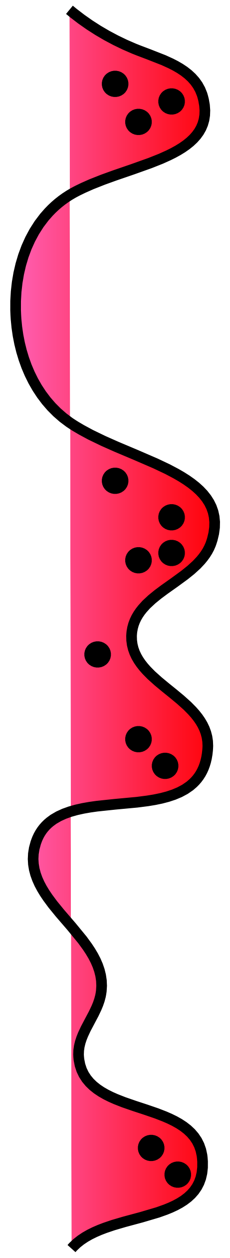
$|\psi(X)| = \sqrt{p_\theta(X)}$

$X = [x_1, \ldots, x_6]$

θ

$$E_{\theta'} = \mathbf{E}_{X\sim|\psi_{\theta'}|^2}\left[\epsilon_{\theta'}(X)\right]$$

θ'



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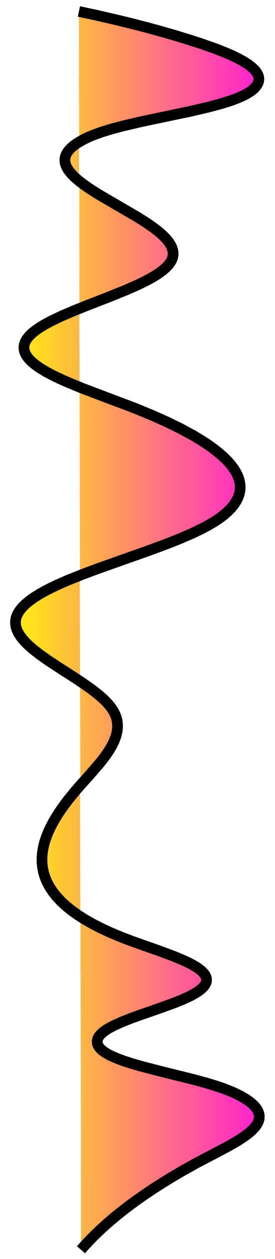


$|\text{Ground State}\rangle = \psi_{\theta^\star}(X)$

$$E_{\theta^\star} = \mathbf{E}_{X\sim|\psi_{\theta^\star}|^2}\left[\epsilon_{\theta^\star}(X)\right]$$

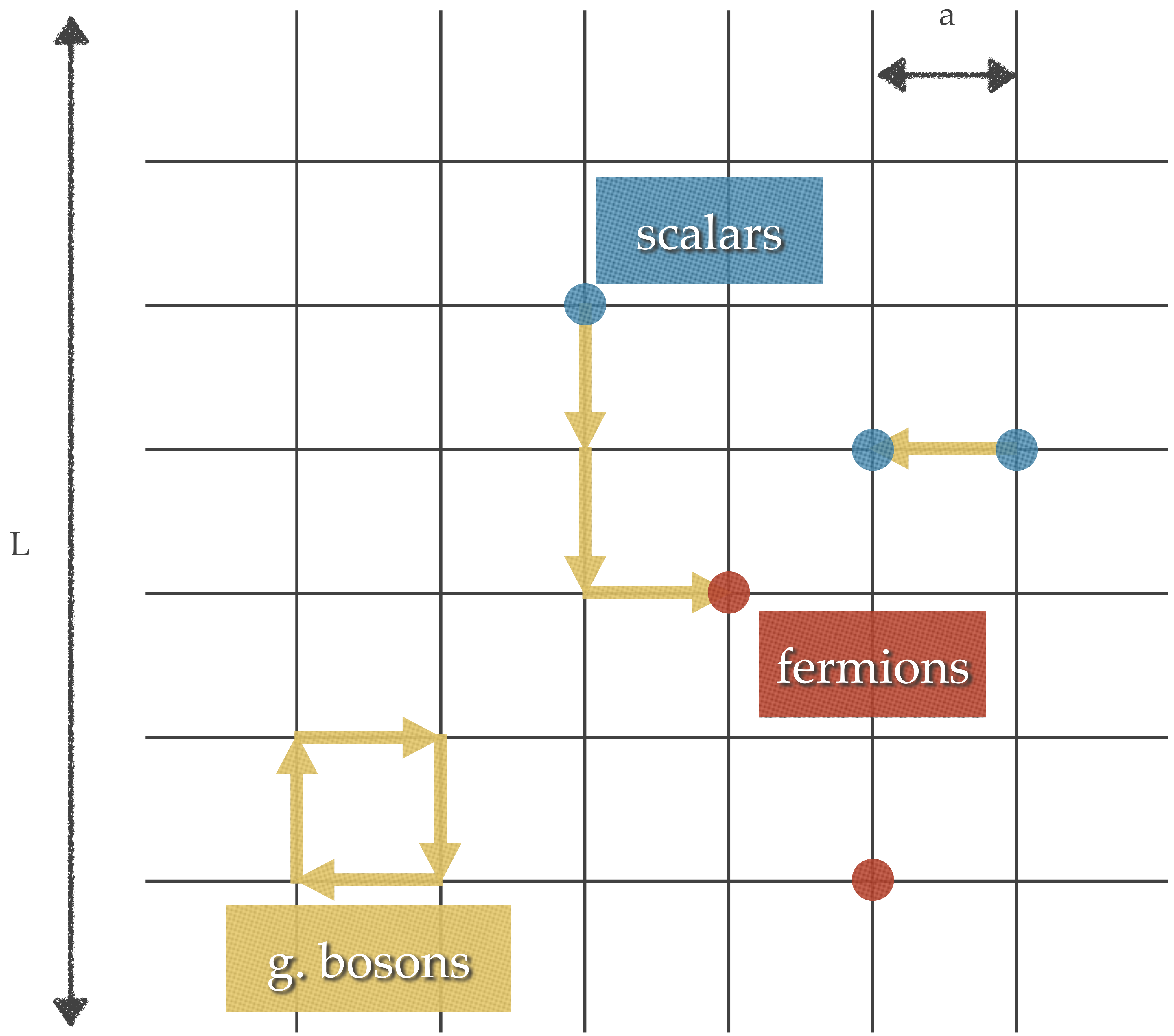
dependence on hidden layer units α

α	1	2	5	10	20	50	HT (exact)
$\lambda = 0.2$	3.137(2)	3.137(2)	3.140(2)	3.138(2)	3.137(2)	3.135(2)	3.134
$\lambda = 0.5$	3.313(2)	3.312(2)	3.308(2)	3.307(2)	3.302(2)	3.305(2)	3.297
$\lambda = 1.0$	3.544(3)	3.544(2)	3.541(3)	3.528(2)	3.519(2)	3.520(2)	3.516
$\lambda = 2.0$	3.914(3)	3.910(3)	3.892(3)	3.872(3)	3.857(3)	3.859(3)	3.854

[Rinaldi et al., arxiv:2108.02942]

Path Integral Monte Carlo

Lattice Gauge Theory Primer



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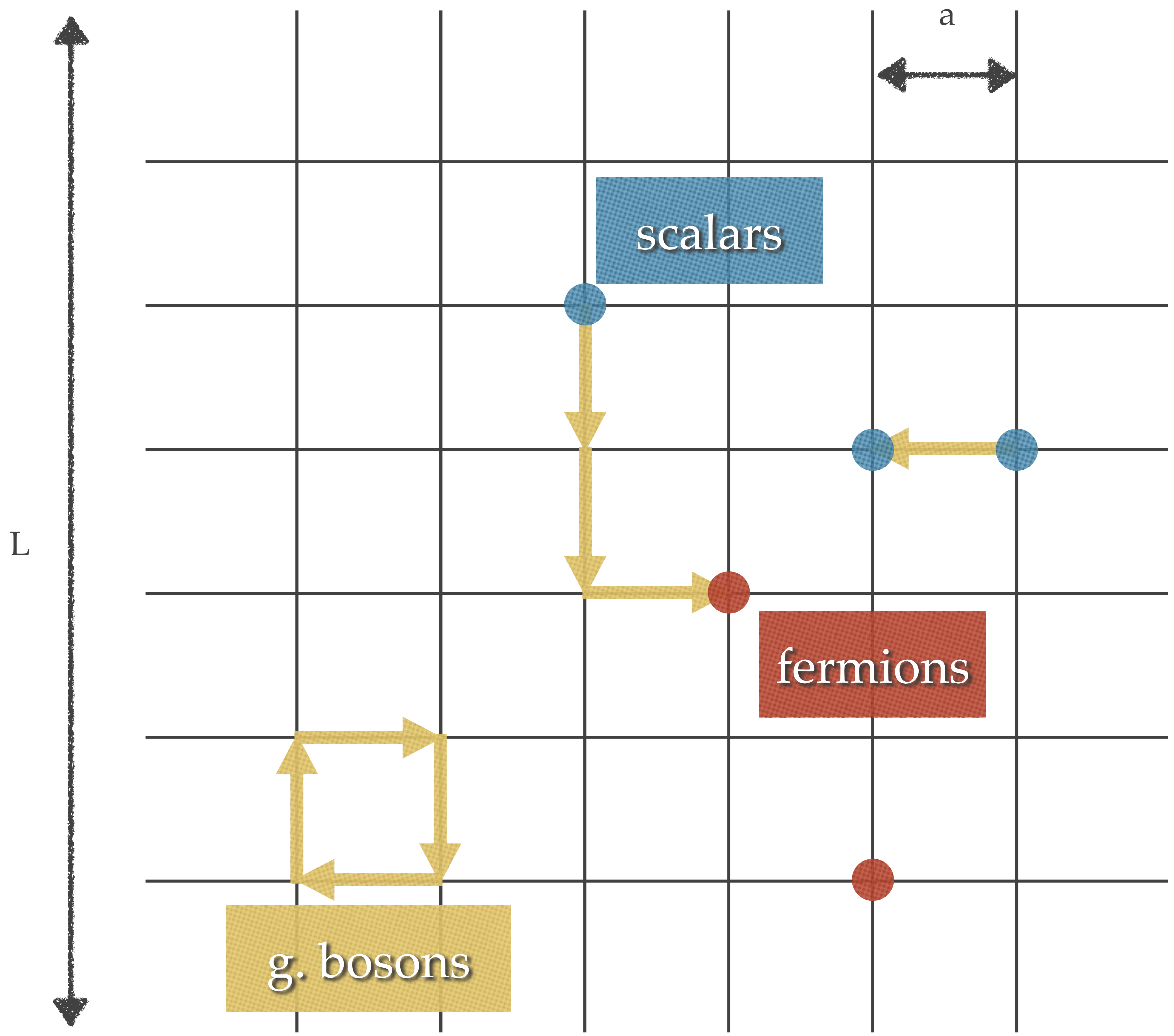

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  - lattice spacing “a”
  - lattice size “L”

[Rinaldi et al., arxiv:2108.02942]

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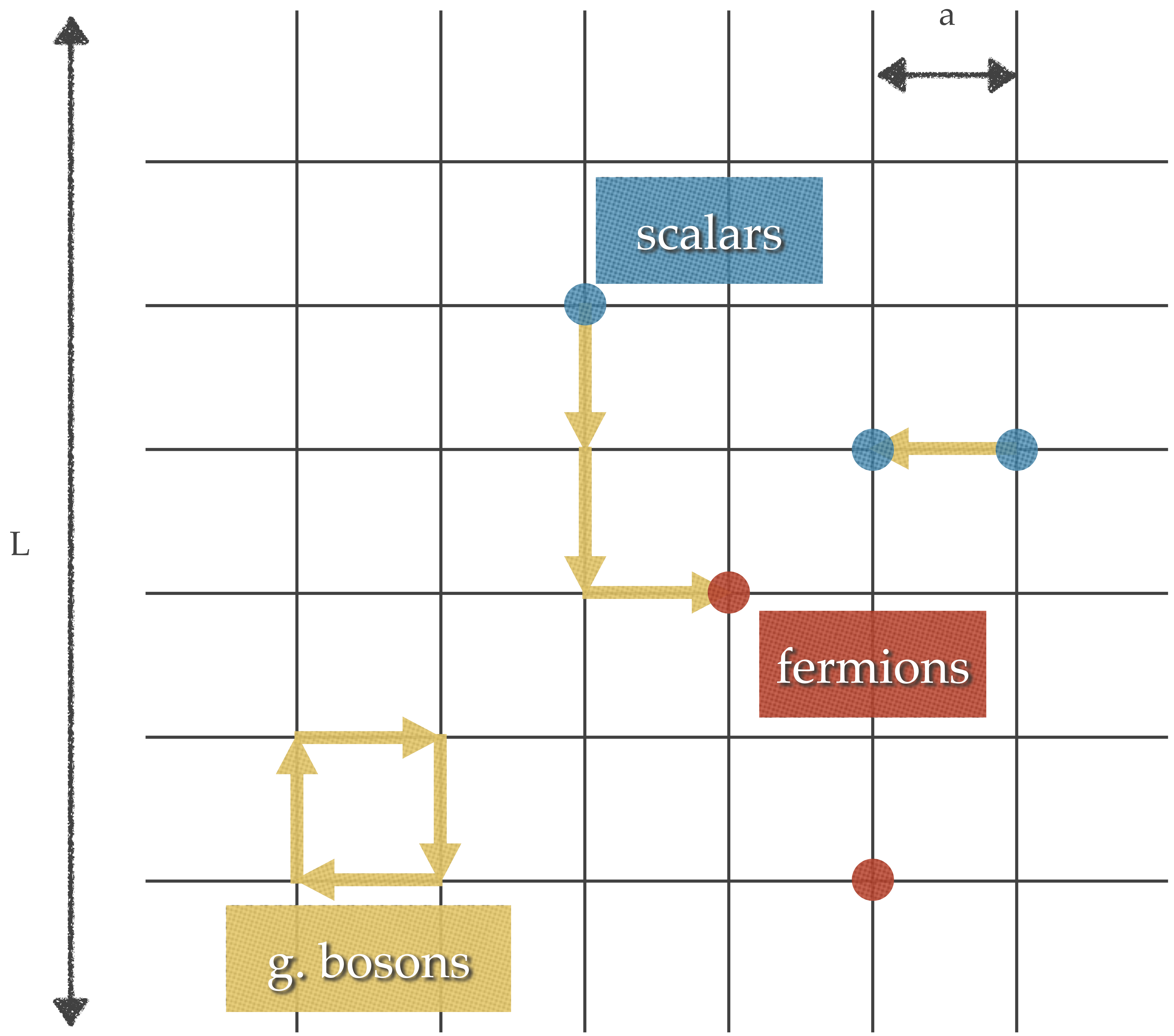

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  - not a model!
  - no simplifications

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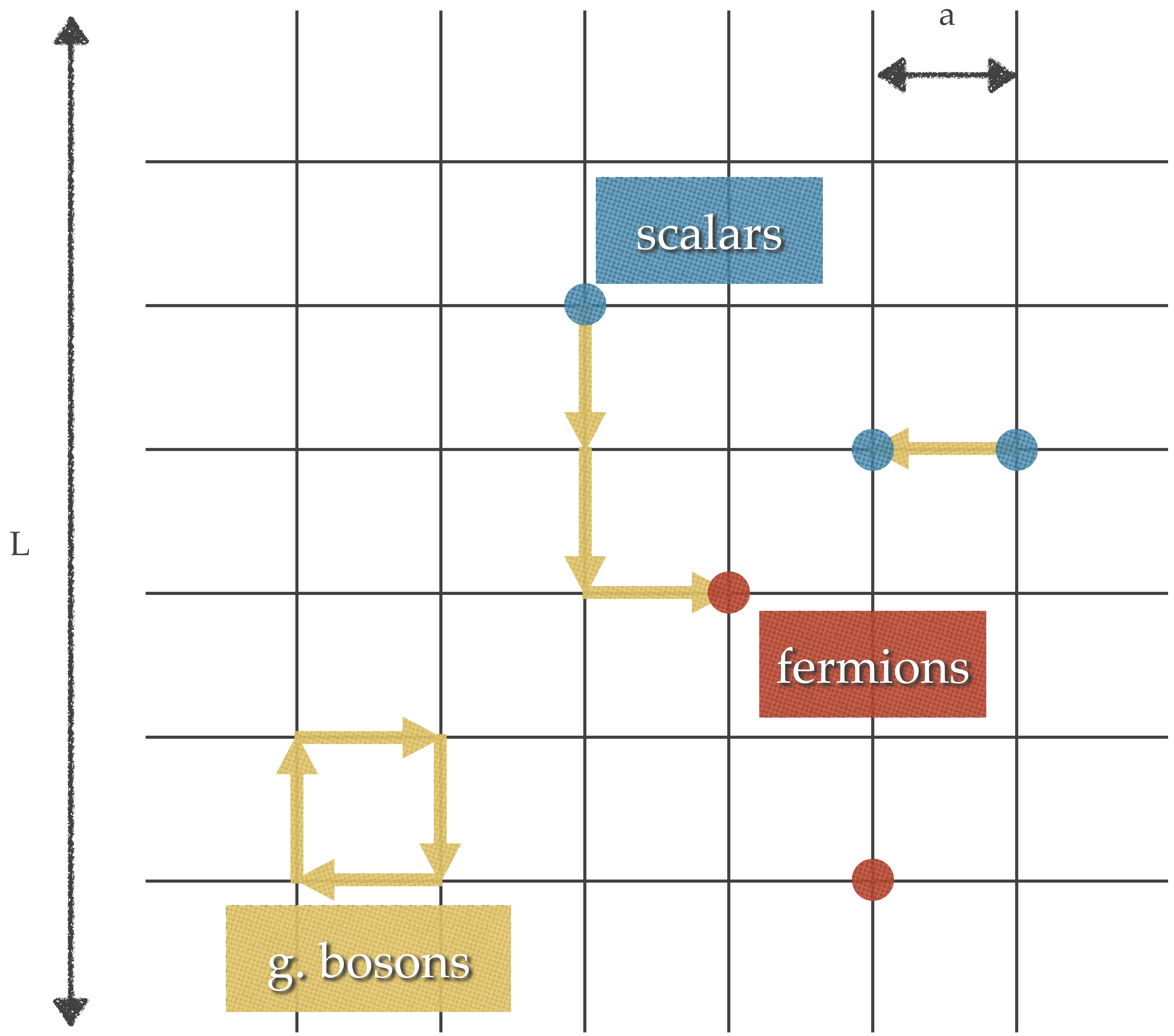

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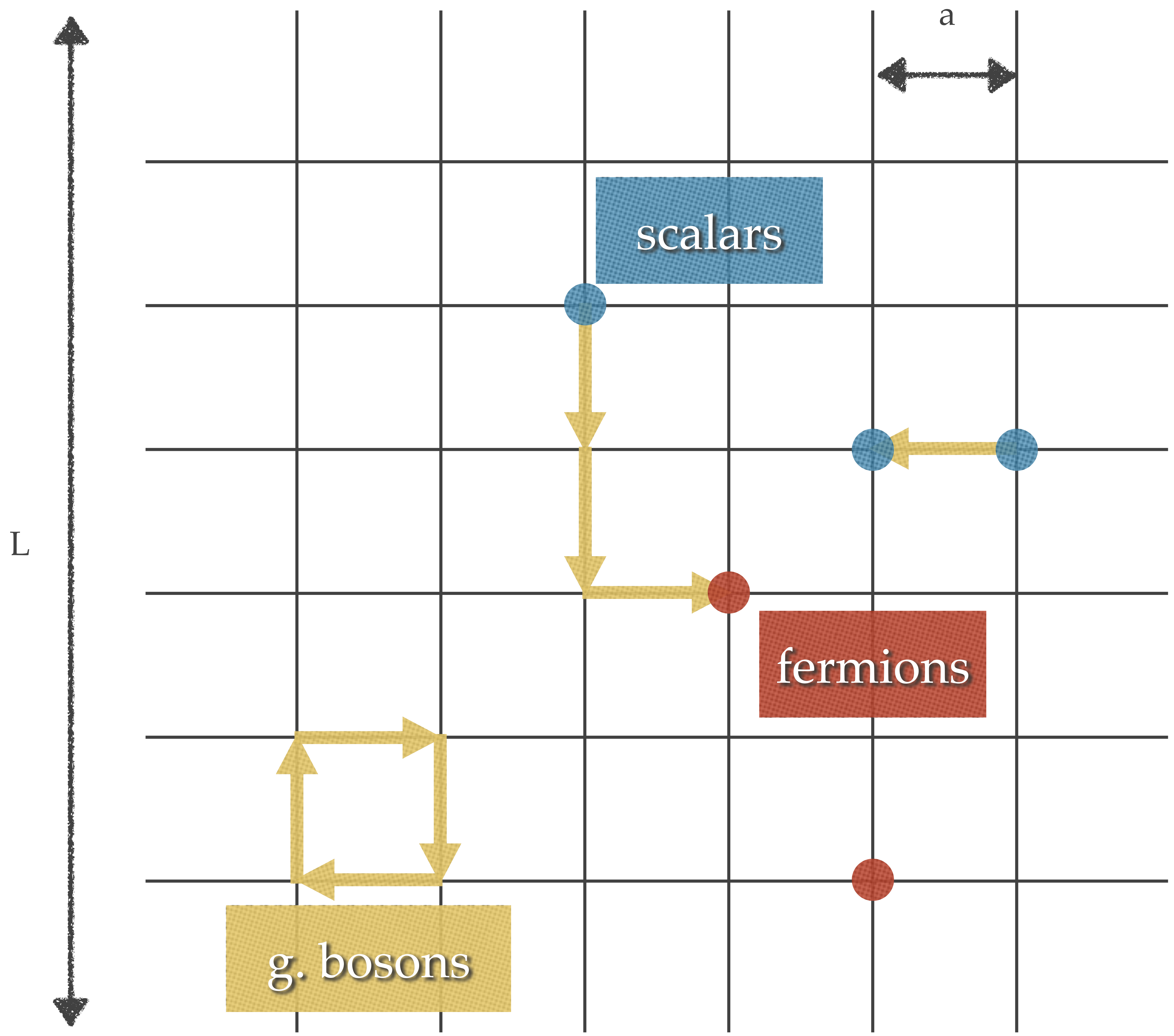

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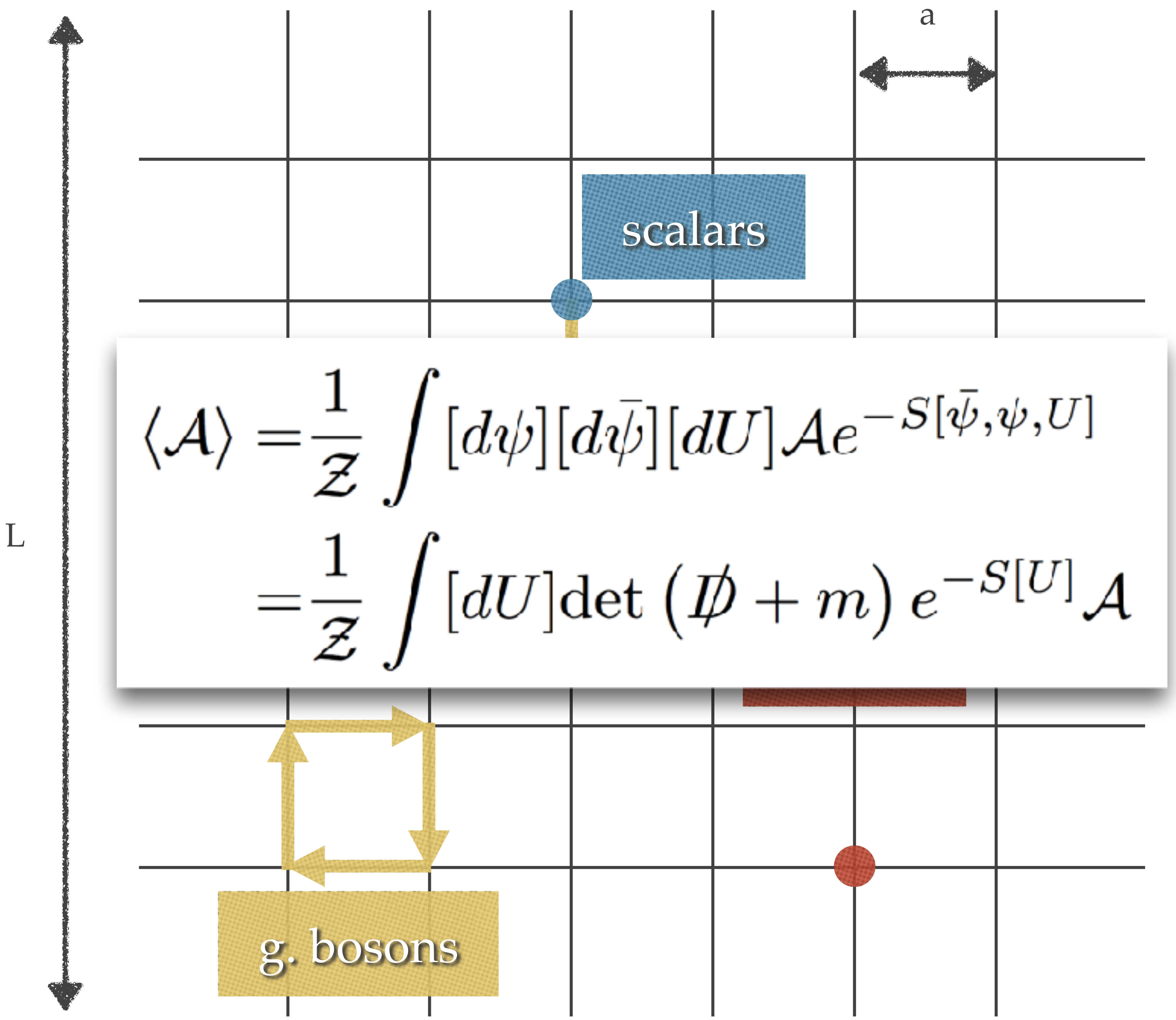
  - Systematic
  - Statistical

[Rinaldi et al., arxiv:2108.02942]

Path Integral Monte Carlo

Lattice Gauge Theory Primer



$$\langle \mathcal{A} \rangle = \frac{1}{\mathcal{Z}} \int [d\psi][d\bar{\psi}][dU] \mathcal{A} e^{-S[\bar{\psi},\psi,U]}$$
$$= \frac{1}{\mathcal{Z}} \int [dU] \det \left(\not{D} + m \right) e^{-S[U]} \mathcal{A}$$

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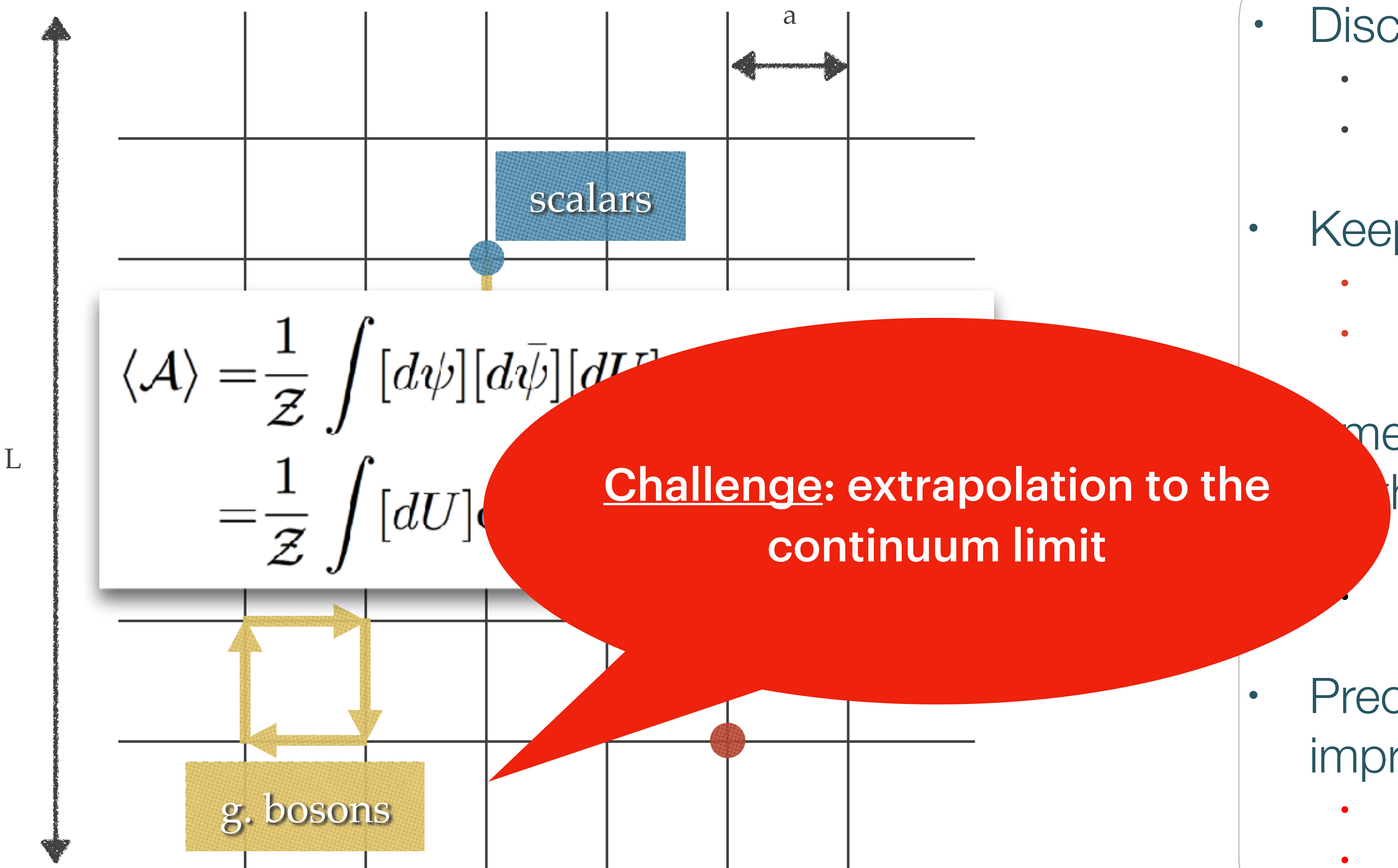

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Results

Small-scale: N=2, D=2

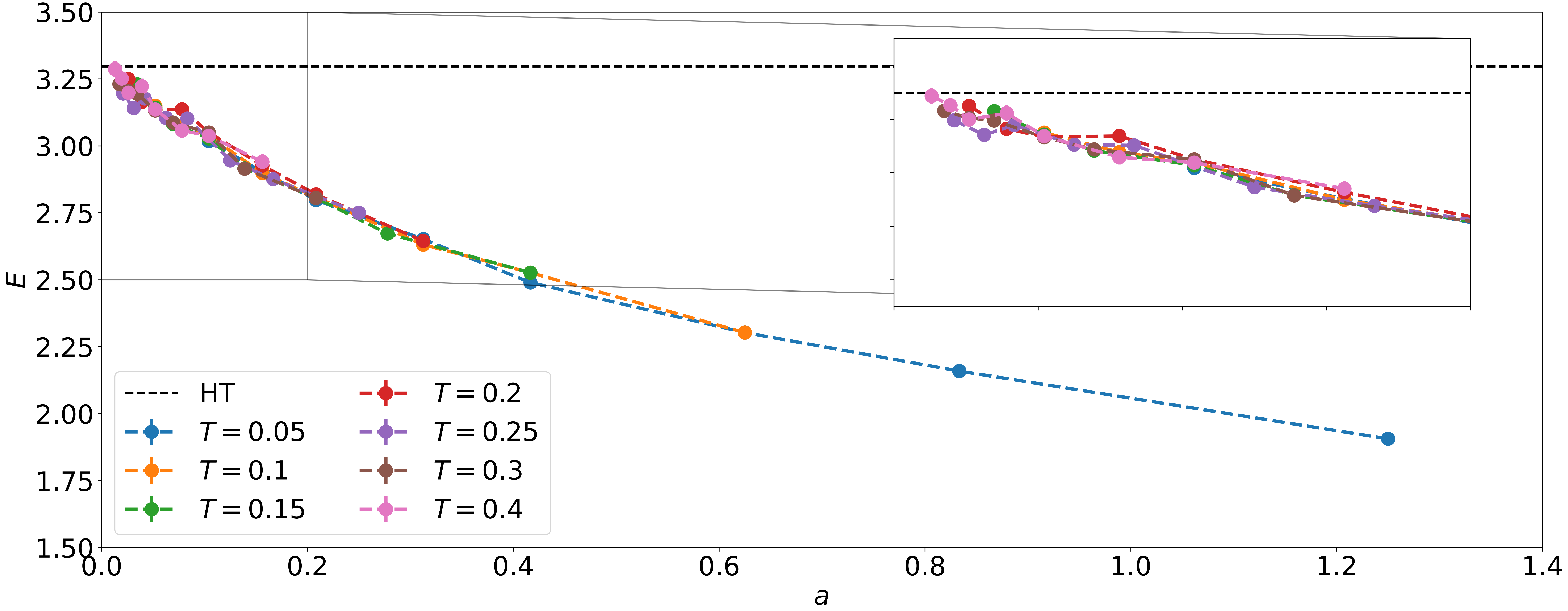
No truncation Λ

Parameters:

- Temperature
- Number of lattice sites

Observables:

- Energy



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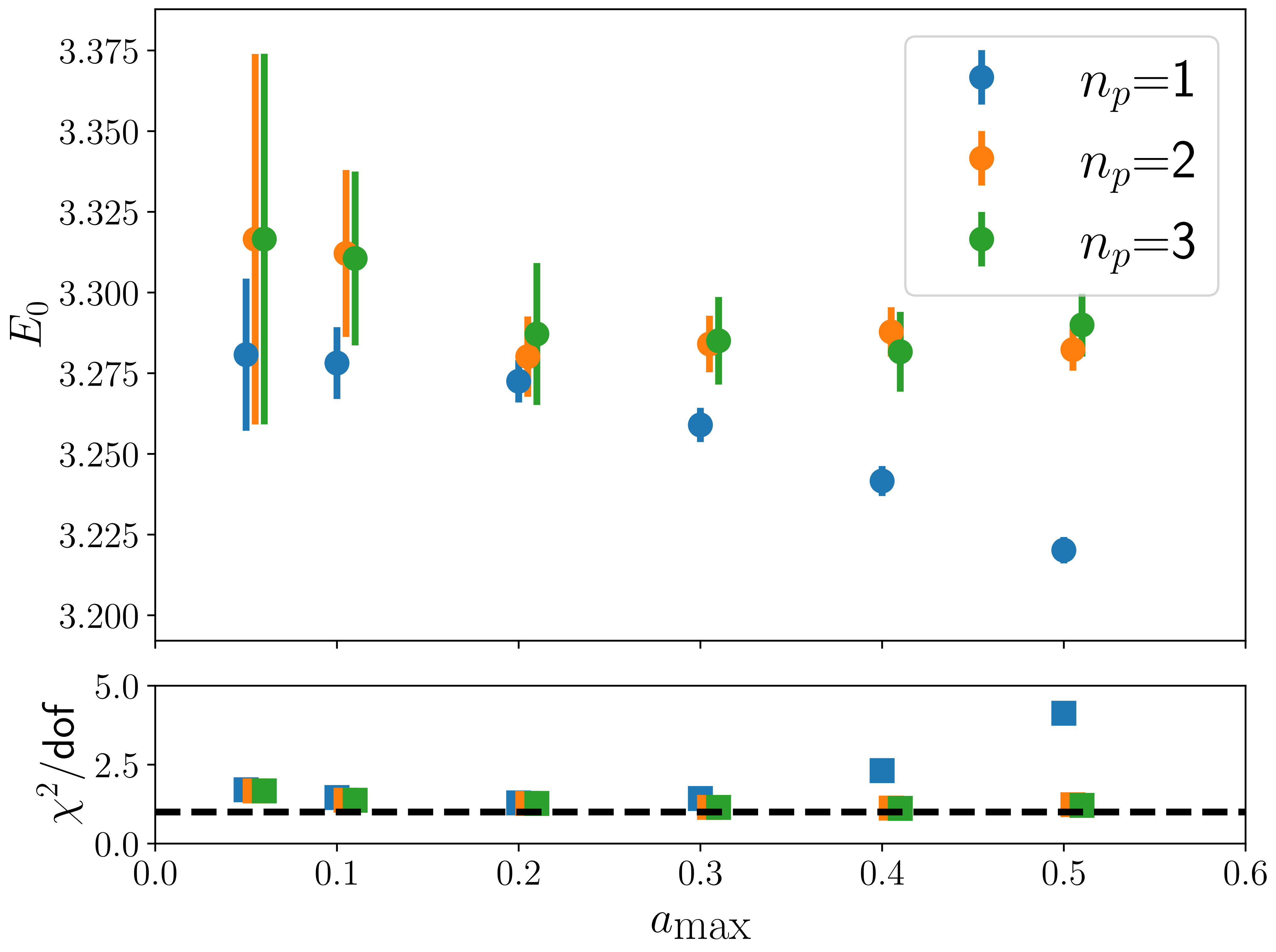
Global Extrapolation



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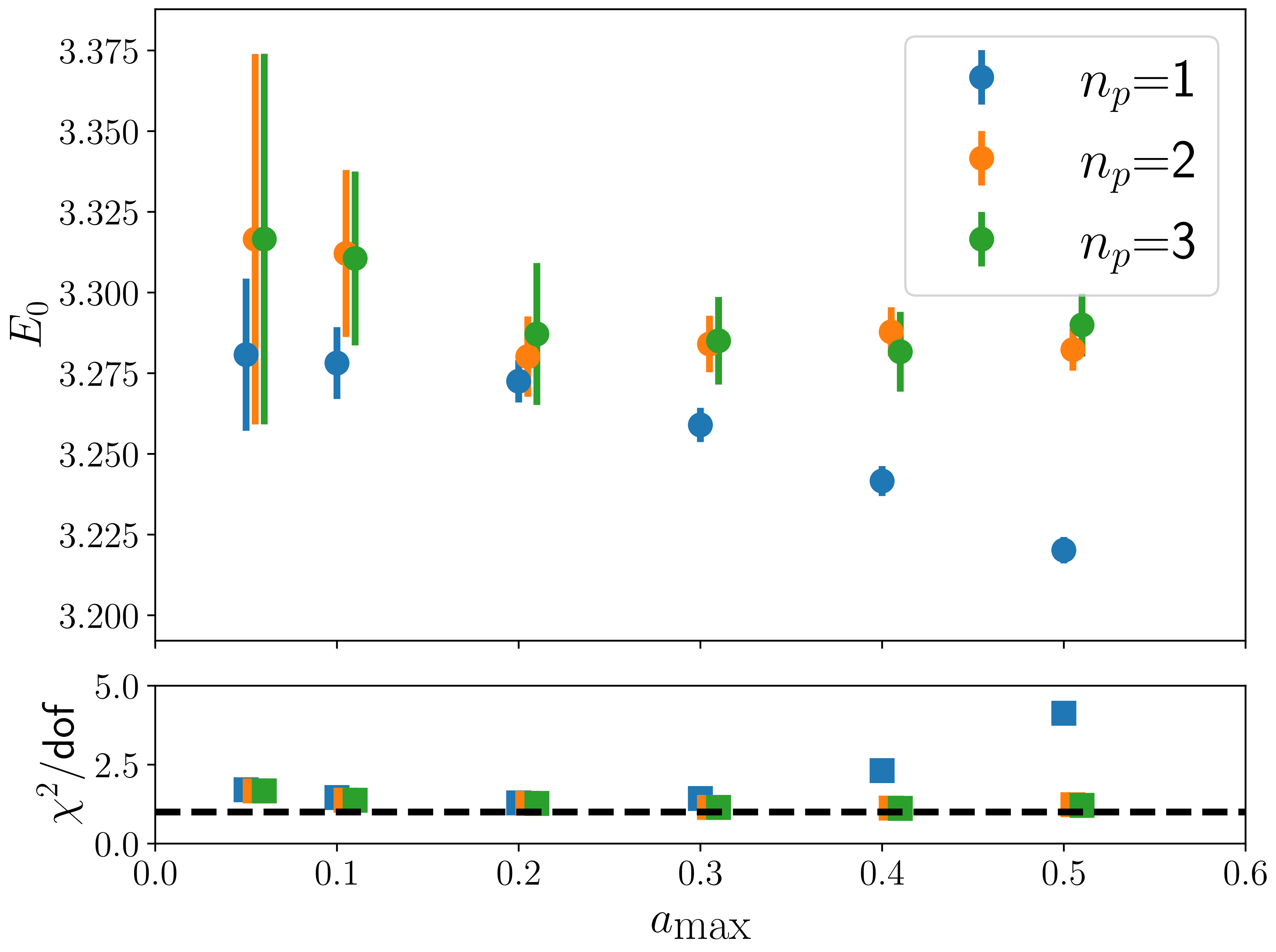
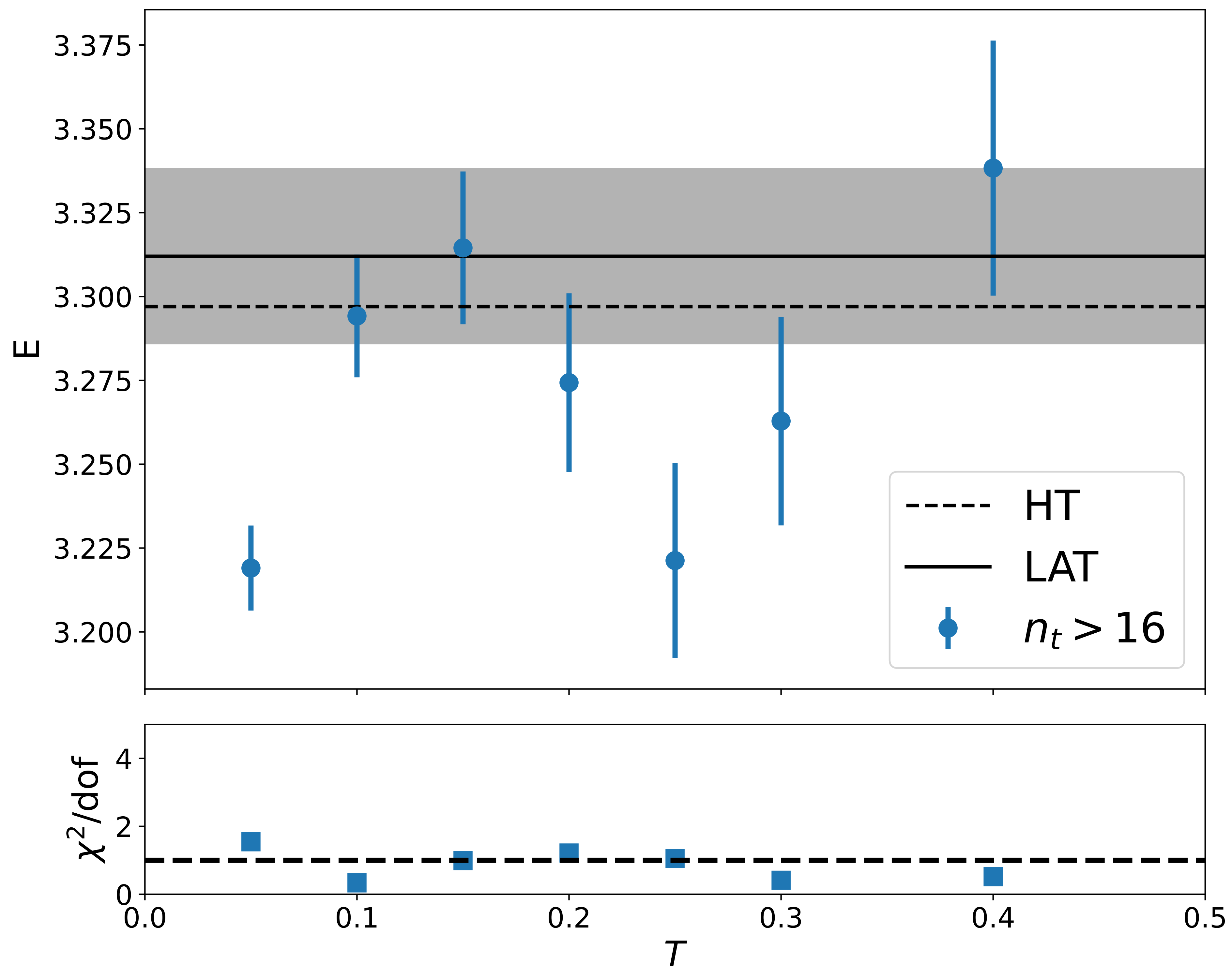


Local Extrapolation



[Rinaldi et al., arxiv:2108.02942]

Comparison

Ground state energy

Bosonic Model

D=2	HT	VQE	DL	MC
N=2	✅	Λ = 2,4	✅	✅
N=3	‼️	❌	✅	✅
N>3	❌	❌	✅	✅

Supersymmetric Model

D=2	HT	VQE	DL	MC
N=2	✅	Λ = 4	✅	✅
N=3	✅	❌	✅	✅
N>3	‼️	❌	✅	✅

[Rinaldi et al., arxiv:2108.02942]

Comparison

Benchmarking different methods

Bosonic Model

SU(2)

	$\lambda = 0.5$	$\lambda = 1.0$	$\lambda = 2.0$
$E_{0,\mathrm{HT}}$	3.297	3.516	3.855
$E_{0,\mathrm{DL}}$	3.302(2)	3.519(2)	3.857(3)
$E_{0,\mathrm{MC}}$	3.312(26)	3.497(33)	3.847(30)
$E_{0,\mathrm{VQE}}$	3.309	3.547	3.933

SU(3)

	$\lambda = 0.5$	$\lambda = 1.0$	$\lambda = 2.0$
$E_{0,\mathrm{DL}}$	8.824(7)	9.432(7)	10.426(8)
$E_{0,\mathrm{MC}}$	8.836(38)	9.381(38)	10.236(41)

Supersymmetric Model

SU(2)

	$\lambda = 0.5$	$\lambda = 1.0$	$\lambda = 2.0$
$E_{0,\mathrm{HT}}$	0.000	0.000	0.000
$E_{0,\mathrm{DL}}$	0.009(5)	0.014(6)	0.034(7)
$E_{0,\mathrm{VQE}}$	0.027	0.079	0.177

[Rinaldi et al., arxiv:2108.02942]

Conclusions
and roadmap

- ✓ Quantum simulations and deep learning can be used for addressing **Quantum Gravity** problems, using the holographic duality
- ✦ Hybrid quantum-classical algorithms can be used on current quantum hardware to study small-size matrix models. On the road to larger systems!
- ✦ Fast sampling from generative models allows an efficient representation of the ground state of matrix models
- ➡ Finding efficient parametrized quantum circuits for supersymmetric matrix models is very important: study new PQC construction methods (big industry right now)
- ➡ Using machine learning or tensor network approximations to simplify quantum simulations could be crucial with current resources: lead to simpler PQC??
- ➡ Error-mitigation will be important on real quantum hardware

LATTICE QUANTUM FIELD THEORY – MATHEMATICS

$$\mathcal{L}_{QCD} = -\frac{1}{4}F^2 + \bar{\psi}(i\not{D} + m)\psi$$

MICROSCOPIC THEORY OF FIELDS

ψ: quark field

U: gauge field

$$\langle \mathcal{O} \rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}\psi \, \mathcal{D}\bar{\psi} \, \mathcal{D}U \; e^{-S[\bar{\psi},\psi,U]} \mathcal{O}$$

QUANTUM FEYNMAN PATH INTEGRAL

Physical observable

DISCRETIZE — Makes integral finite dimens.

$$\{U_1, U_2, U_3, \ldots, U_N\}$$

MARKOV CHAIN MONTE CARLO

Sampling

$$\approx \frac{1}{N} \sum_{i=1}^{N} \mathcal{O}\left[U_i\right] + O\left(\frac{1}{\sqrt{N}}\right)$$

IMPORTANCE SAMPLING

Estimator

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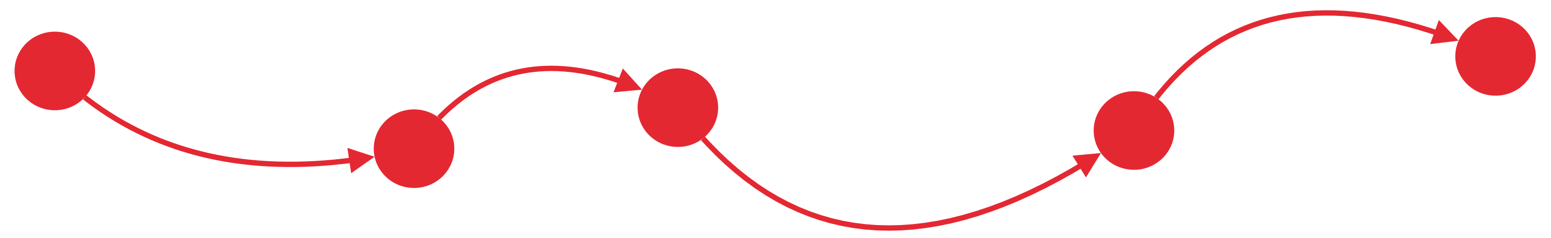

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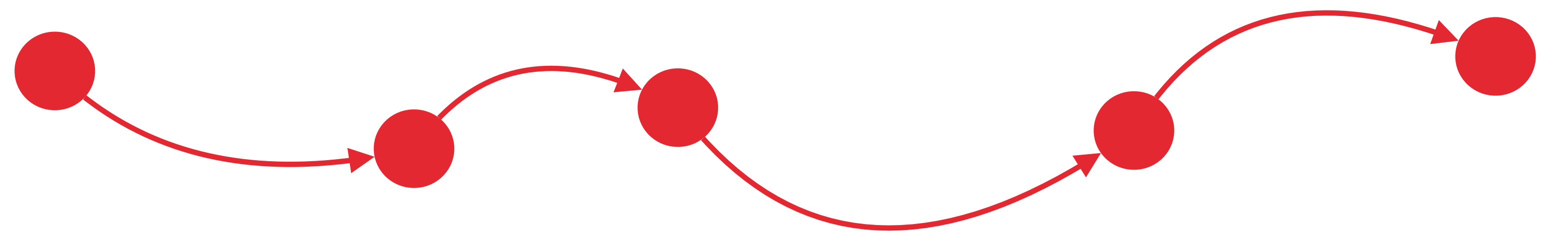

move in configuration space with prob.

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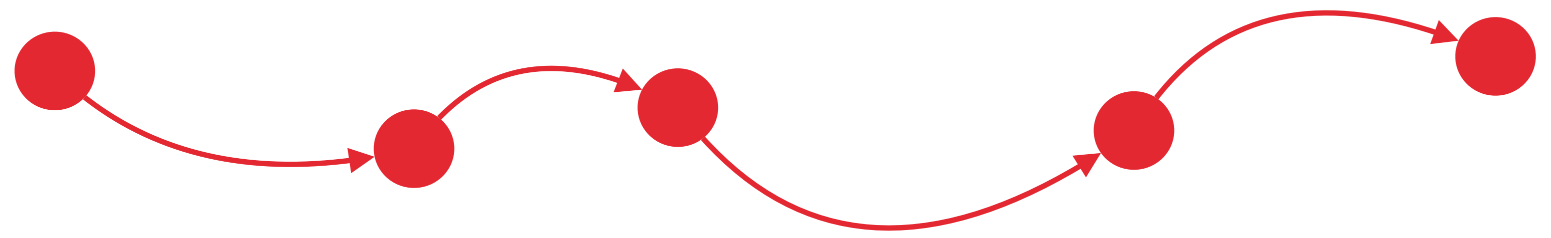

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statistical error

IMPORTANCE SAMPLING

Estimator