

# Nuclear Forces in the Medium: Insight from the Equation of State

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# OUTLINE:

- ❑ **Can the nuclear complexity emerge from low-energy QCD?**
  - **How do we predict nuclear structure from fundamental interactions?**
  - **Can modern theories provide all answers?**
  
- ❑ **Recent work with neutron matter and neutron-rich systems**
  
- ❑ **Work in progress and future plans at the UI nuclear theory group**

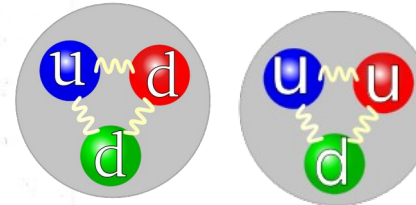
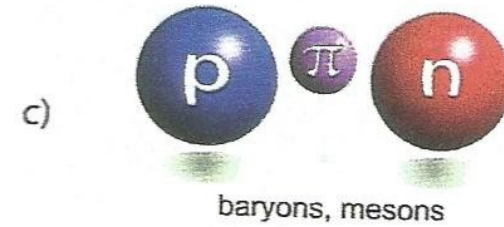
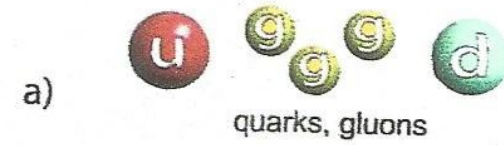
Our (still incomplete) knowledge of the nuclear force is the result of decades of struggle.

**Nuclear Forces:  
A hierarchy of scales.**

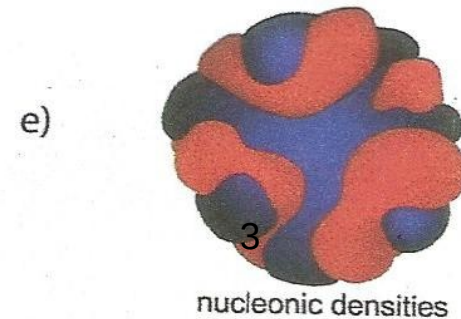
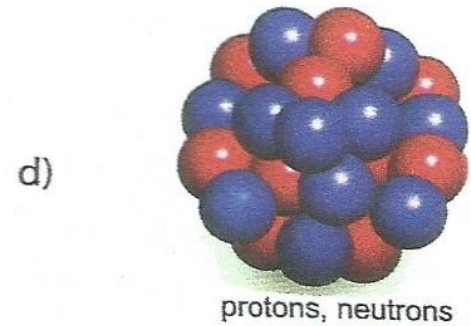
**Scale determines the appropriate degrees of freedom**

*Physics of Hadrons*

Degrees of Freedom



*Physics of Nuclei*



# The nuclear force: a very brief historical perspective

1960s-70s: The OBE model was developed, followed by more sophisticated meson-exchange models.

1980s: Meson theory became very popular. The goal of deriving the nuclear force from QCD led to the development of quark models.

1980s-1990's: The era of high-precision NN potentials based on meson theory or purely phenomenological (supplemented with phenomenological 3NFs).

Effective Field Theory (EFT) for low-energy QCD: Weinberg 1979, 1990, 1991.

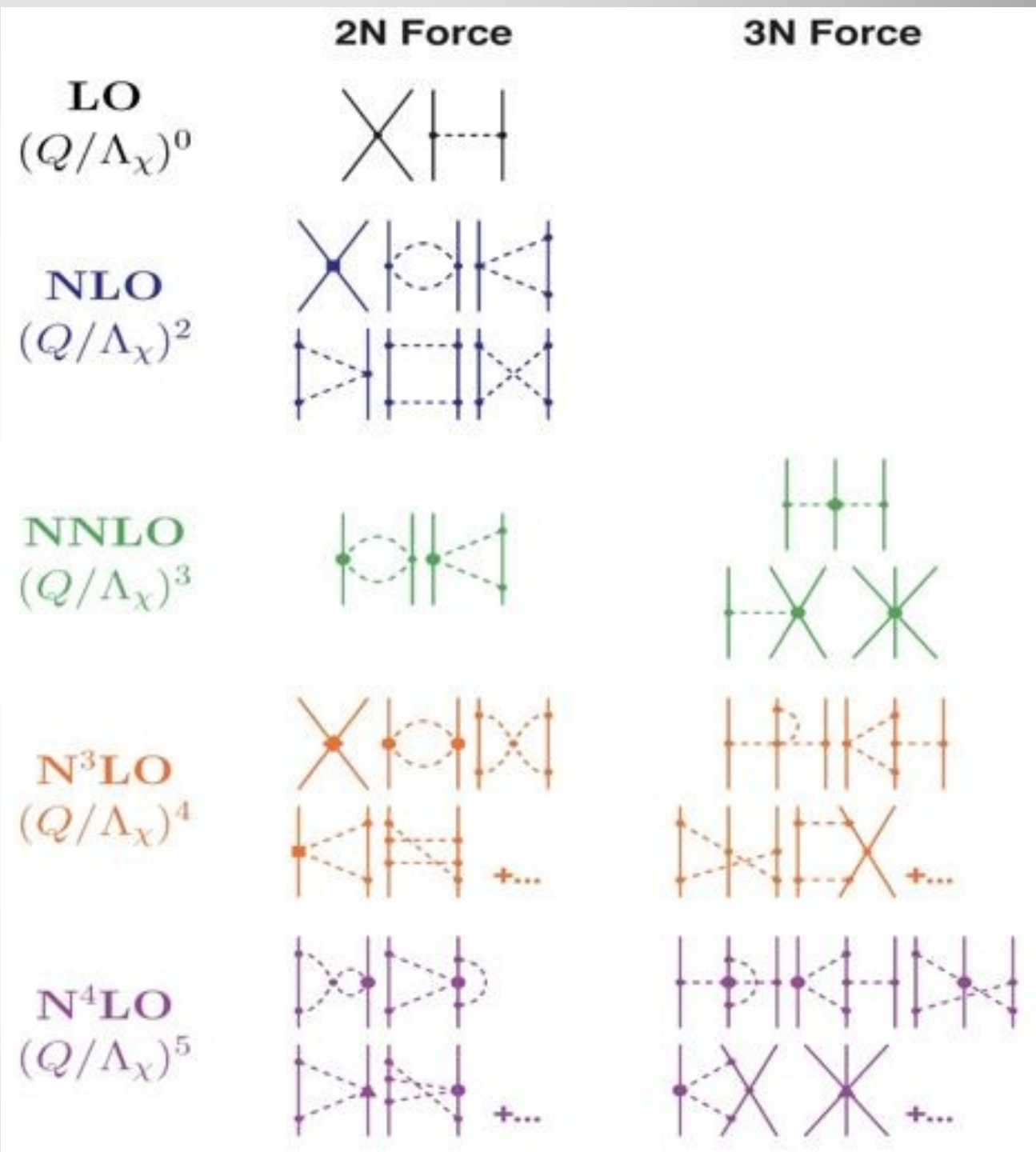
## Since more than 2 decades:

For low energies, nuclear chiral EFT has become the most favorable approach to construct nuclear two- and few-body forces in a systematic way.

Based upon the symmetries of low-energy QCD, while using degrees of freedom relevant for low-energy nuclear physics. Predictions can be improved systematically.

Together with an organizational scheme to rank-order the various diagrams (power counting), **nuclear two- and few-body forces can be developed in a controlled hierarchy.**

# Hierarchy of nuclear forces in Chi-EFT $\longrightarrow$



**....Gaining insight from nuclear matter:**

**An example.....**

# EXPLORING THE RELATION BETWEEN SYMMETRIC NUCLEAR MATTER AND NUCLEI WITH CHIRAL FORCES

## Symmetric Nuclear matter (SNM):

- An extrapolation from finite nuclei: infinite system with saturation density and energy approximately equal to

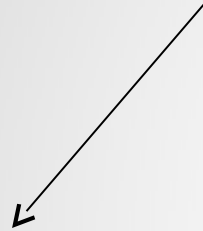
$$\rho = 0.16 \text{ fm}^{-3} \text{ and } \frac{E}{A} = -16 \text{ MeV}$$

- Traditionally seen as a “test bench” for nuclear many-body theories
- What do microscopic approaches predict for such relationship?

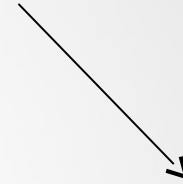


## Framing the issue:

### Chiral 2N and 3N forces (N2LO and N3LO)



Predict realistic B.E. and radii for a wide range of nuclei, but unable to saturate SNM. (Huether et al., 2019)

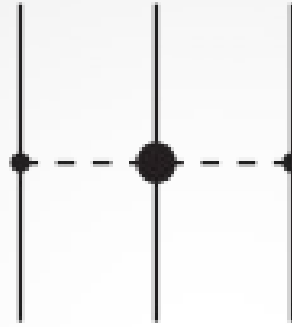
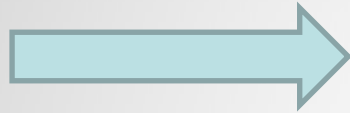


If the SNM constraint is included (along with  $^3\text{H}$  B.E. constraint), medium-mass nuclei are systematically underbound, radii are too large. (Hoppe et al., 2019)

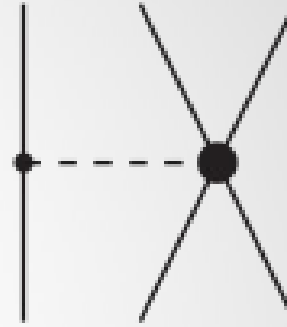
Next, we explore this apparent inconsistency by considering various scenarios.

**2NF: high-quality NN chiral potentials up to N4LO  
(Entem, Machleidt, and Nosyk, 2017)**

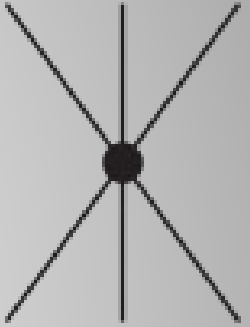
**3NF**



$C_i$



$C_D$



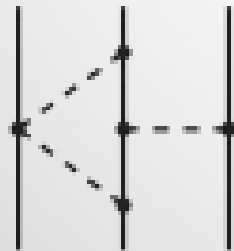
$C_E$

**Diagrams of the leading chiral 3NF**

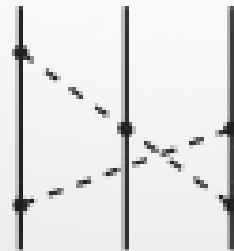
**Subleading 3NF: Some typical diagrams:**



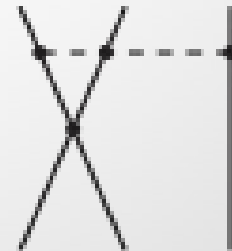
(a)



(b)



(c)



(d)



(e)

[1] Fits to  $^3\text{H}$  B.E. + SNM

[2] Fits to  $^3\text{H}$  and  $^{16}\text{O}$  B.E.

Source	chiral order	$\Lambda$ (MeV)	$C_D$	$C_E$
Ref. [1]	N <sup>2</sup> LO	450	(a) 2.25	0.07
			(b) 2.50	0.1
			(c) 2.75	0.13
	N <sup>3</sup> LO	450	0.0	-1.32
			0.25	-1.28
			0.50	-1.25
Ref. [2]	N <sup>2</sup> LO	450	10.0	0.90
	N <sup>3</sup> LO	450	9.0	-0.152

[1] Drischler, Hebeler, Schwenk, PRL**122**, 042501 (2019)

[2] Huether *et al.*, PLB **808**, 135651 (2020)

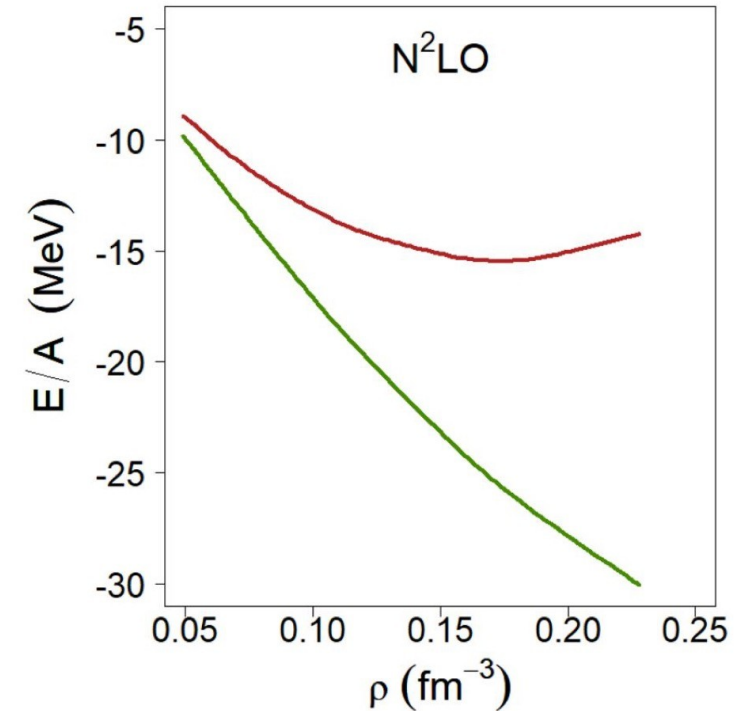
**E/A in SNM vs. density.**

**Red:  $c_D, c_E$  from [1];**

**Green:  $c_D, c_E$  from [2]**

**Both sets of LECs predict the  ${}^3\text{H}$  B.E. accurately.**

**The  ${}^{16}\text{O}$  constraint generates a 3NF that dramatically lowers the EoS at higher densities**

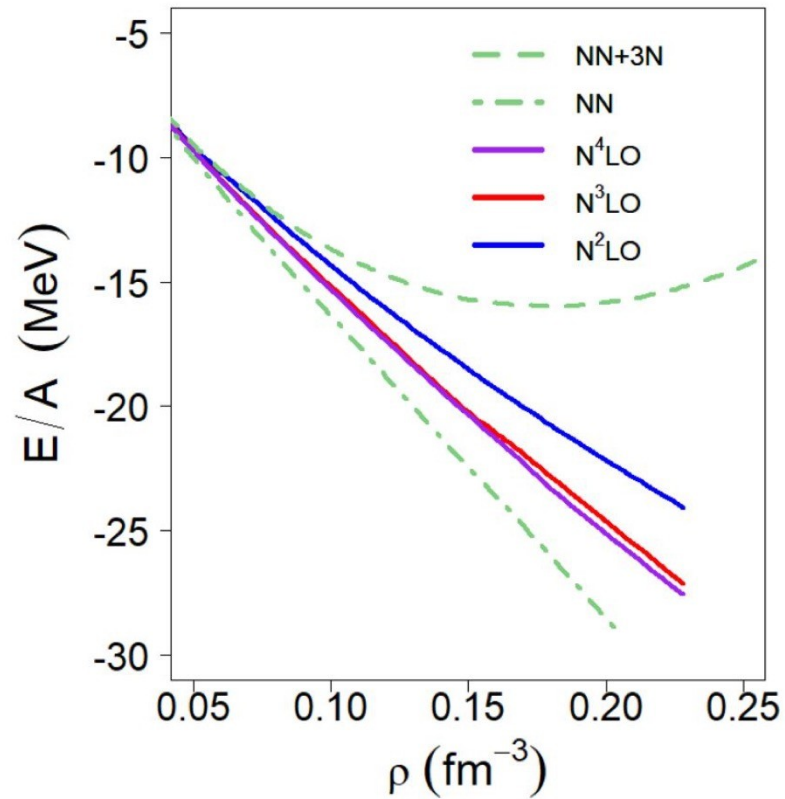


[1] Drischler, Hebeler, Schwenk, PRL122, 042501 (2019)

[2] Huether *et al.*, PLB808, 135651 (2020)

# Ground state energy and charge radii predicted with our EoS (red curve)

<b>Nucleus</b>	<b><math>\Lambda</math> (MeV)</b>	<b>(<math>c_D, c_E</math>)</b>	<b>E/A (MeV)</b>	<b><math>r_{ch}</math> (fm)</b>	<b>E/A (exp.)</b>	<b><math>r_{ch}</math> (exp.)</b>
<b><math>^{16}\text{O}</math></b>	<b>450</b>	<b>(a)</b>	<b>-6.83</b>	<b>2.89</b>	<b>-7.98</b>	<b>2.73</b>
		<b>(b)</b>	<b>-6.92</b>	<b>2.88</b>		
		<b>(c)</b>	<b>-7.01</b>	<b>2.86</b>		
<b><math>^{40}\text{Ca}</math></b>	<b>450</b>	<b>(a)</b>	<b>-7.60</b>	<b>3.57</b>	<b>-8.55</b>	<b>3.49</b>
		<b>(b)</b>	<b>-7.73</b>	<b>3.55</b>		
		<b>(c)</b>	<b>-7.86</b>	<b>3.53</b>		
<b><math>^{48}\text{Ca}</math></b>	<b>450</b>	<b>(a)</b>	<b>-7.75</b>	<b>3.63</b>	<b>-8.67</b>	<b>3.48</b>
		<b>(b)</b>	<b>-7.88</b>	<b>3.61</b>		
		<b>(c)</b>	<b>-8.02</b>	<b>3.59</b>		
<b><math>^{208}\text{Pb}</math></b>	<b>450</b>	<b>(a)</b>	<b>-6.87</b>	<b>5.58</b>	<b>-7.87</b>	<b>5.50</b>
		<b>(b)</b>	<b>-7.03</b>	<b>5.54</b>		
		<b>(c)</b>	<b>-7.19</b>	<b>5.50</b>		



Dash-dot green: Hebeler force [3]  
 Dashed green: Hebeler force + 3NF

SRG-evolved low-momentum interaction  
 (starting from EM(2003) “1.8/2.0” + 3NF  
 fitted to  ${}^3\text{H}$  B.E. and the  ${}^4\text{He}$   $r_p$ )

Good description of closed shell nuclei  
 from  ${}^4\text{He}$  to  ${}^{78}\text{Ni}$  (Simonis *et al.*, 2017)

Can we construct a realistic, bare 2NF,  
 as soft the SRG-evolved from [3]?

Blue, red, purple:

The softest (unpublished) NN potentials  
 of EMN ( $\Lambda = 400 \text{ MeV}$ ) from third to fifth order

[3] Hebeler et al., PRC83, 031301  
 (2011)

**To summarize the observations:**

**The value of  $c_D$  from fits to  $^{16}\text{O}$  (and triton) has the effect of lowering the EoS at all densities.**

**A 3NF which binds the triton and includes the SNM constraint provides insufficient binding in medium-mass nuclei. The EoS is still on the repulsive side. If the triton constraint is relaxed, the resulting couplings allow for a better description of medium-mass nuclei while substantially overbinding the triton.**

**This suggests the need for a 2NF-3NF combination that provides sufficient attraction at lower densities while still enabling saturation.**

- Large differences exist among the 3NF couplings fitted through different observables.
- A simultaneously optimal combination of  $c_D$ ,  $c_E$  for few-nucleon systems, medium-mass nuclei, and nuclear matter, has not been found.

**Scenario 1: a “super-soft 2NF combined with a repulsive, strongly density dependent 3NF**

**Scenario 2: A moderately soft 2NF is combined with a 3NF which is attractive at low density but turns repulsive at and above saturation.**



# **MOVING ON TO OTHER SYSTEMS ...**

# Neutron-rich matter

Energy per particle for  
isospin asymmetric  
matter:

Parabolic  
approximation

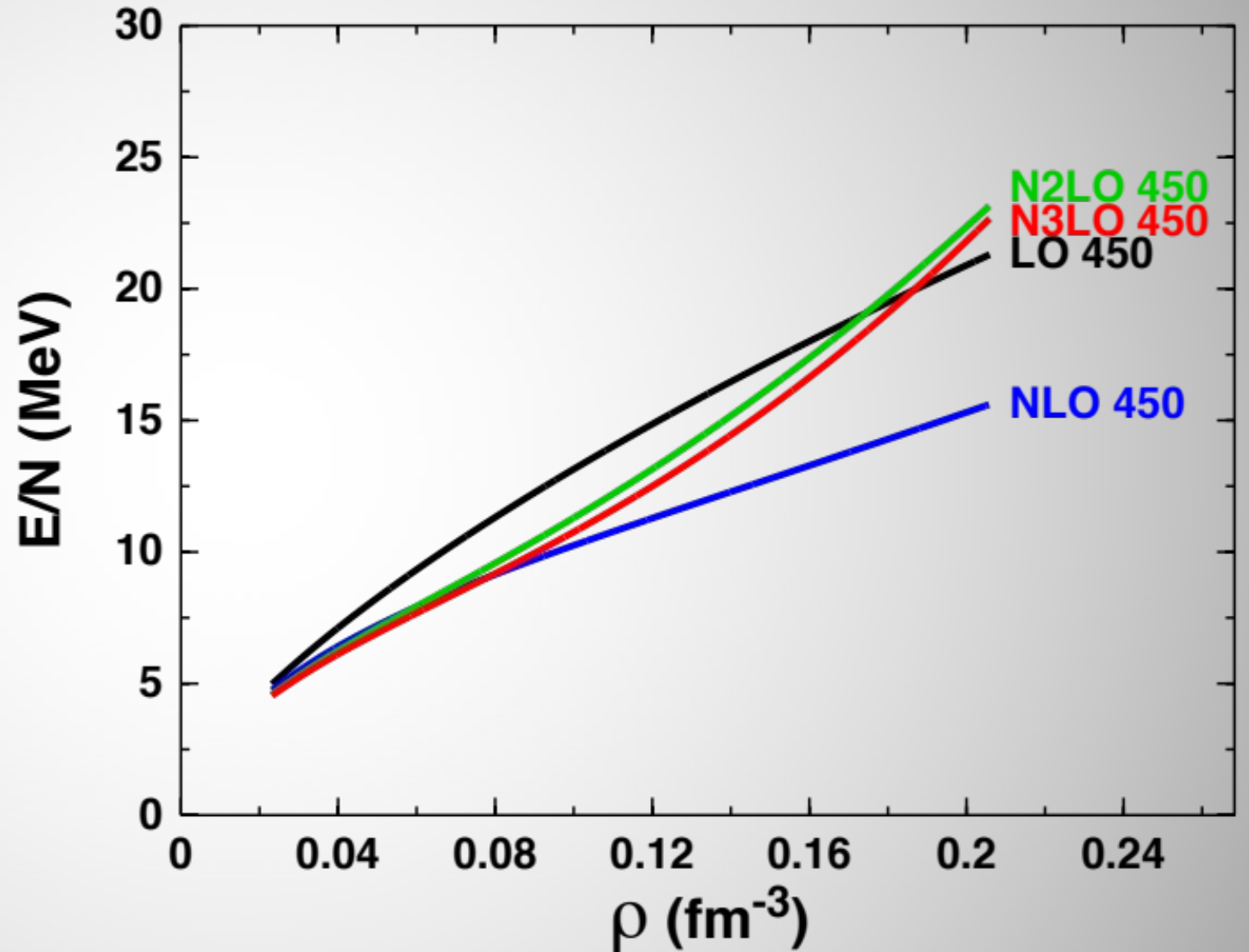
$$e(\rho, \alpha) = e(\rho, 0) + e_{sym} \alpha^2$$

$$e_{sym}(\rho) = e(\rho, 1) - e(\rho, 0) \quad \alpha = \frac{\rho_n - \rho_p}{\rho_n + \rho_p}$$

$$e_{sym}(\rho) = e_{sym}(\rho_0) + L \frac{\rho - \rho_0}{3\rho} + \frac{K}{2} \frac{(\rho - \rho_0)^2}{(3\rho)^2} + \dots$$

# Pure neutron matter:

Energy per neutron as a function of density at the specified chiral orders.



$$\mathbf{J} = e_{sym}(\rho_0)$$

$$\mathbf{e}_{nm}(\rho_0) = (16.6 \pm 1.2) \text{ MeV}$$

$$\mathbf{L} = (55.7 \pm 9.1) \text{ MeV}$$

$$\mathbf{J} = (32.6 \pm 1.5) \text{ MeV}$$

$$\mathbf{P}_{nm}(\rho_0) = (2.97 \pm 0.48) \text{ MeV} / \text{fm}^3$$

From a Bayesian analysis by Drischler et al., 2020:

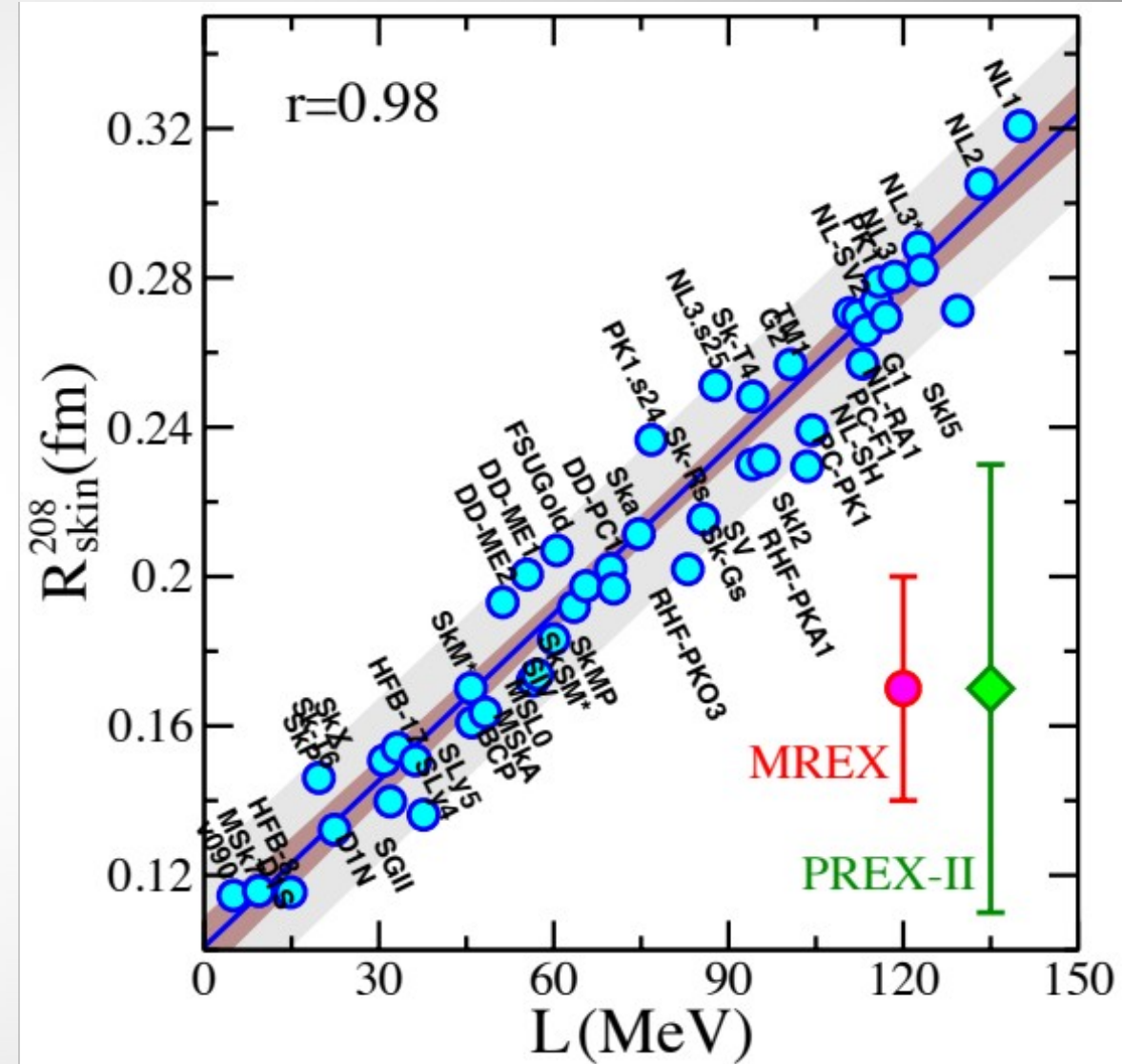
$$\mathbf{J} = (31.7 \pm 1.1) \text{ MeV}$$

$$\mathbf{L} = (59.8 \pm 4.1) \text{ MeV}$$

# Neutron skin of $^{208}\text{Pb}$ :

$$S = (0.168 \pm 0.010) \text{ fm}$$

As to be expected from



**PREX II values:**

*...This result challenges myriad of experimental measurements and theoretical predictions....*

arXiv:2101.03193

(Reed, Fattoyev, Horowitz, & Piekarewicz)

*Or rather: myriad of experimental measurements and theoretical predictions challenge this result... (F.S.)*

$$J = 38.09 \pm 4.73 \text{ MeV}$$

$$L = 106 \pm 37 \text{ MeV}$$

$$P_{NM}(\rho_0) \approx \frac{1}{3} L \rho_0 = (3.45 - 7.15) \text{ MeV fm}^{-3}$$

**!!!**

$$S = 0.29 \pm 0.07 \text{ fm}$$

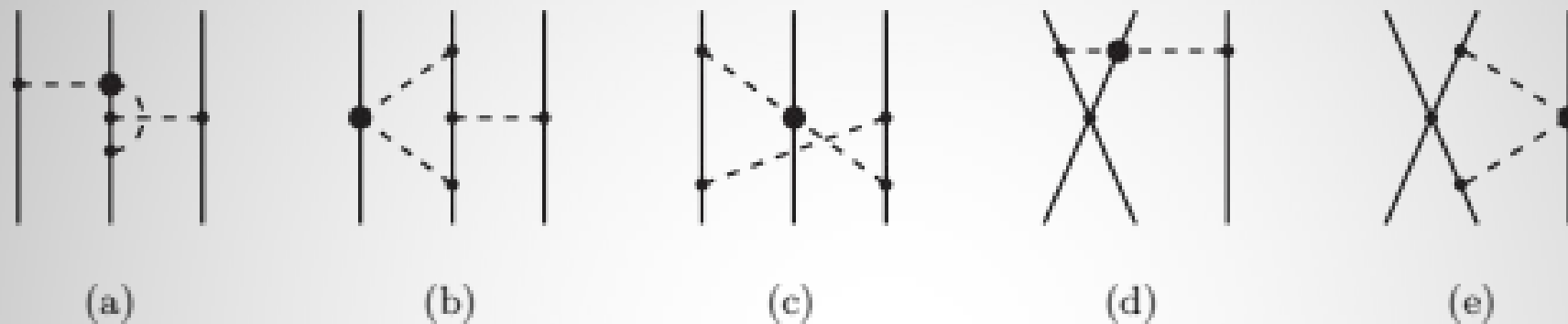
**3NF at N3LO: limited degree of flexibility**

**Some components of the 3NF at N4LO may be necessary**

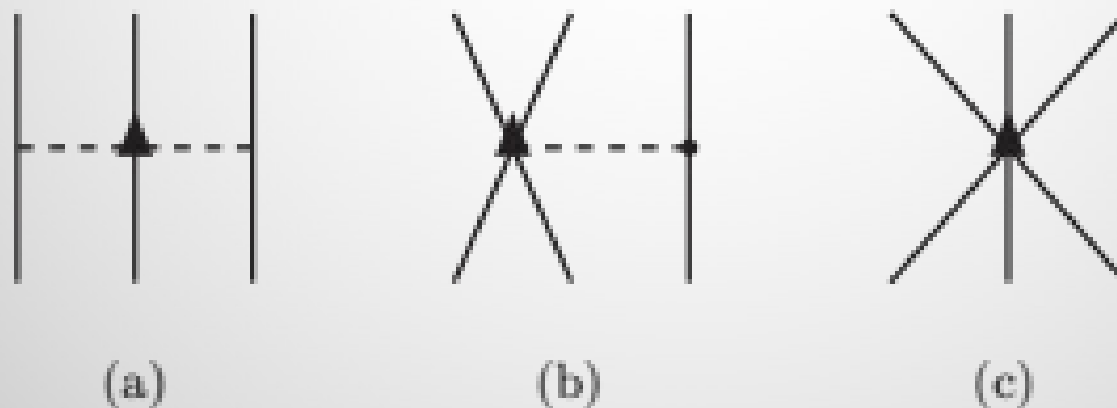
**Slower convergence of chiral EFT than expected?**

**See Girlanda et al. PRC99, 054003 (2019):  
Possible solution of the “ $A_y$  puzzle” from 3NFs at N4LO?**

## One-loop 3NF diagrams appearing at N4LO

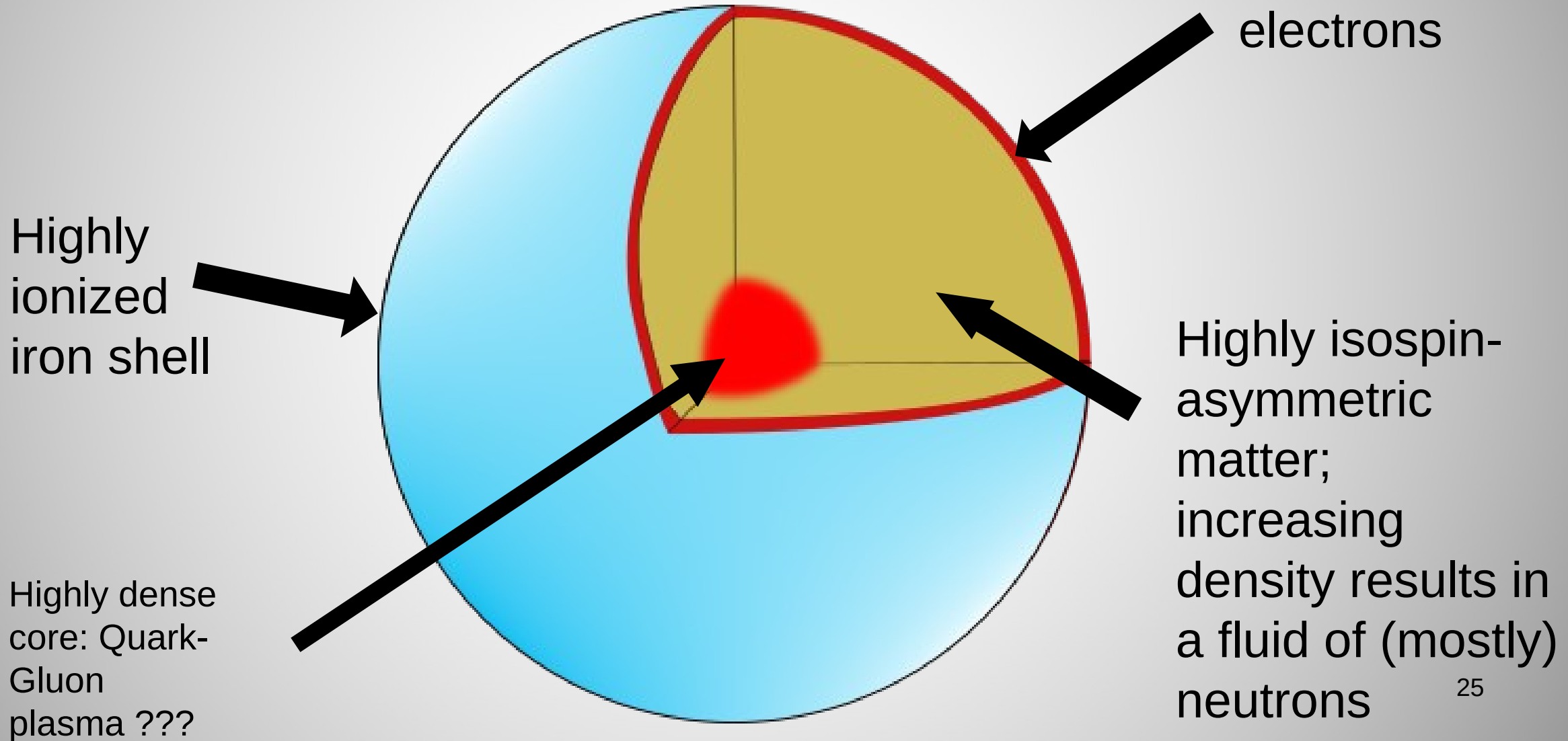


## 3NF tree diagrams appearing at N4LO

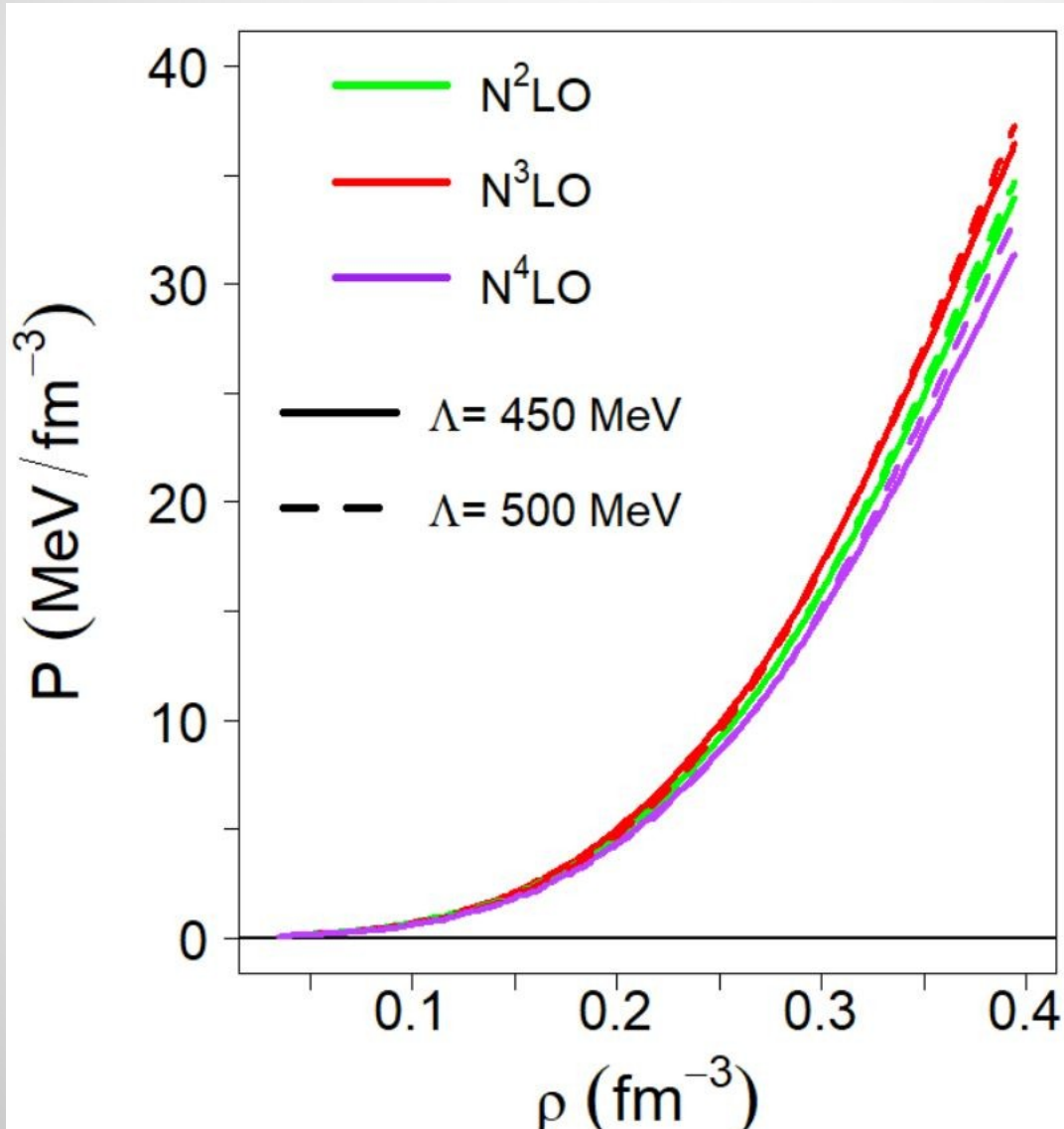




# The EoS and Neutron Stars



# Our predictions for the pressure in beta-stable matter as a function of density:



**Chiral Effective Field Theory is a low-energy theory. Our chiral EoS is applicable within a limited range of Fermi momenta.**

**The complete EoS is created by matching three contributions:**

**Subnuclear  
crystal EoS**

**U**

**Microscopic  
EoS in beta  
equilibrium**

**U**

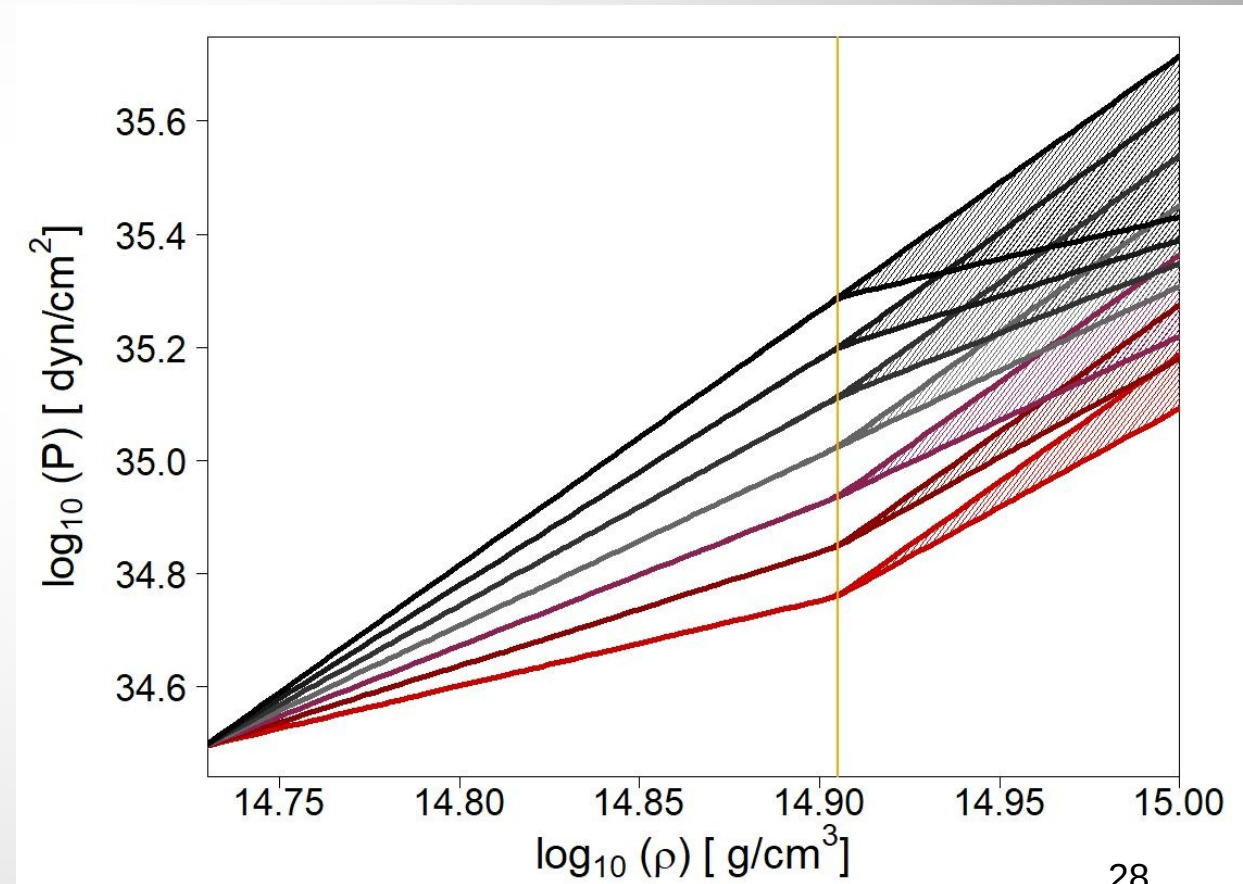
**High density  
polytropic  
continuation**

# High Density Polytropic Continuation

At high densities (above approximately  $2\rho_0$ ) we use (piecewise) polytropes over a broad range of adiabatic indices:

$$P(\rho) = a \rho^\Gamma$$

In this way, we wish to explore the sensitivity of the radius of the average-mass NS to the corresponding spreading of the pressure.



# Neutron star Mass-Radius Relation

Chirally constrained  
EoS + polytropic  
extension



TOV  
Equation



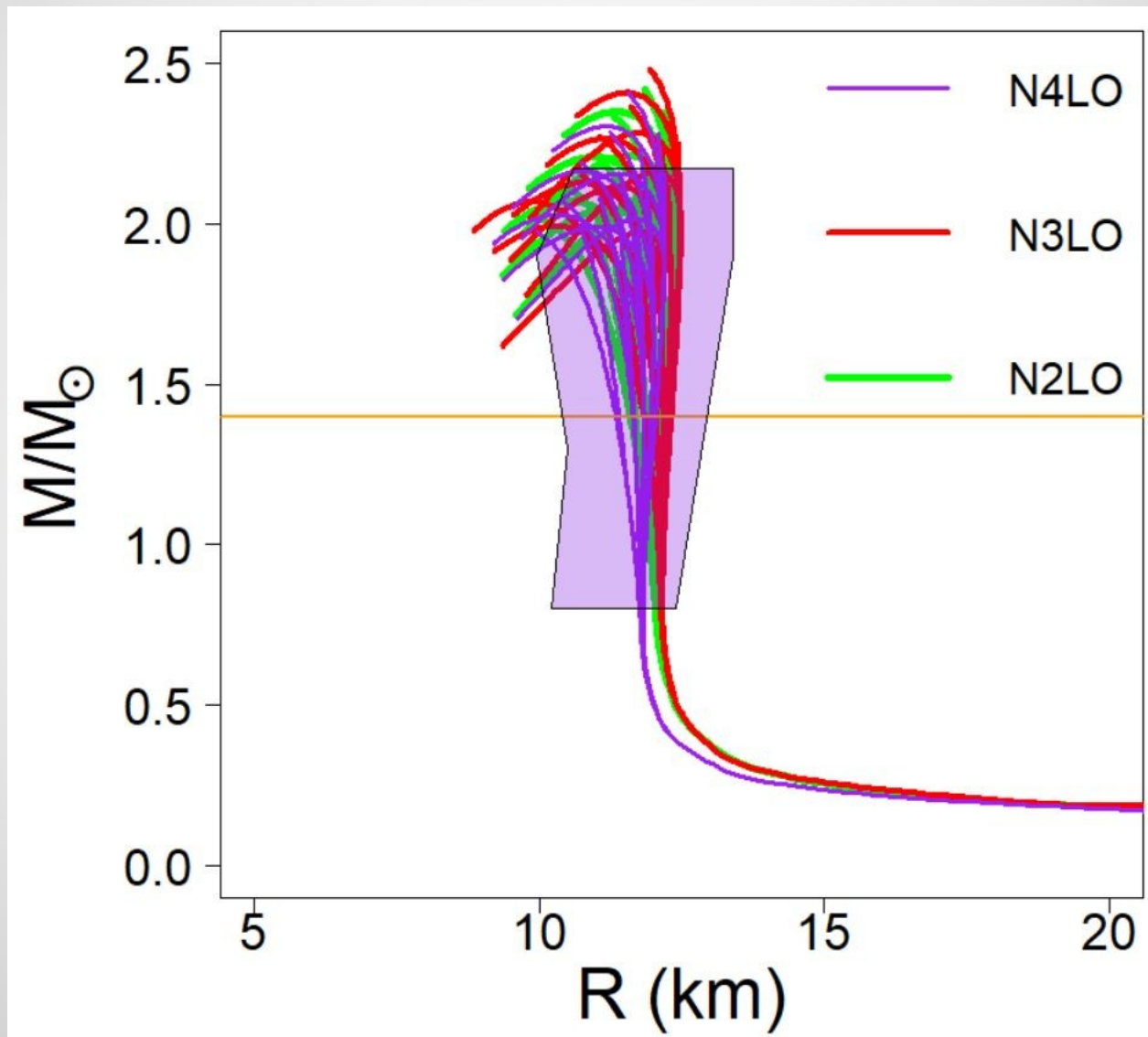
Mass-  
radius  
relation

Maximum mass at least  $1.97 M_{\odot}$

Constraints:

Causality Constraint:  $\frac{dP}{dE} < 1$

## Mass-radius relation at various order of chi-EFT



**Estimates for the radius of the canonical-mass neutron star,  
M = 1.4 solar masses:**

**Our predictions:**

**R = (10.8 – 12.8) km**

**LIGO-Virgo:  
(Annala et al., 2018)**

**R < 13.6 km ( $S \leq 0.25 fm$ )**

**PREX II :**

**R = (13.33 – 14.26) km**

**Also, threshold densities for DU  
consistent with PREX II are very low.**

## ....CONCLUSIONS UP TO THIS POINT:

- **After decades of struggle, the 2N force is under control.**
- **Will open questions in nuclear structure be resolved with chiral EFT? Not clear. Although the approach is conceptually pleasing, the proliferation of many-nucleon force contributions with higher orders may be a serious challenge.**
- **As for the neutron matter EoS, we will continue to proceed microscopically and systematically.**
- **Experimental programs (such as at FRIB or Jlab), together with strong theoretical support, are crucial for further progress.**



# Work in Progress/Future Plans

- **What can we learn from hard electron scattering on nuclei (measured at Jlab)?**
  - **Spectral functions and momentum distributions.**
  - **Short range correlations.**
  - **“Measuring” momentum distributions?**

# Motivations

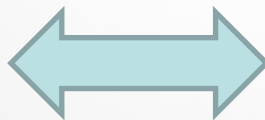
Known since a long time:

Nuclear force: short-range repulsion, medium-range attraction.

Therefore, mean-field picture has limitations.

Nuclear wave functions overlap strongly:  
correlations

Short-range correlations (SRC)  
= nucleon dynamics at short distances



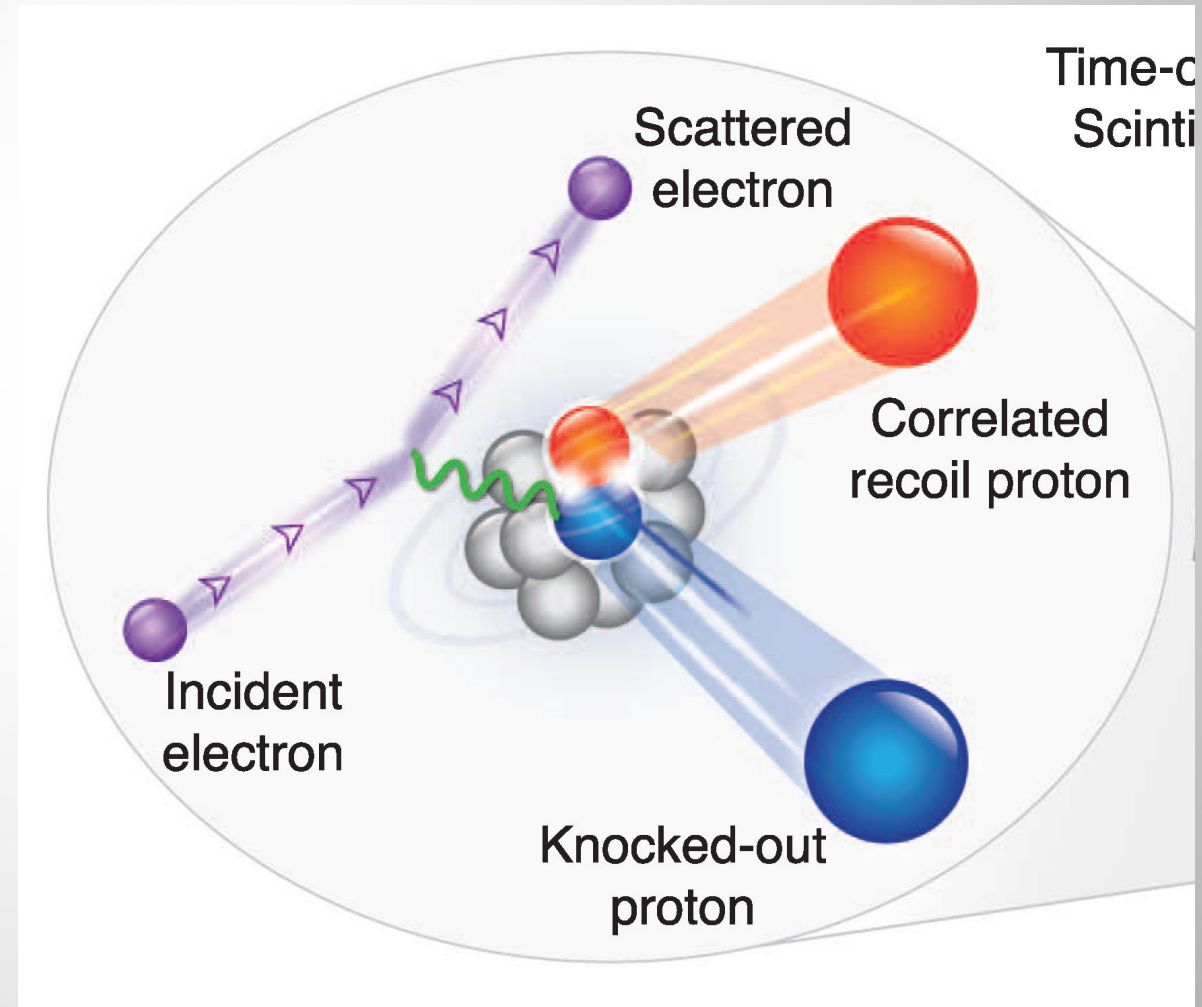
Responsible for the high-momentum components in the wave function

The experimental program at Jlab with  $A(e,e')X$   
(and other) measurements at high momentum-transfer (CLAS Coll.,  
Jefferson Lab Hall A, B, C Coll., etc..) stimulated additional interest

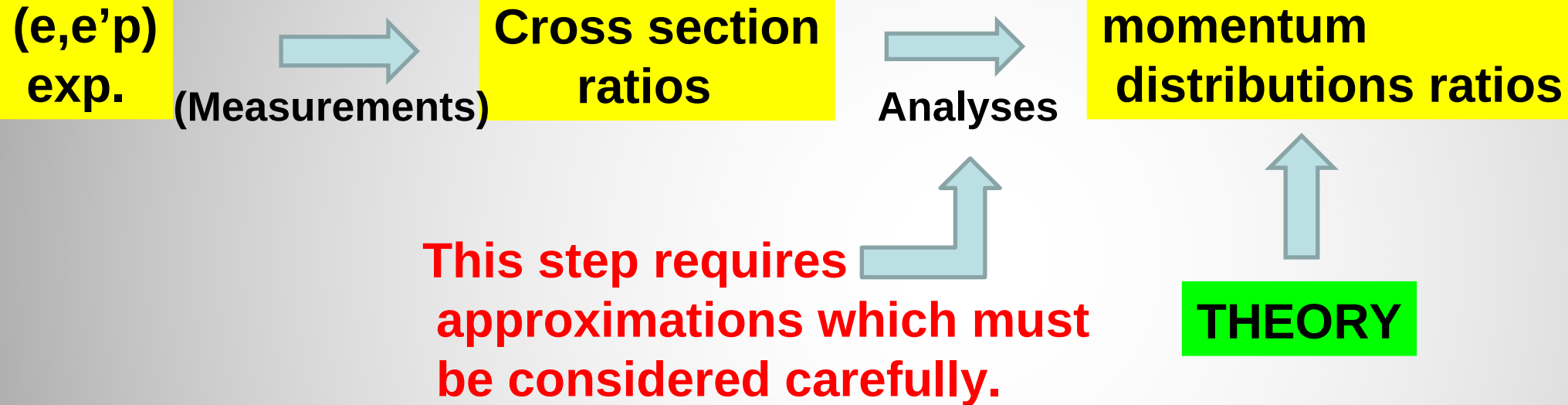
$A(e,e')X$  inclusive  
 $A(e,e'p)$  exclusive  
 $A(e,e'pN)$  exclusive

Frankfurt&Strikman (1981),  
Egiyan et al. (2003,2006),  
Fomin et al. (2012),  
Tang et al. (2003),  
Piassetzky et al. (2006),  
Shneor et al. (2007),  
Subedi et al. (2008),  
Baghdasaryan et al. (2010)  
Hen et al. (2014),  
Korover et al. (2014),  
Monaghan et al. (2014),  
Makek et al. (2016)

**R. Cruz-Torres et al. (2019)**



**Therefore, this is an important and timely issue**



**Intriguing question:**

**What does a nucleon “do” in a nucleus?**

**Can we “see” a high-momentum proton in a nucleus?**

**1980's:**

The bremsstrahlung reaction



was proposed as a way to determine off-shell aspects of the NN interactions.

**1998-2000:**

Fearing & Scherer show the

*“Impossibility of Measuring Off-Shell Amplitudes”*

**2020:**

F.S. et al.: Should we carefully revisit similar issues as those raised by Fearing *et al.* with regard to constraining the off-shell nature of the NN interaction? (In progress.)

Thank you