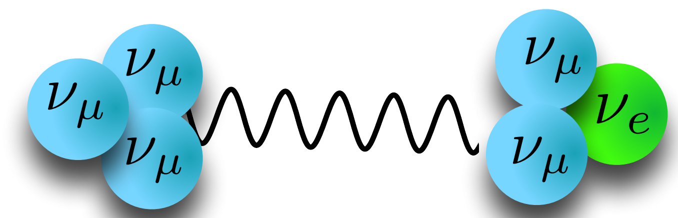
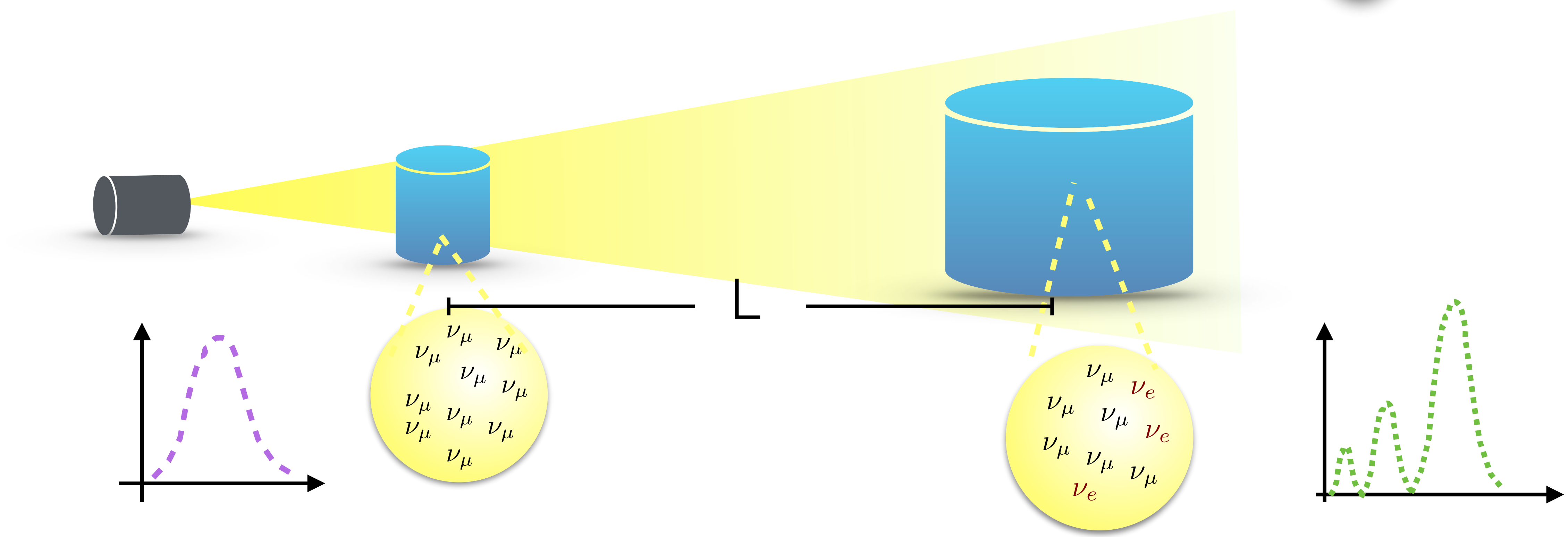
Addressing Neutrino-Oscillation Physics

$$P_{\nu_\mu \to \nu_e}(E, L) \sim \sin^2 2\theta \sin^2 \left(\frac{\Delta m^2 L}{4E}\right) \to \Phi_e(E, L)/\Phi_\mu(E, 0)$$





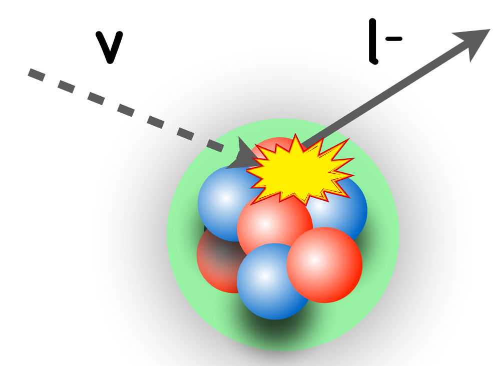
Detectors measure the neutrino interaction rate:

$$N_e(E_{\rm rec}, L) \propto \sum_i \int \Phi_e(E, L)\sigma_i(E) f_{\sigma_i}(E, E_{\rm rec}) dE$$

Reconstructed ν energy

Cross Section

Smearing matrix



A quantitative knowledge of σ(E) and $f_\sigma(E)$ is crucial to precisely extract ν oscillation parameters

To study neutrinos we need nuclei

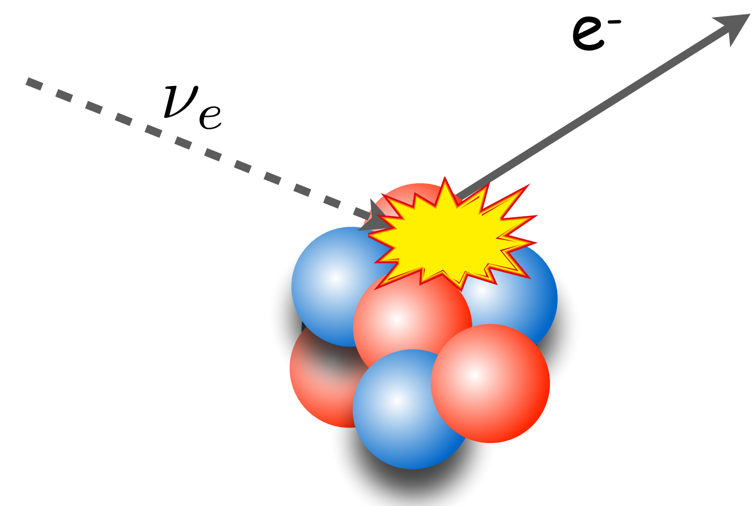
? Where does Nuclear Physics come into play

Number of Interactions = $\sigma \times \Phi \times N$

Cross Section

Neutrino Flux

Targets

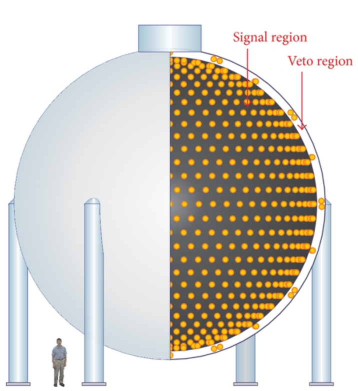


Utilize heavy target in neutrino detectors to maximize interactions→ understand nuclear structure

Carbon

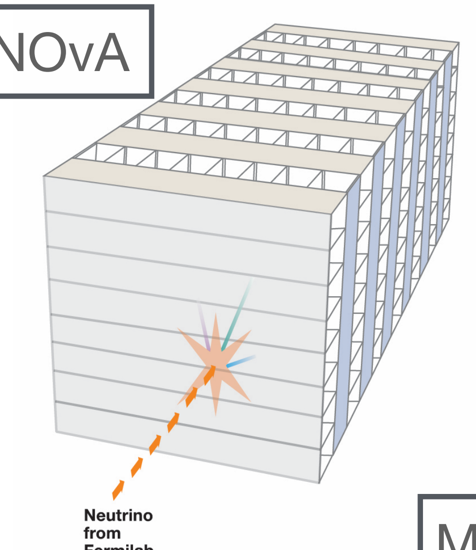
Oxygen

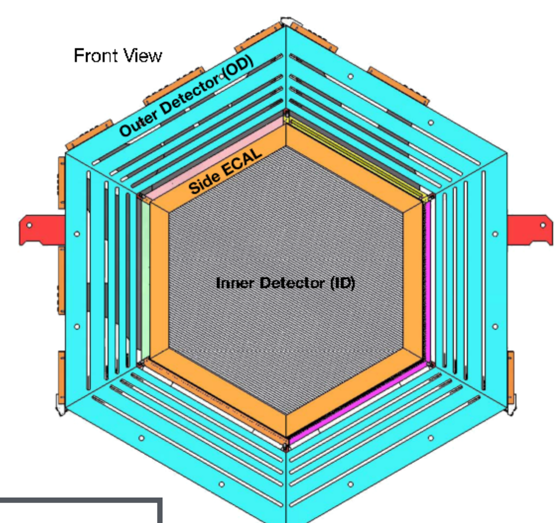
Argon



MiniBooNE

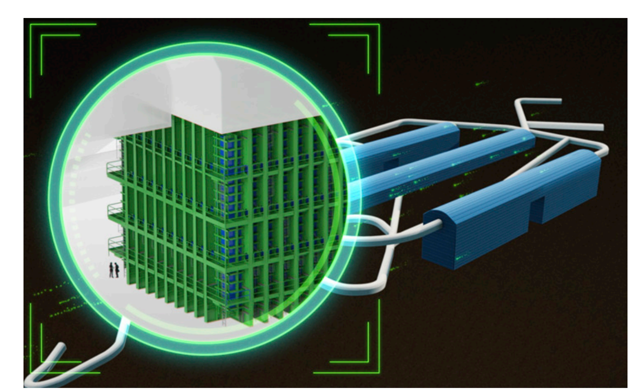
NOvA





MINERvA

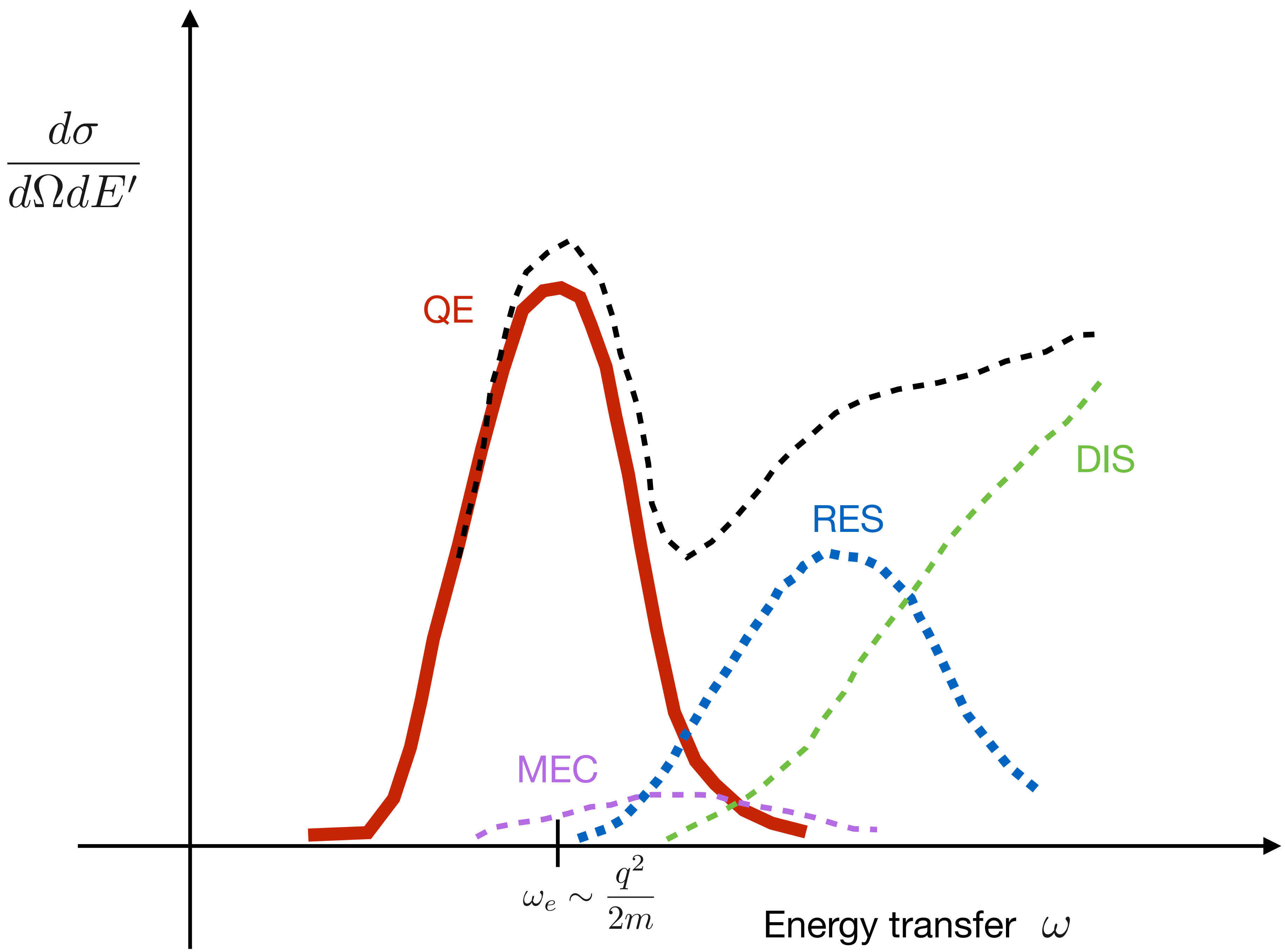
S-K

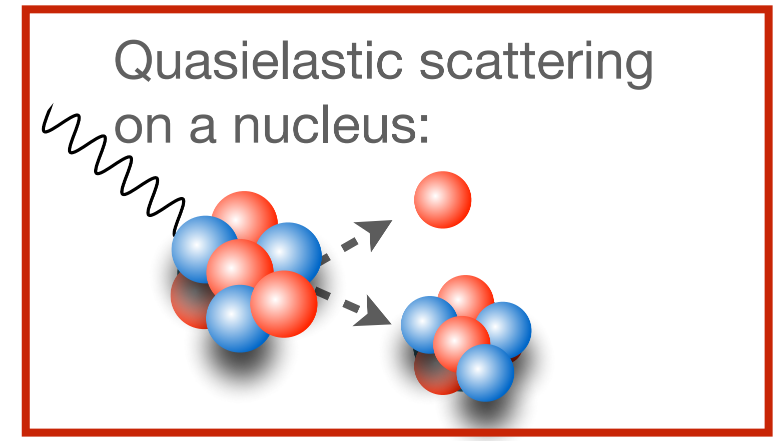


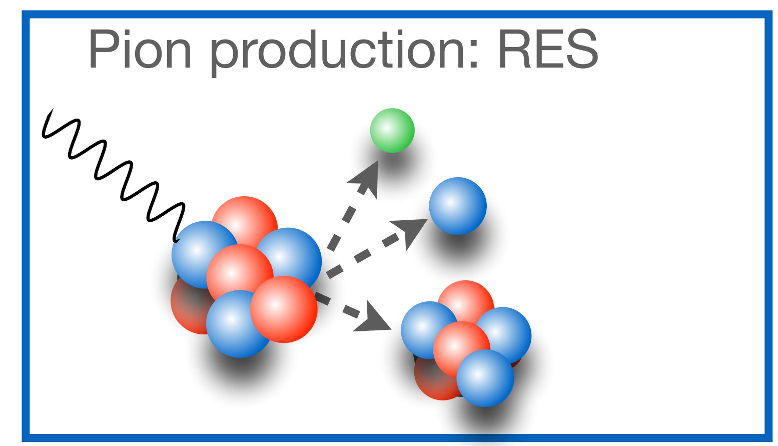
DUNE

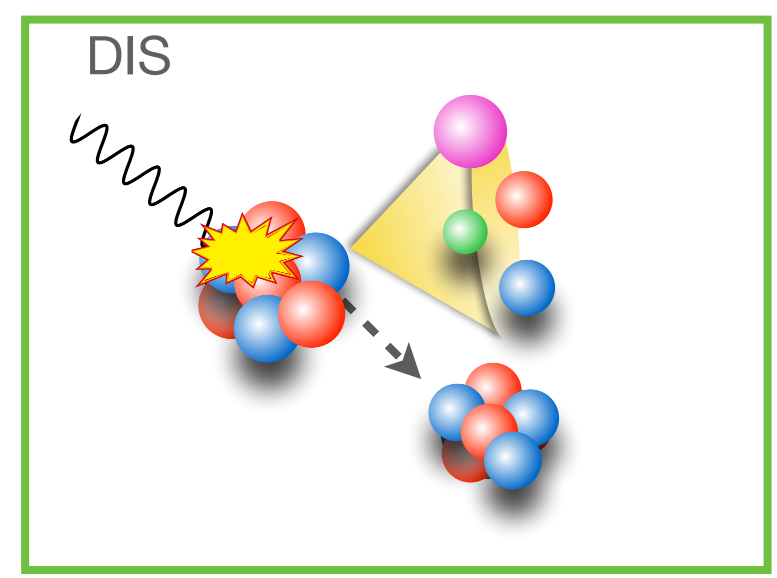
Lepton-nucleus cross section

Different reaction mechanisms contributing to lepton-nucleus cross section —fixed value of the beam energy (monochromatic)





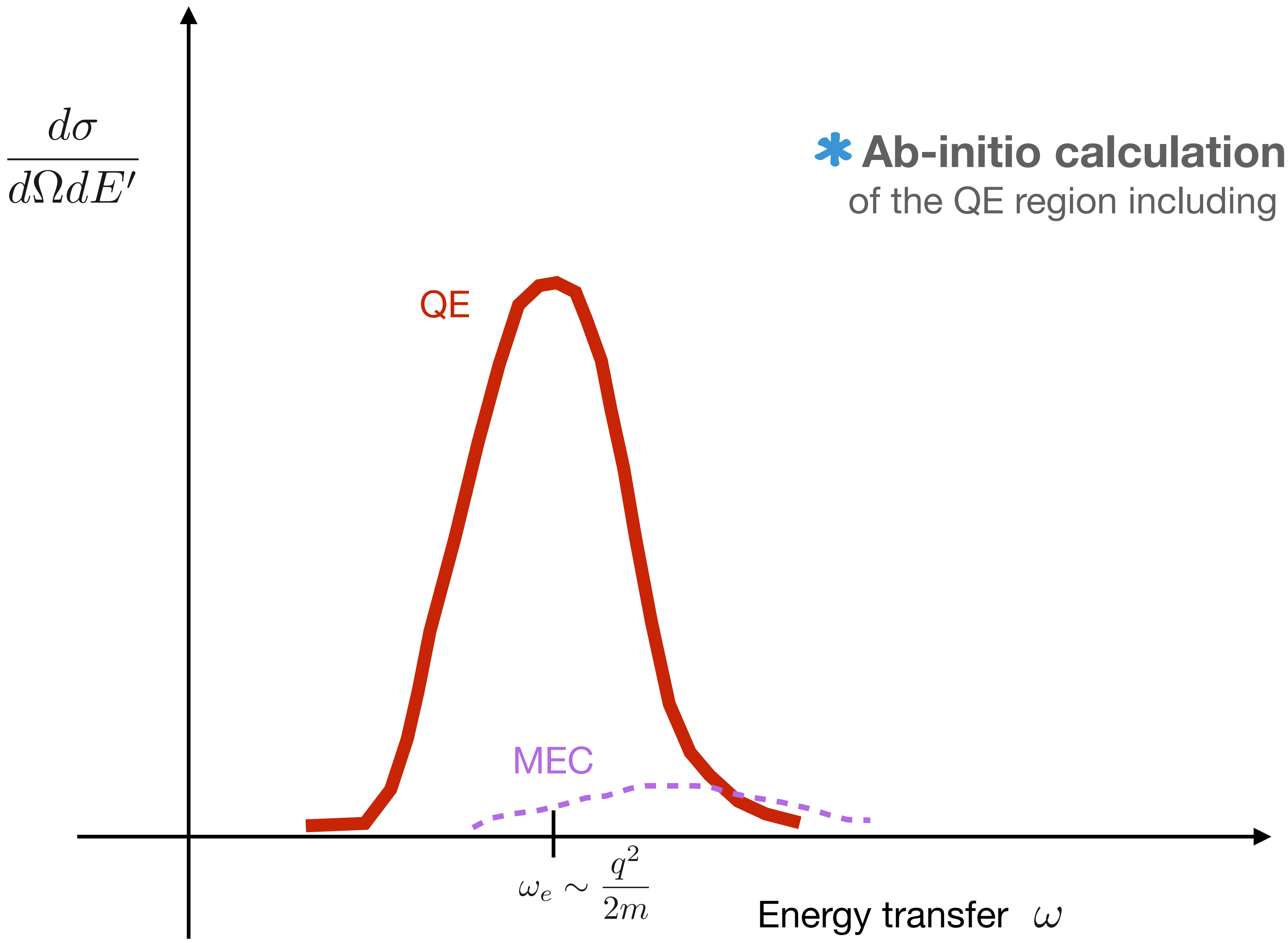




In neutrino experiments these contributions are not nicely separated

Outline of the talk

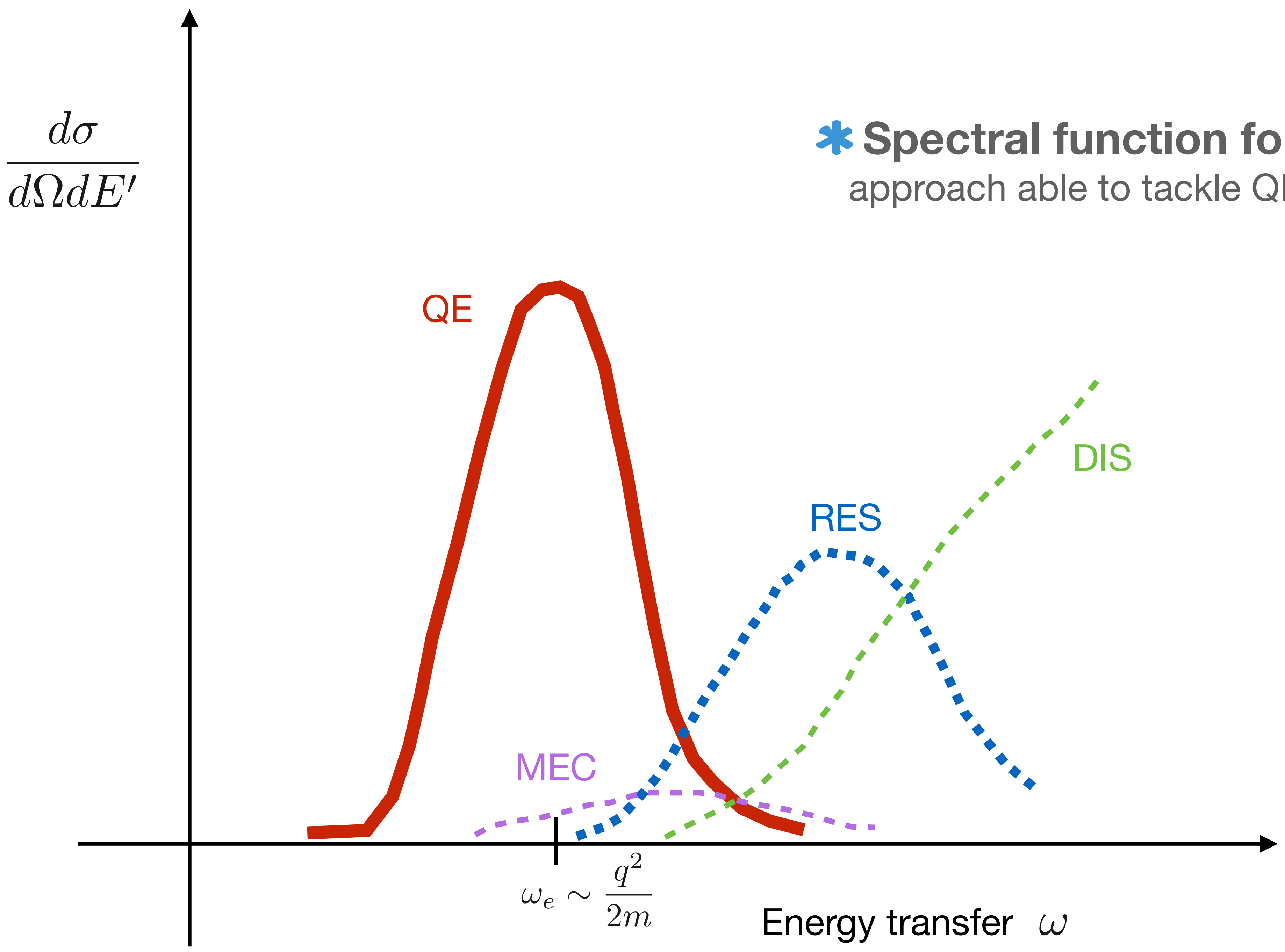
1st Part of the Presentation



* **Ab-initio calculations (QMC)** accurate predictions of the QE region including one- and two-body currents

Outline of the talk

2nd Part of the Presentation

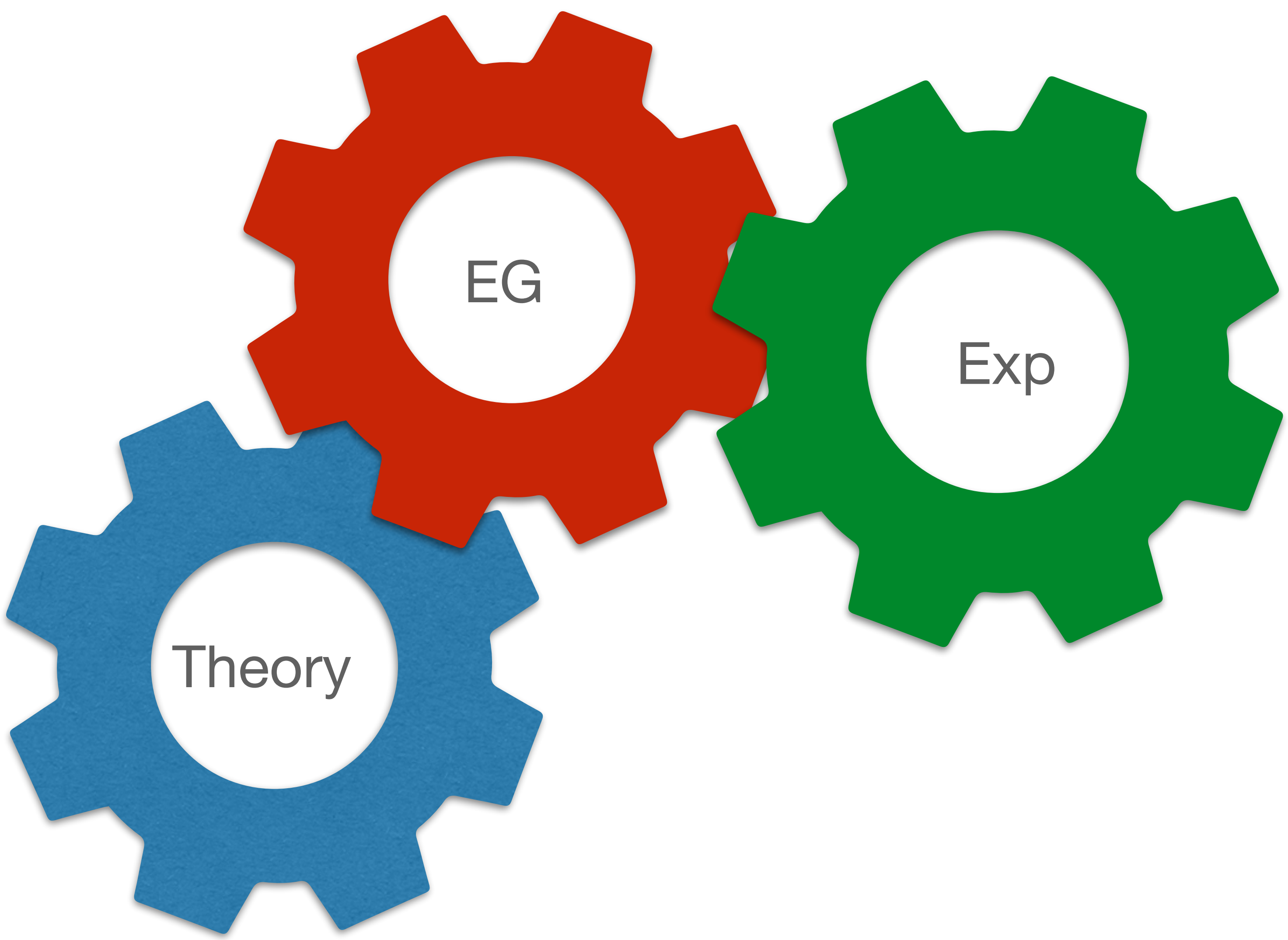


* **Spectral function formalism:** more approximate approach able to tackle QE, dip and π-production regions.

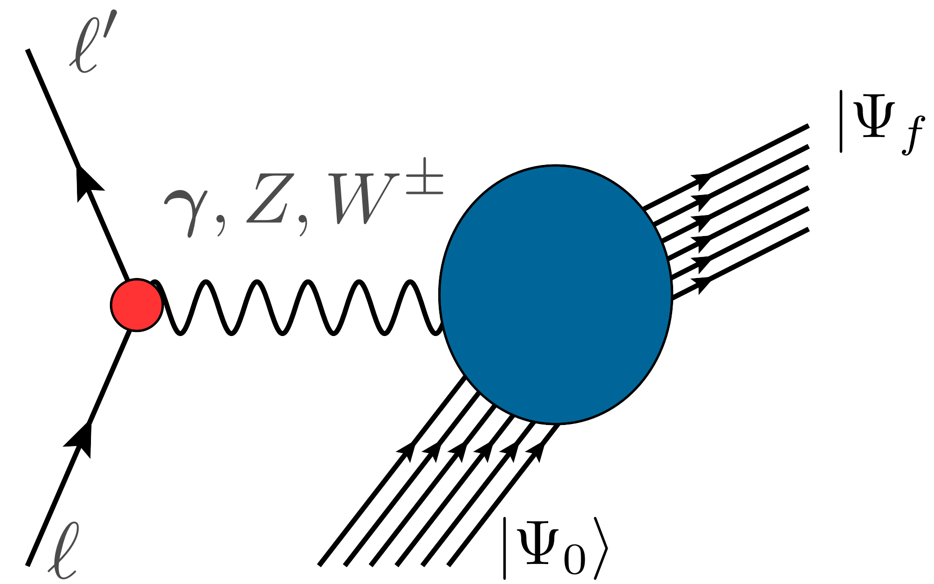
Outline of the talk

3nd Part of the Presentation

- **Intra-nuclear cascade:** propagating particles produced at the interaction vertex through the nucleus



Theory of lepton-nucleus scattering



Inclusive cross section lepton scatters off a nucleus and the hadronic final state is undetected

$$d\sigma \propto L^{\alpha\beta} R_{\alpha\beta}$$

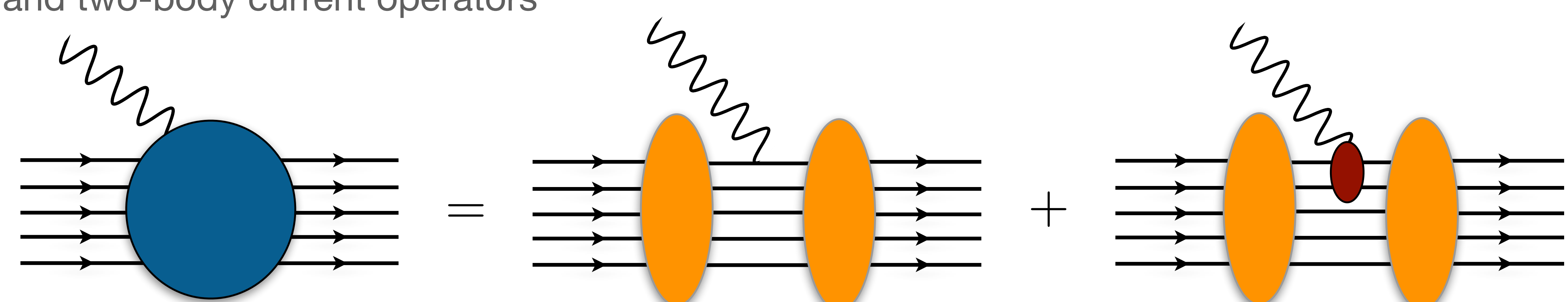
Nuclear response to the electroweak probe:

$$R_{\alpha\beta}(\omega, \mathbf{q}) = \sum_f \langle 0|J_\alpha^\dagger(\mathbf{q})|f\rangle\langle f|J_\beta(\mathbf{q})|0\rangle\delta(\omega - E_f + E_0)$$

The initial and final wave functions describe many-body states:

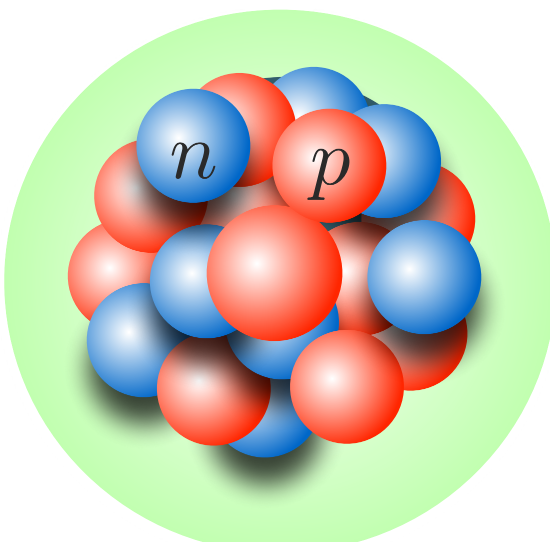
$$|0\rangle = |\Psi_0^A\rangle \;, |f\rangle = |\Psi_f^A\rangle, |\psi_p^N, \Psi_f^{A-1}\rangle, |\psi_k^\pi, \psi_p^N, \Psi_f^{A-1}\rangle \ldots$$

One and two-body current operators



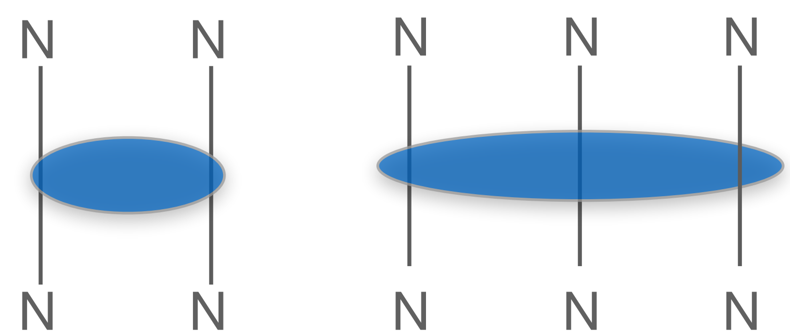
The basic model of nuclear theory

At low energy, the effective degrees of freedom are pions and nucleons:



$$H = \sum_i \frac{\mathbf{p}_i^2}{2m} + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk} + \dots$$

1-body **2-body** **3-body**

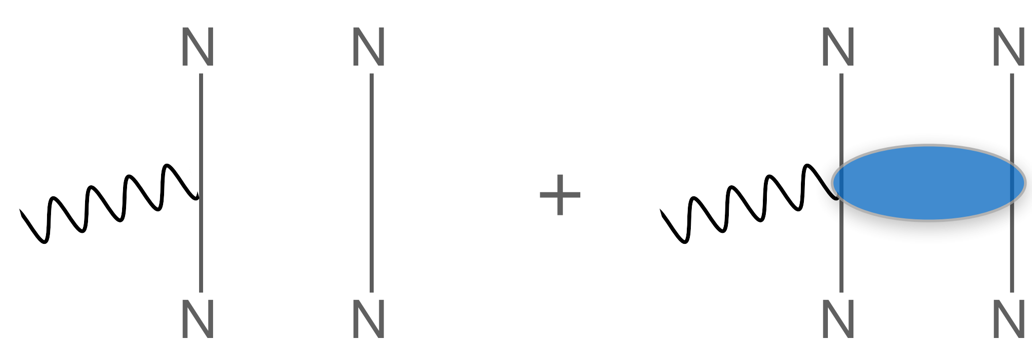


The electromagnetic current is constrained by the Hamiltonian through the **continuity equation**

$$\boldsymbol{\nabla} \cdot \mathbf{J}_{\rm EM} + i[H, J^0_{\rm EM}] = 0 \qquad [v_{ij}, j_i^0] \neq 0$$

The above equation implies that the current operator includes one and two-body contributions

$$J^\mu(q) = \sum_i j_i^\mu + \sum_{i<j} j_{ij}^\mu + \dots$$



Green's Function Monte Carlo approach

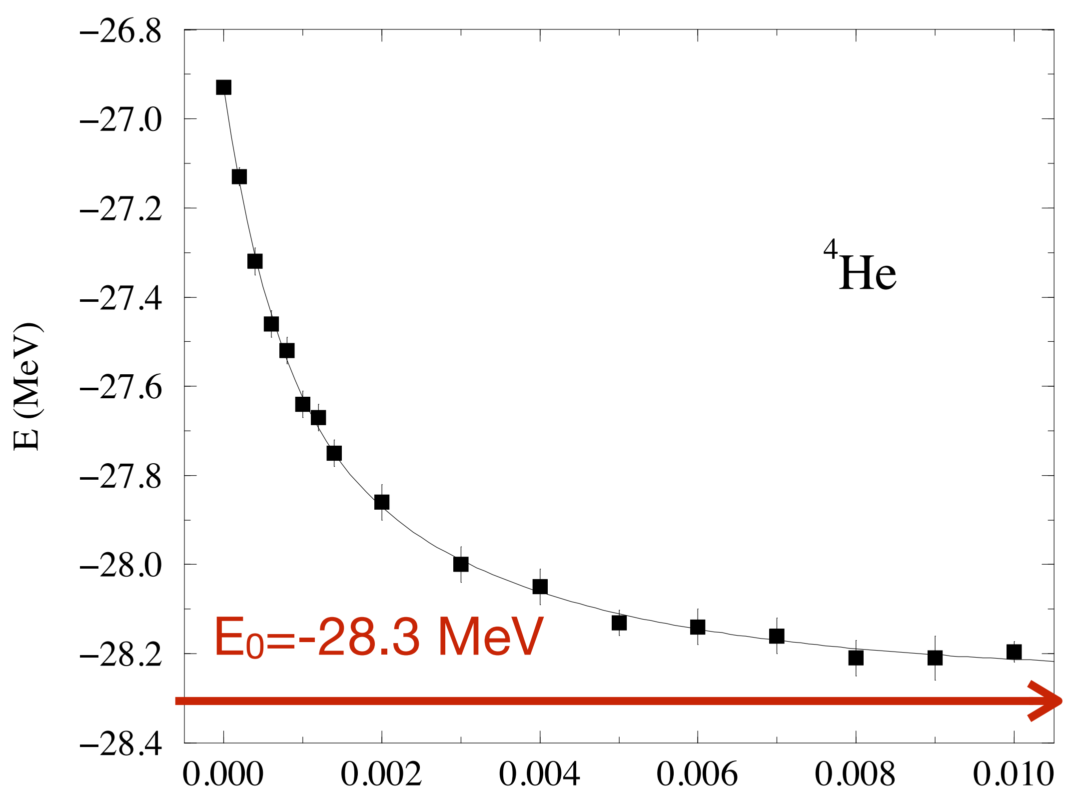
We want to solve the Schrödinger equation

$$H\Psi(\mathbf{R}; s_1 \dots s_A, \tau_1 \dots \tau_A) = E\Psi(\mathbf{R}; s_1 \dots s_A, \tau_1 \dots \tau_A)$$

Any trial wave function can be expanded in the complete set of eigenstates of the the Hamiltonian according to

$$|\Psi_T\rangle = \sum_n c_n |\Psi_n\rangle \qquad H|\Psi_n\rangle = E_n|\Psi_n\rangle$$

QMC techniques **projects out the exact lowest-energy state:** $e^{-(H-E_0)\tau}|\Psi_T\rangle \to |\Psi_0\rangle$



B. Pudliner et al., PRC **56**, 1720 (1997)

The computational cost of the calculation is $2^A \times A!/(Z!(A-Z)!)$

$$|S\rangle = \begin{pmatrix} s_{\uparrow\uparrow\uparrow} \\ s_{\uparrow\uparrow\downarrow} \\ s_{\uparrow\downarrow\uparrow} \\ s_{\uparrow\downarrow\downarrow} \\ s_{\downarrow\uparrow\uparrow} \\ s_{\downarrow\uparrow\downarrow} \\ s_{\downarrow\downarrow\uparrow} \\ s_{\downarrow\downarrow\downarrow} \end{pmatrix}$$

Integral Transform Techniques

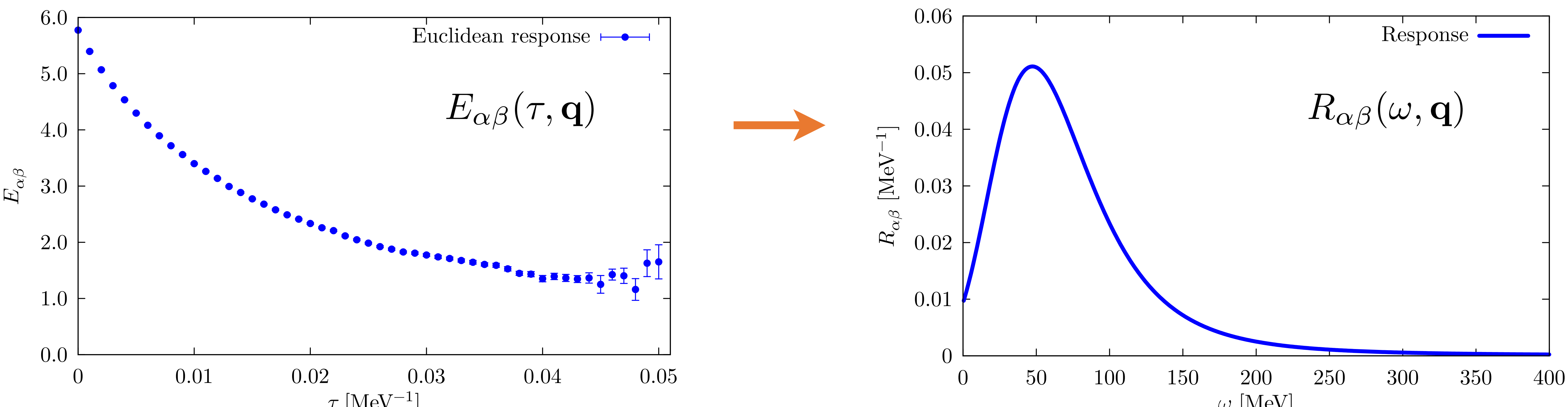
Nuclear response function in principle involve evaluating a number of transition amplitudes:

$$R_{\alpha\beta}(\omega,\mathbf{q}) = \sum_f \langle 0|J^\dagger_\alpha(\mathbf{q})|f\rangle\langle f|J_\beta(\mathbf{q})|0\rangle\delta(\omega - E_f + E_0)$$

Valuable information can be obtained from the integral transform of the response function

$$E_{\alpha\beta}(\sigma,\mathbf{q}) = \int d\omega K(\sigma,\omega)R_{\alpha\beta}(\omega,\mathbf{q}) = \langle\psi_0|J^\dagger_\alpha(\mathbf{q})K(\sigma,H-E_0)J_\beta(\mathbf{q})|\psi_0\rangle$$

Inverting the integral transform is a complicated problem

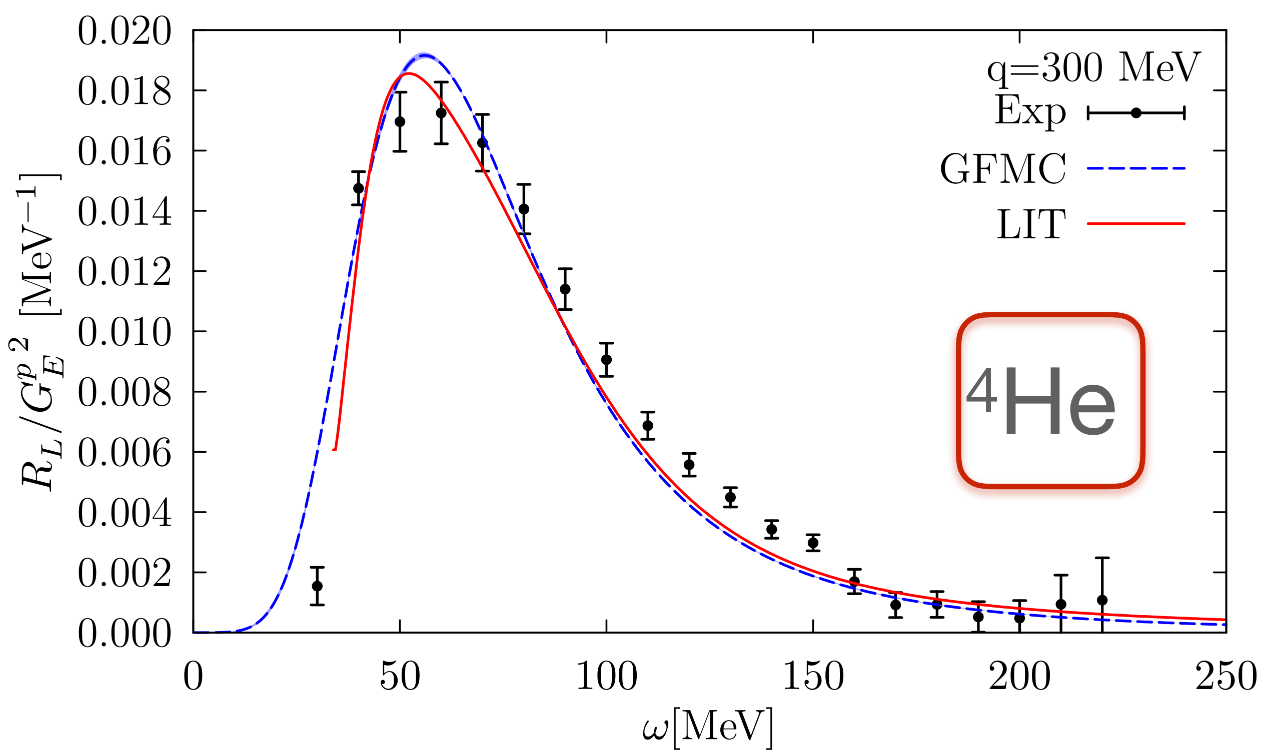


Integral Transform Techniques

- The Lorentz integral transform (LIT)

$$K(\sigma,\omega) = \frac{1}{(\omega-\sigma_R)^2+\sigma_I^2}$$

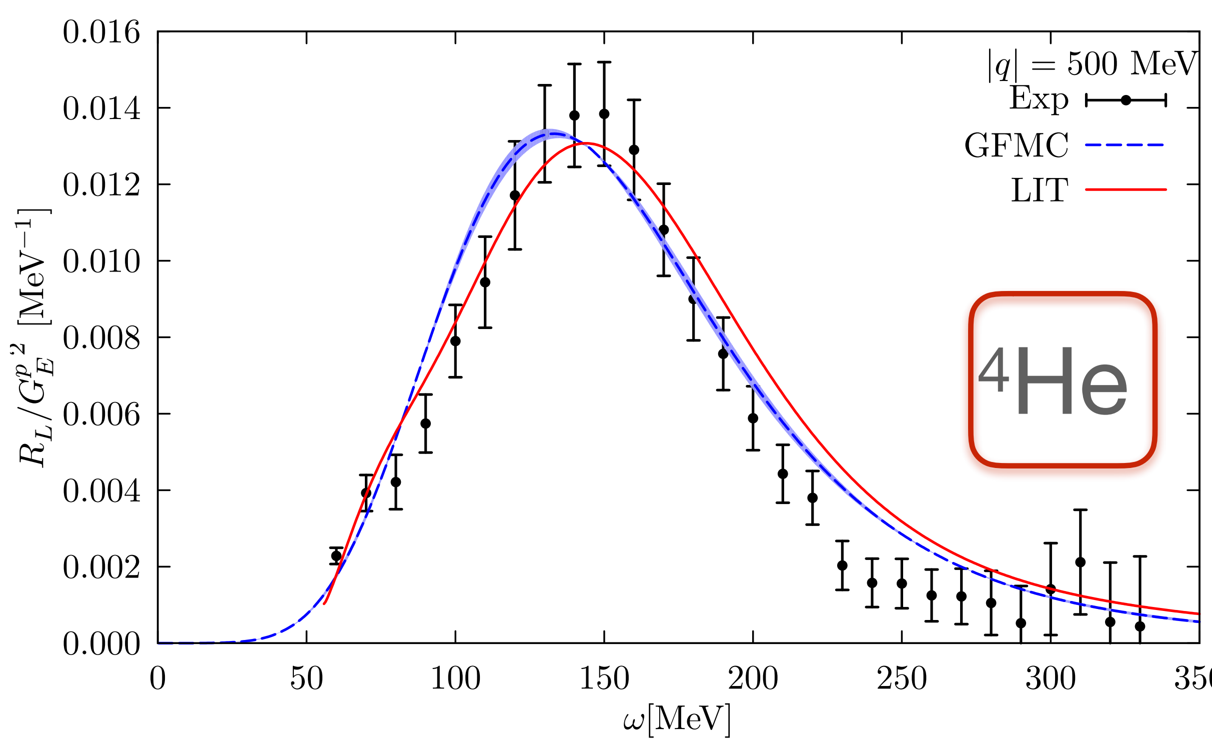
has been successfully exploited in the calculation of nuclear responses:
Using HH: V. D. Efros et al., Phys Lett B 338, 130 (1994)
Using CC: Bacca et al., PRC 76, 014003 (2007), PRL 111, 122502 (2013)



- The Laplace integral transform

$$K(\sigma,\omega) = e^{-\omega\sigma}$$

of the nuclear responses is computed within GFMC and inverted using bayesian techniques: Maximum Entropy
A. Lovato et al, Phys.Rev.Lett. 117 (2016), 082501, Phys.Rev. C97 (2018), 022502

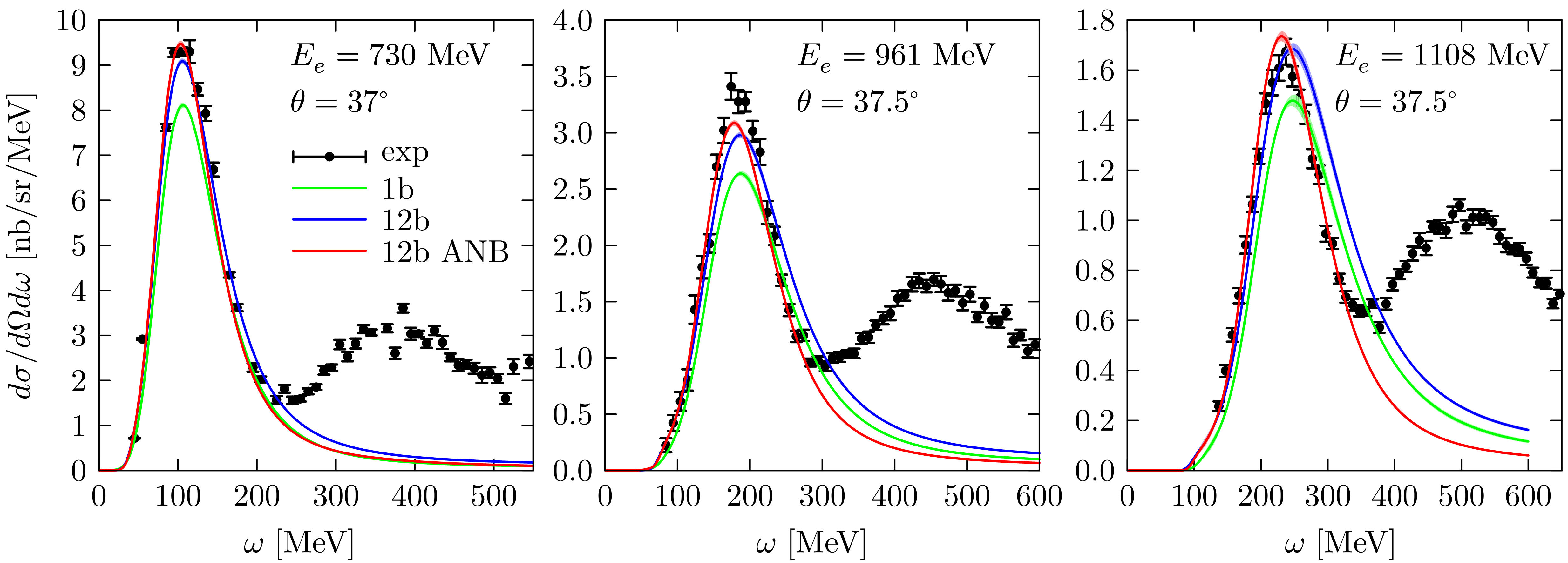


GFMC electron ^{4}He-cross sections

Virtually exact results for nuclear electroweak responses in the **quasi-elastic region** up to moderate values of **q**.
Initial and final state interactions fully accounted for.

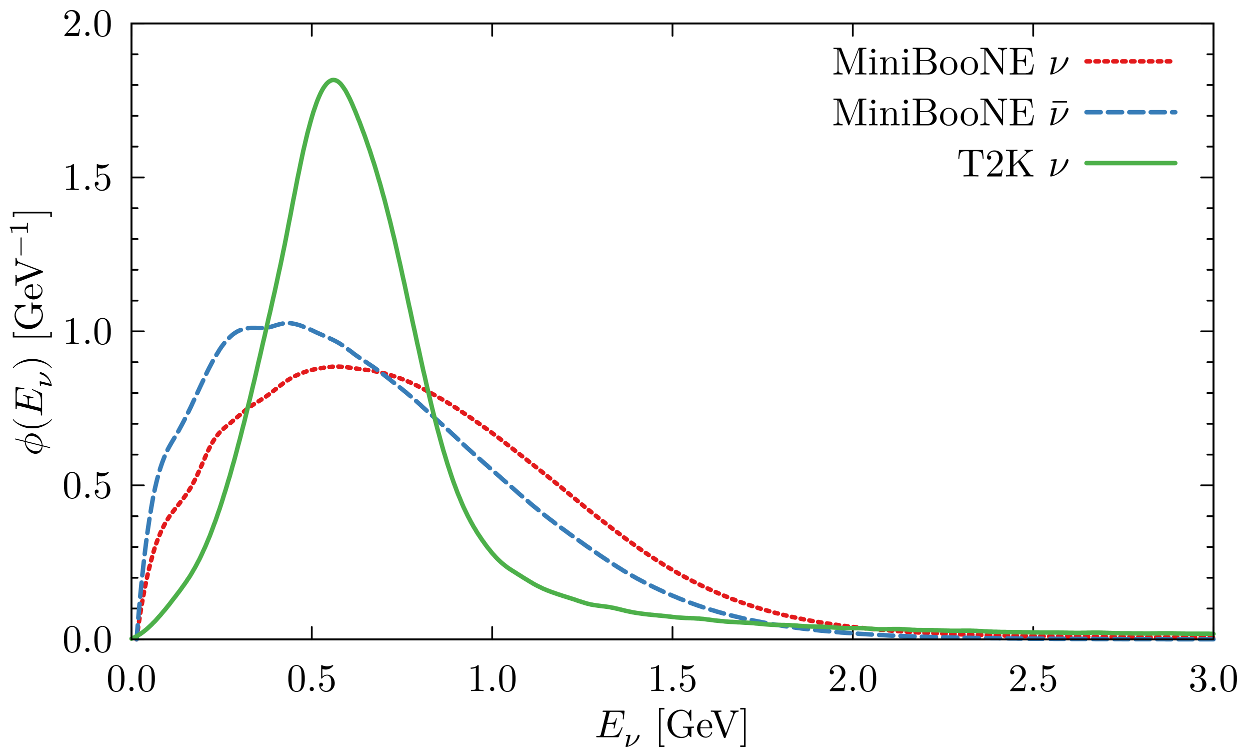
Computational cost grows exponentially with the number of particles: currently limited to ^{12}C

✐ N.R, W. Leidemann, et al PRC 97 (2018) no.5, 055501



- Very good agreement in the quasielastic region when: one- and two-body currents are included
- Peak on the right: π production can not be described within this approach

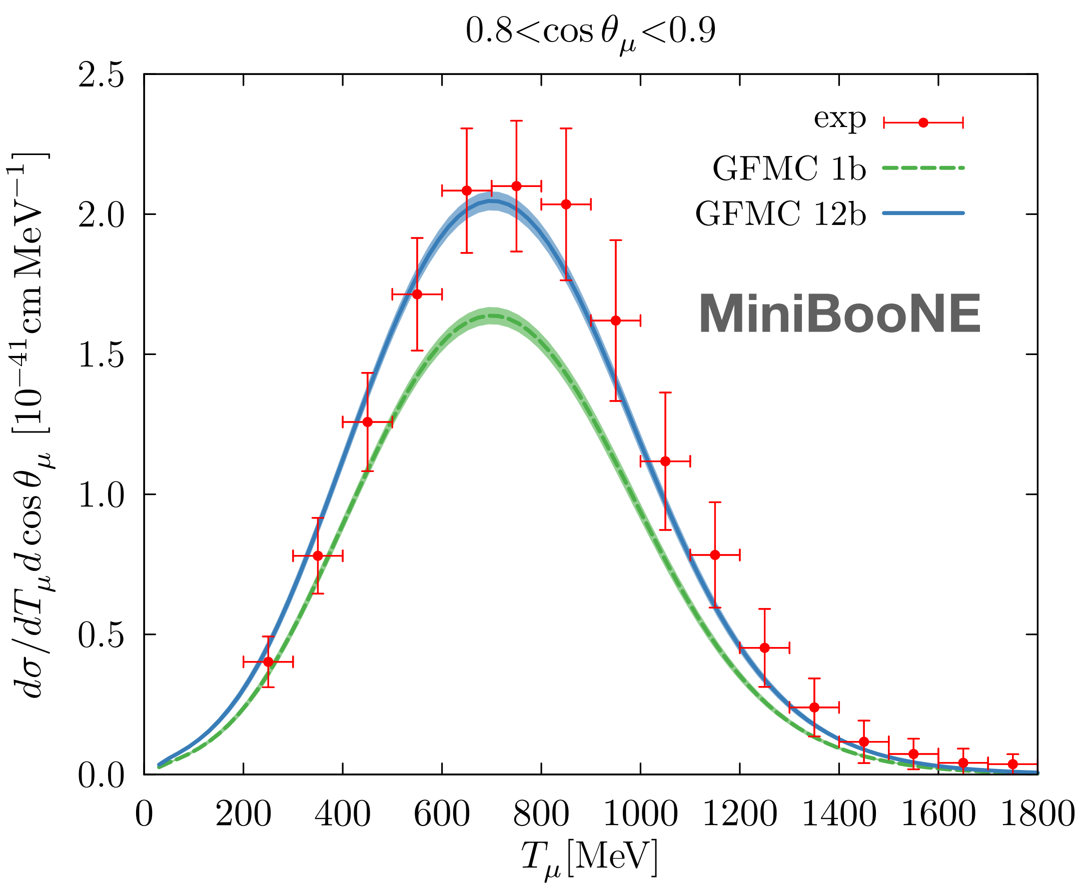
GFMC CC ν_μ ^{12}C-cross sections

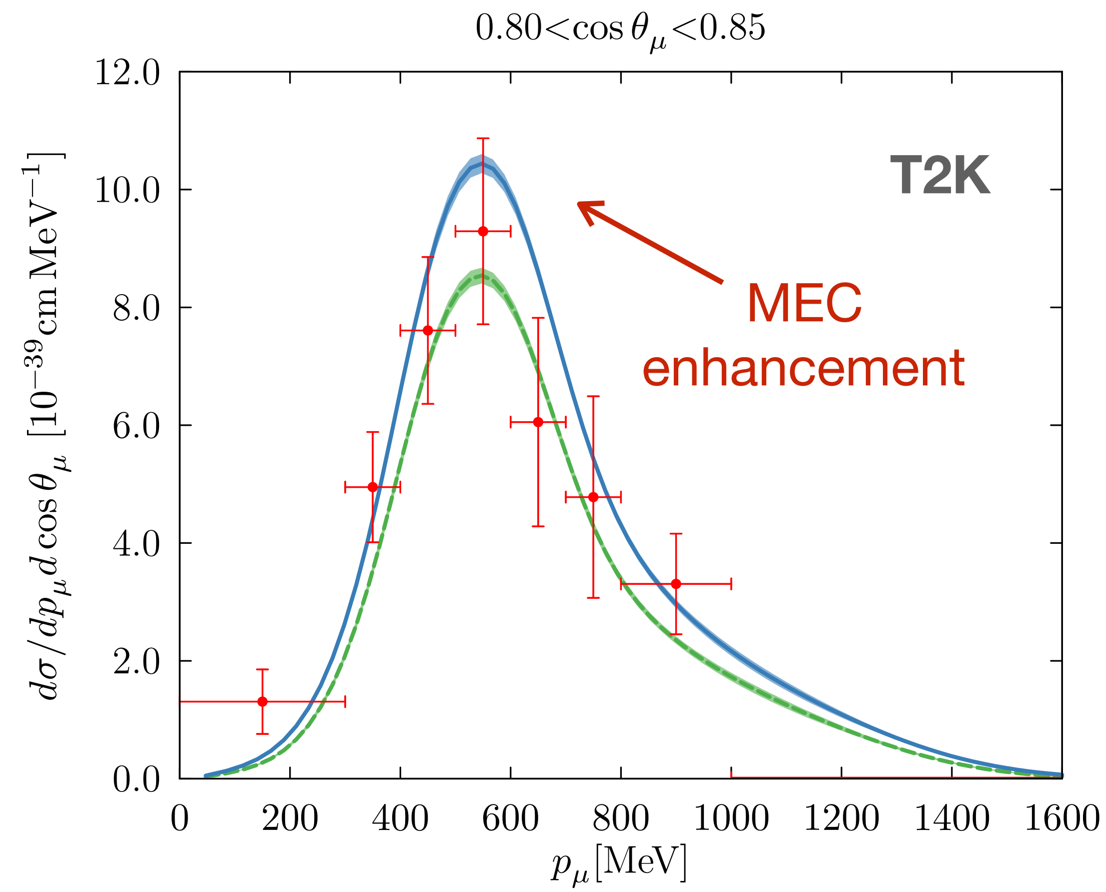


First microscopic calculation of neutrino-nucleus cross section

$$\left\langle \frac{d\sigma}{dT_\mu \, d\cos\theta_\mu} \right\rangle = \int dE_\nu \, \phi(E_\nu) \, \frac{d\sigma(E_\nu)}{dT_\mu \, d\cos\theta_\mu}$$

✐A.Lovato, NR et al, Phys.Rev.X 10 (2020) 3, 031068

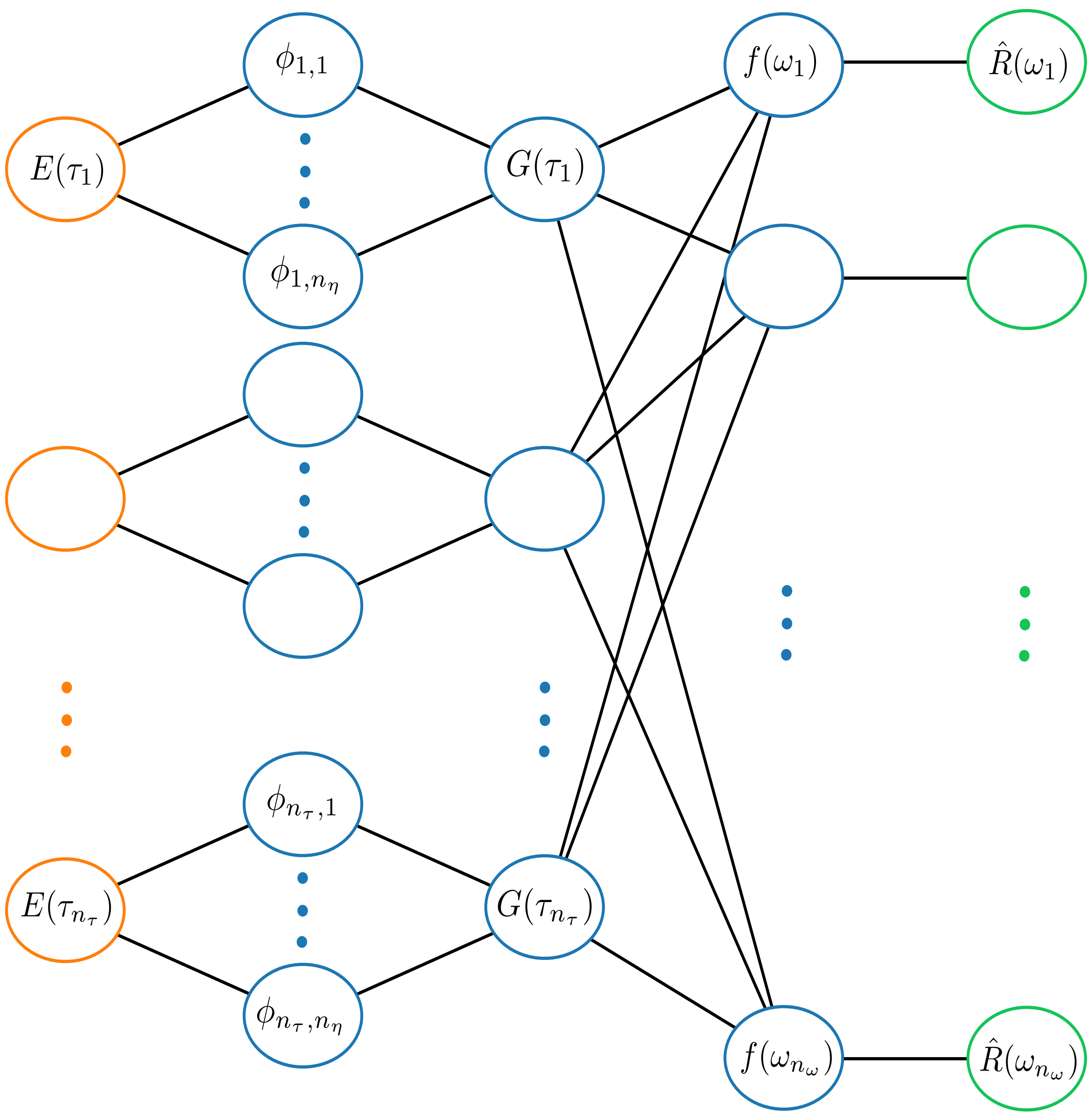




Machine learning-based inversion of R(q,ω)

$$E(\mathcal{T}) = K(\Omega, \mathcal{T})R(\Omega) \quad \rightarrow \quad R(\Omega) = K(\Omega, \mathcal{T})^{-1}E(\mathcal{T})$$

Inversion is unstable because of exponentially small tails in the kernel for large τ



We define a Gaussian kernel basis functions

$$\phi(x, \mu, \sigma) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

We contract the Gaussian unit by weights to obtain the output associated to ω_i

$$f(\omega_i) = \sum_{j=1}^{n_\tau} W_{i,j} \sum_{k=1}^{n_\eta} \phi(E(\tau_j), \mu_{j,k}, \sigma_{j,k})$$

$$i = 1, \ldots, n_\omega$$

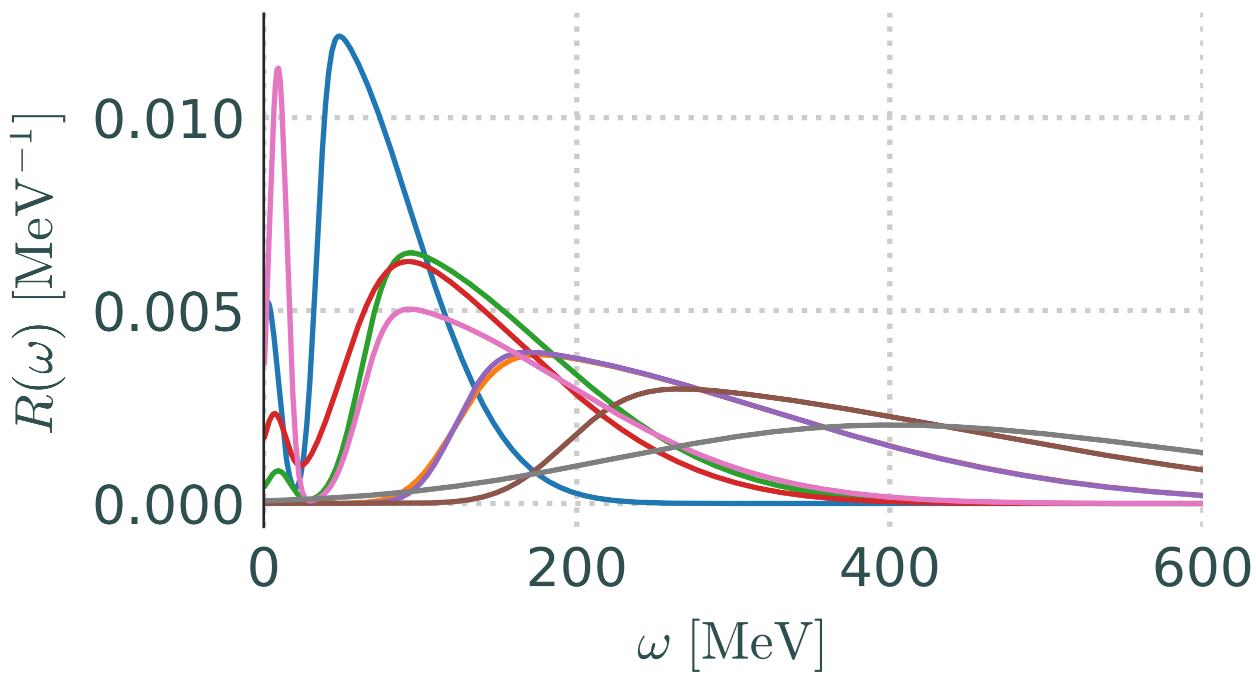
the training parameters are: $\theta = (\mu, \sigma, \mathbf{W})$

The response functions are obtained by exponentiating $f(\omega_i)$

✐ K.Raghavan, et al, arxiv:2010.12703

Machine learning-based inversion of R(q,ω)

Training data examples of response functions



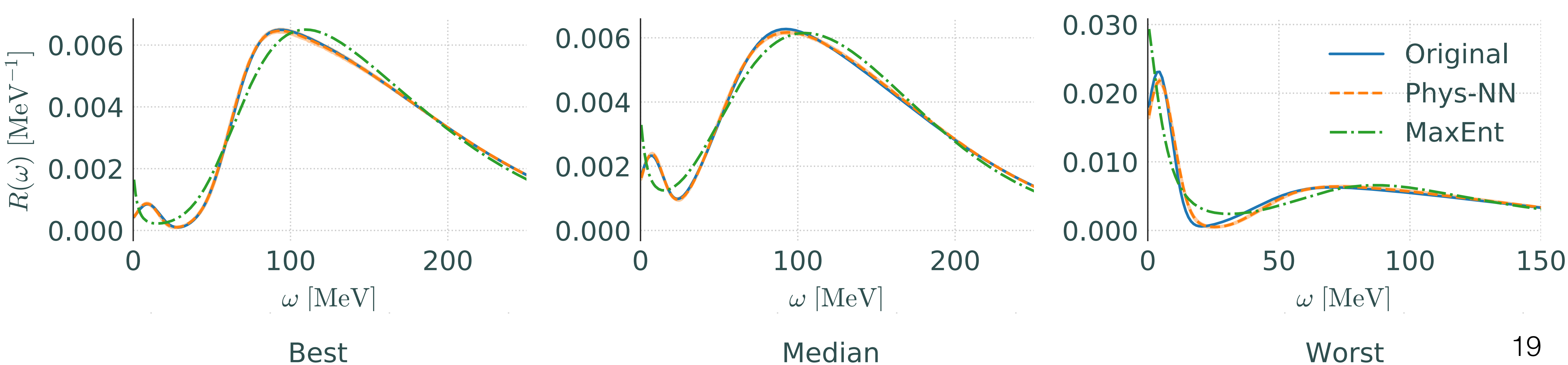
The parameters are found by the supervised learning approach solving

$$\min_{\theta} \frac{1}{|\mathbb{T}|} \sum_{k \in \mathbb{T}} \ell\left(E_k(\mathcal{T}), R_k(\Omega), \hat{R}_k(\Omega;\theta)\right)$$

Using a mini-batch gradient descent to minimize a loss function that is the sum of the response and Euclidean cost

$$\ell(E_k, R_k, \hat{R}_k) = \gamma_R S_R(R_k, \hat{R}_k) + \gamma_E \chi^2_E(E_k, \hat{R}_k)$$

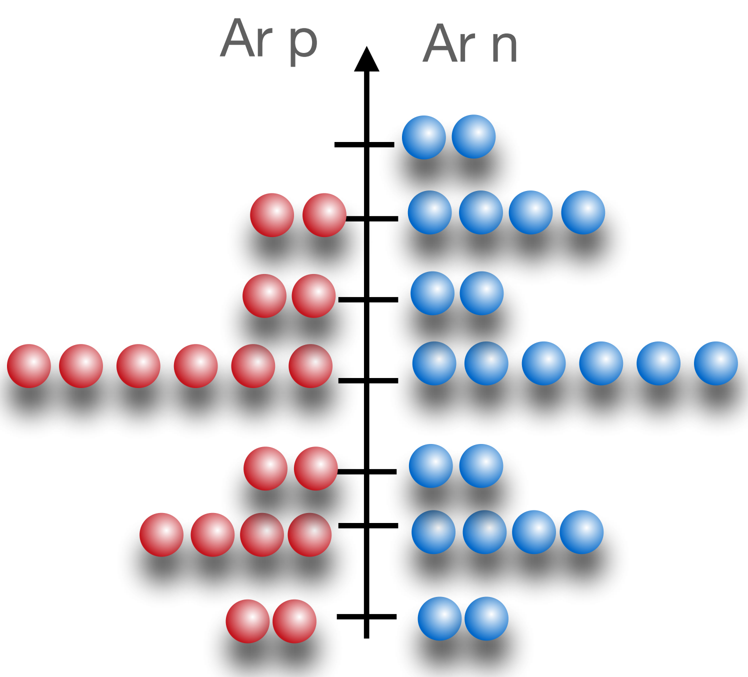
Comparison between the Phys-NN and MaxEnt reconstructions for the two-peak dataset



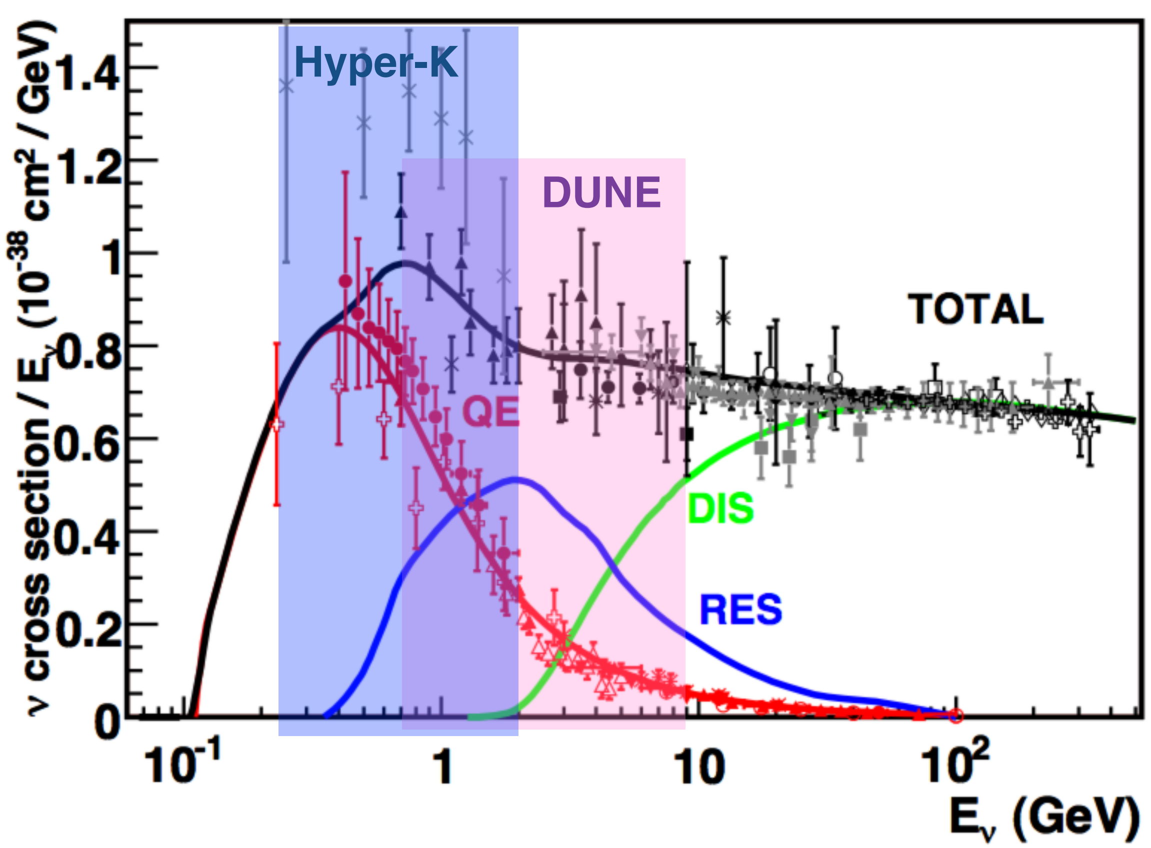
Addressing future precision experiments

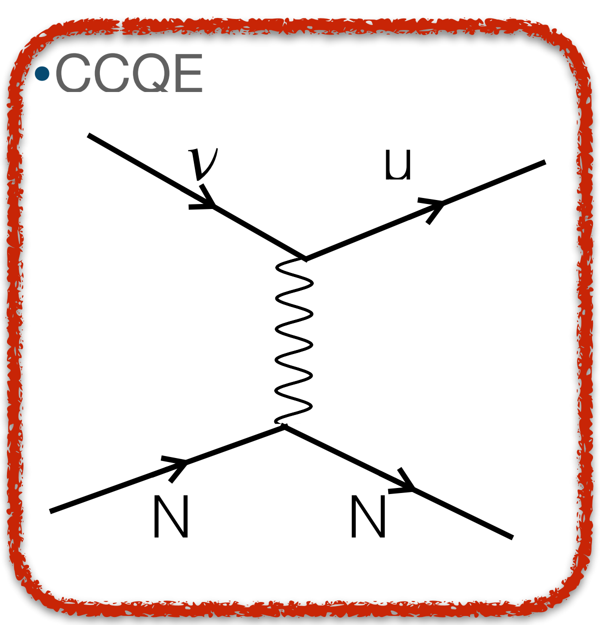
- Liquid Argon TPC Technology

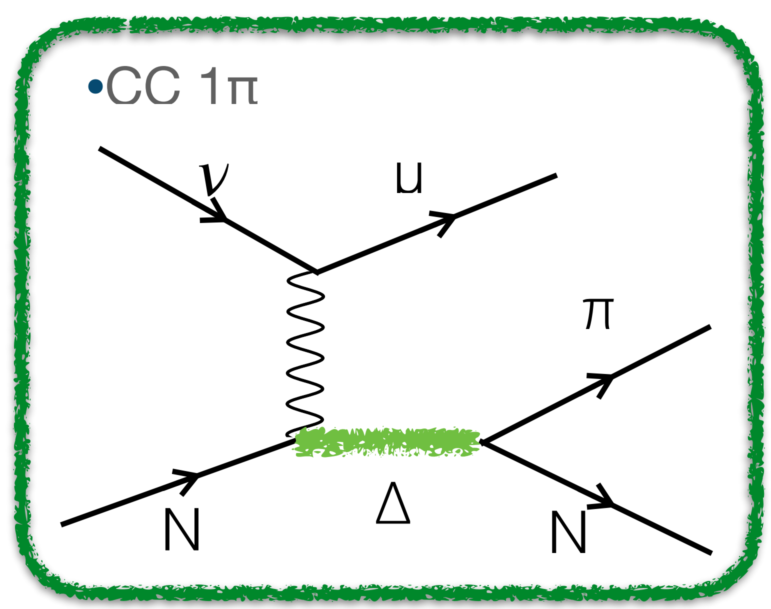
J.A. Formaggio and G.P. Zeller, Rev. Mod. Phys. 84 (2012)

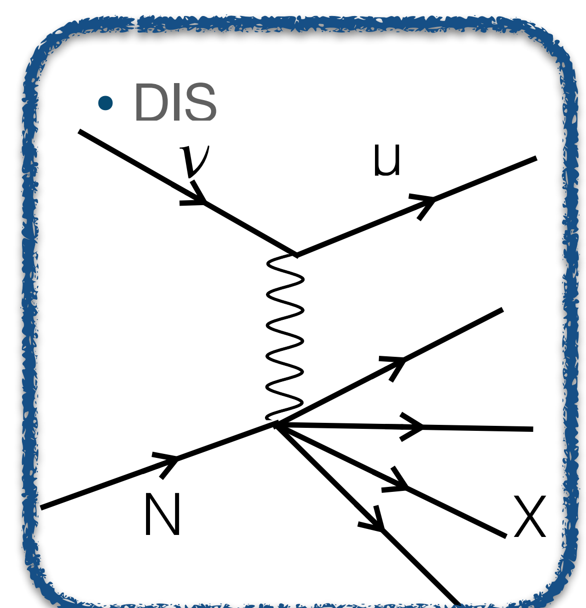


- Ar has a complicated structure, out of the reach of most of the ab initio methods









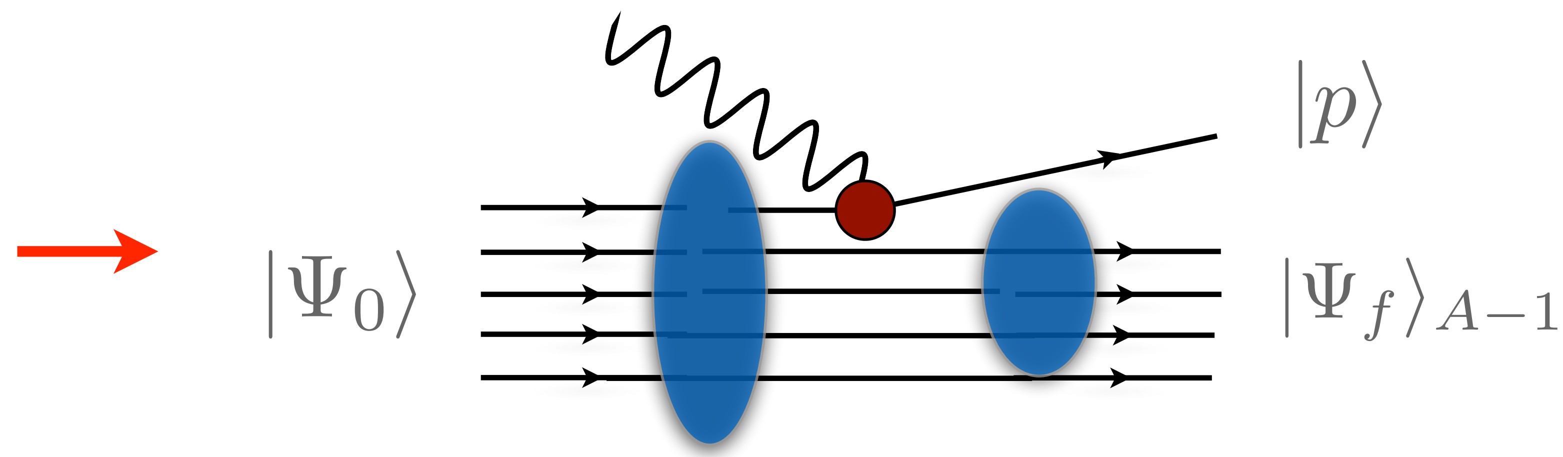
- The dominant reaction mechanism changes dramatically over the region of interest to oscillation experiment

Factorization Scheme and Spectral Function

For sufficiently large values of |**q**|, the **factorization scheme** can be applied under the assumptions

$$|\Psi_f\rangle \to |p\rangle \otimes |\Psi_f\rangle_{A-1}$$

$$J_\alpha = \sum_i j^i_\alpha$$



- The matrix element of the current can be written in the factorized form

$$\langle 0|J_\alpha|f\rangle \to \sum_k \langle 0|[|k\rangle \otimes |f\rangle_{A-1}]\langle k|\sum_i j^i_\alpha|p\rangle$$

- The nuclear cross section is given in terms of the one describing the interaction with individual bound nucleons

$$d\sigma_A = \int dE\, d^3k d\sigma_N P(\mathbf{k}, E)$$

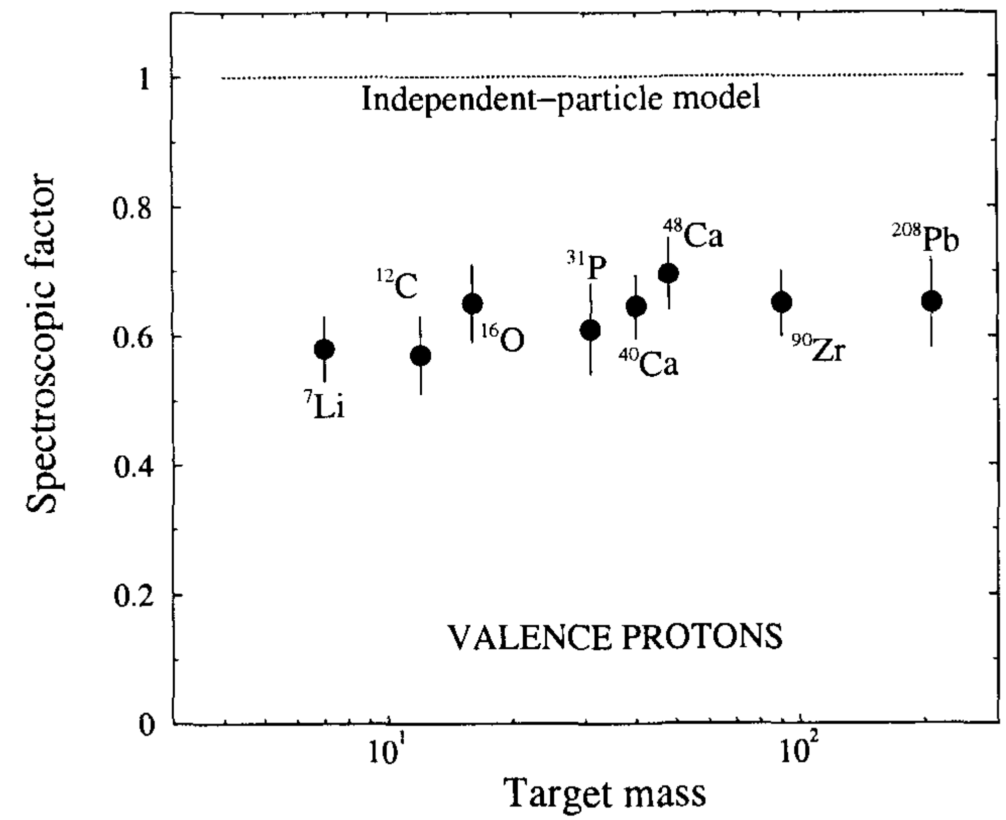
- The intrinsic properties of the nucleus are described by the hole spectral function

The CBF Spectral Function of finite nuclei

- ^{16}O Spectral Function obtained within CBF and using the Local Density Approximation

$$P_{LDA}(\mathbf{k}, E) = P_{MF}(\mathbf{k}, E) + P_{corr}(\mathbf{k}, E)$$

$$\sum_n Z_n |\phi_n(\mathbf{k})|^2 F_n(E - E_n)$$



O. Benhar, A. Fabrocini, and S. Fantoni, Nucl. Phys. A505, 267 (1989).

O. Benhar, A. Fabrocini, S. Fantoni, and I. Sick, Nucl. Phys. A579, 493 (1994)

The CBF Spectral Function of finite nuclei

- ^{16}O Spectral Function obtained within CBF and using the Local Density Approximation

$$P_{LDA}(\mathbf{k},E) = P_{MF}(\mathbf{k},E) + P_{corr}(\mathbf{k},E)$$

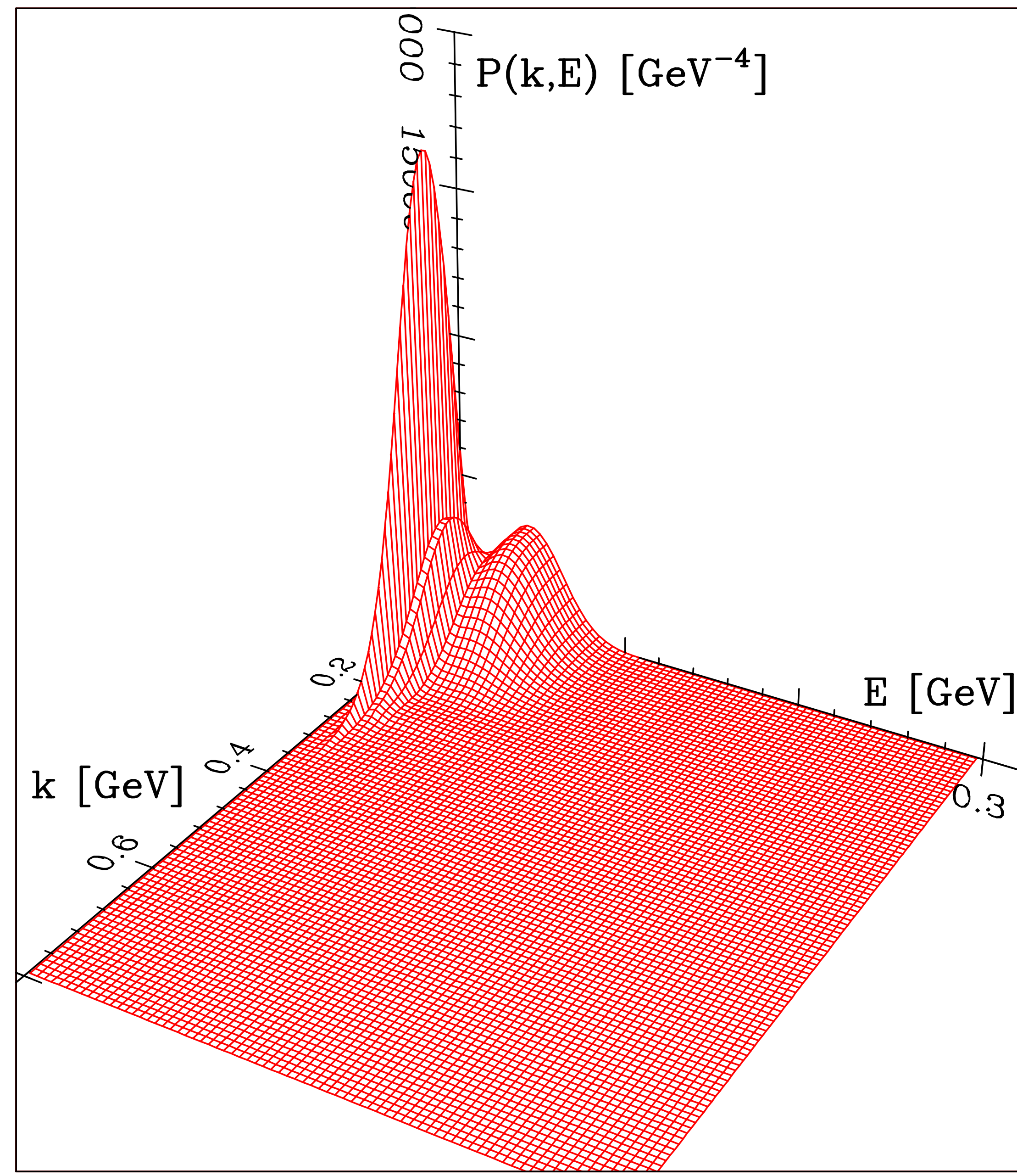
$$\int d^3r P^{NM}_{corr}(\mathbf{k},E;\rho=\rho_A(r))$$

- The one-body Spectral function of nuclear matter:

$$H = \sum_i \frac{\mathbf{p}_i^2}{2m} + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk} + \dots$$

v_{ij}: Argonne v18

V_{ijk}: UIX, IL7



- The Correlated Basis Function approach accounts for correlations induced by the nuclear interactions

$$\Phi_n(x_1 \dots x_A) \rightarrow \mathcal{F}\,\Phi_n(x_1 \dots x_A)$$

O. Benhar, A. Fabrocini, and S. Fantoni, Nucl. Phys. A505, 267 (1989).

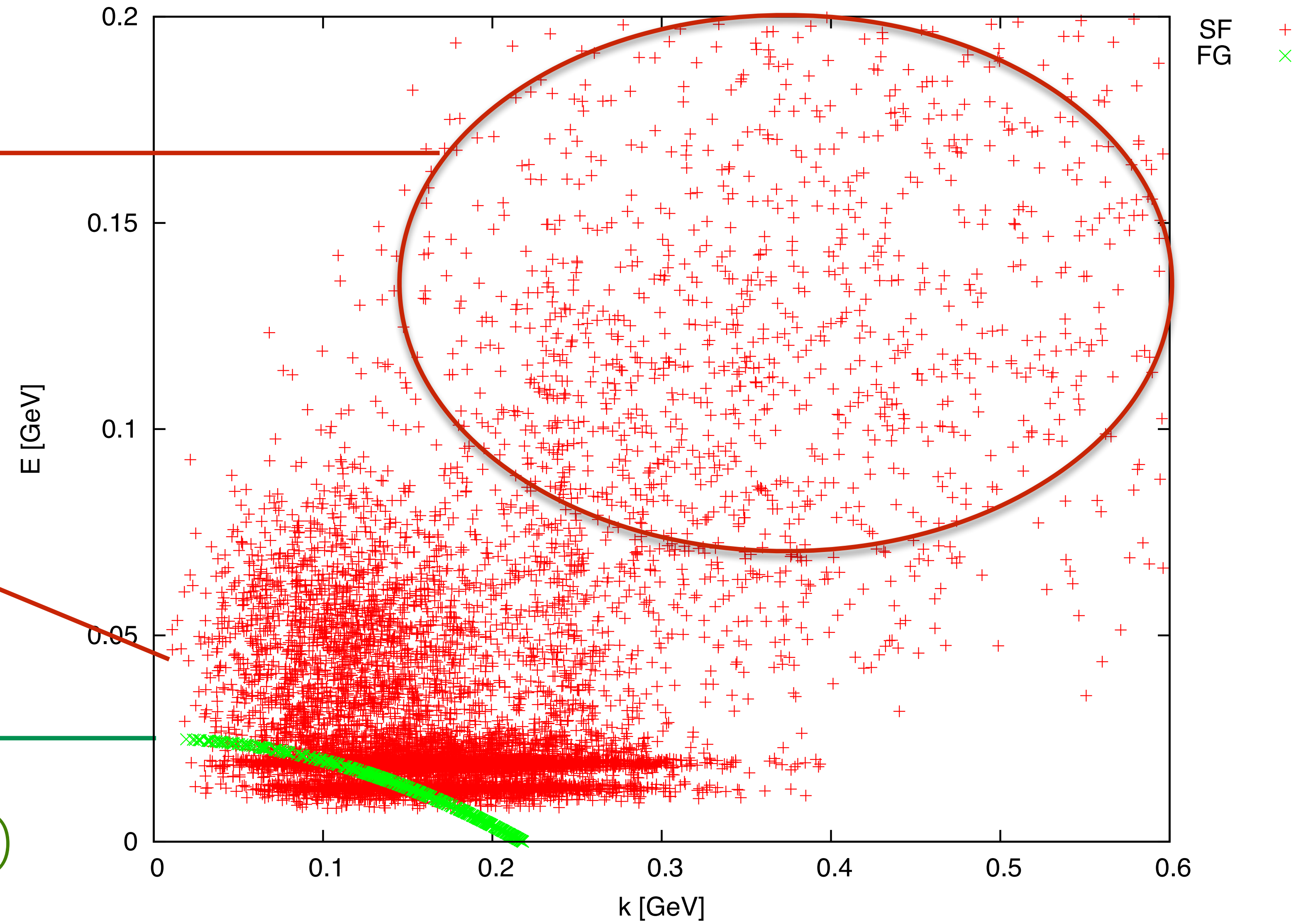
The one-nucleon Spectral Function

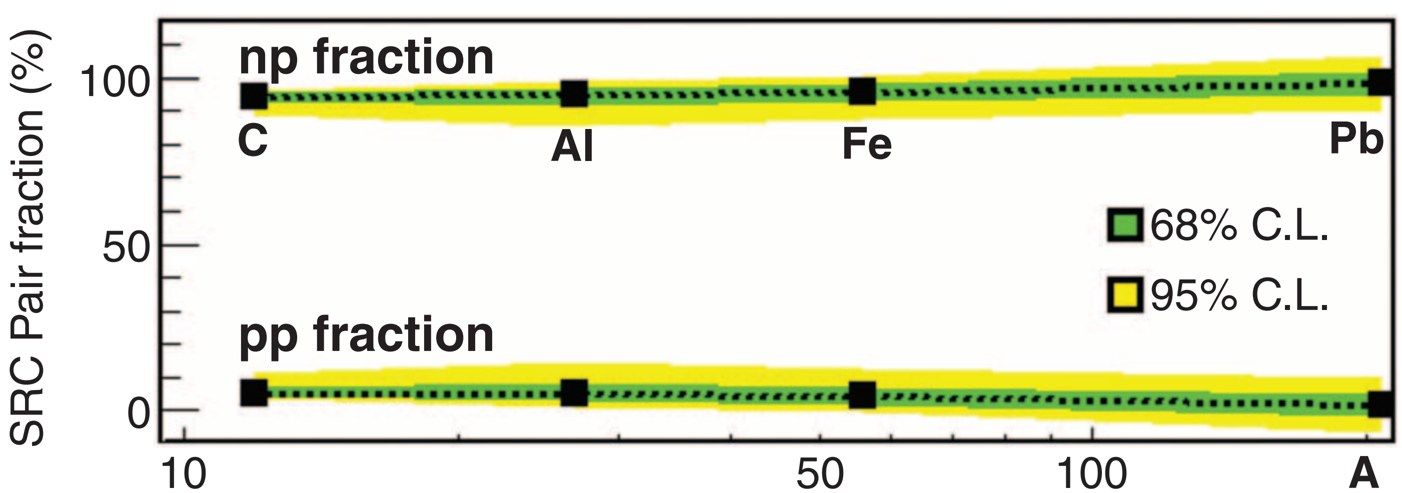
High energy and momentum correlated pairs

Realistic SF: 80% shell model picture, 20% SRC

Fermi gas contribution :

$$P_{\mathrm{FG}}(\mathbf{k}, E) = \delta(E - \epsilon_B)\theta(k_F - |\mathbf{k}|)$$





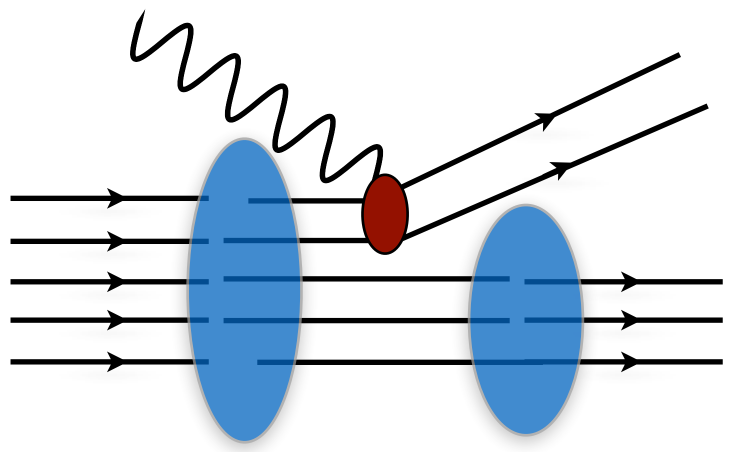
Observed **dominance of np-over-pp** pairs for a variety of nuclei

Consequence of the **tensor component of the nucleon-nucleon interaction**

Extended Factorization Scheme

- Two-body currents are included rewriting the hadronic final state as

$$|f\rangle \to |pp'\rangle_a \otimes |f_{A-2}\rangle$$



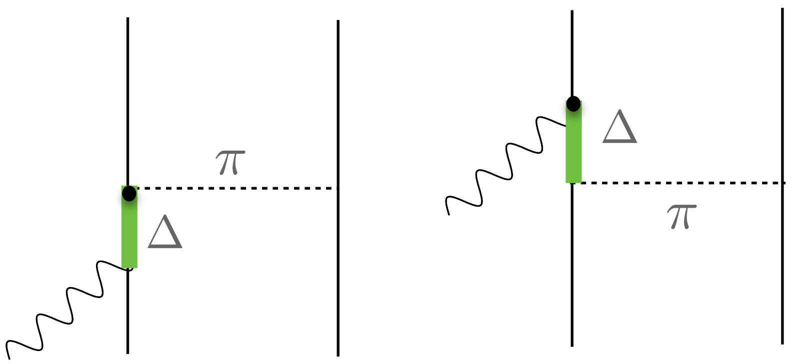
The hadronic tensor for two-body current processes reads

$$W^{\mu\nu}_{2b}(\mathbf{q},\omega) \propto \int dE \frac{d^3k}{(2\pi)^3}\frac{d^3k'}{(2\pi)^3}\frac{d^3p}{(2\pi)^3} P_h(\mathbf{k},\mathbf{k}',E)\, 2\sum_{ij} \langle k\, k'|{j^{\mu}_{ij}}^{\dagger}|p\, p'\rangle_a$$
$$\times \langle p\, p'|j^{\nu}_{ij}|k\, k'\rangle \delta(\omega - E + 2m_N - e(\mathbf{p}) - e(\mathbf{p}')) \,.$$

✐ NR et al, Phys.Rev. C99 (2019) no.2, 025502

✐ NR et al, Phys. Rev. Lett. 116, 192501 (2016)

Relativistic two-body currents

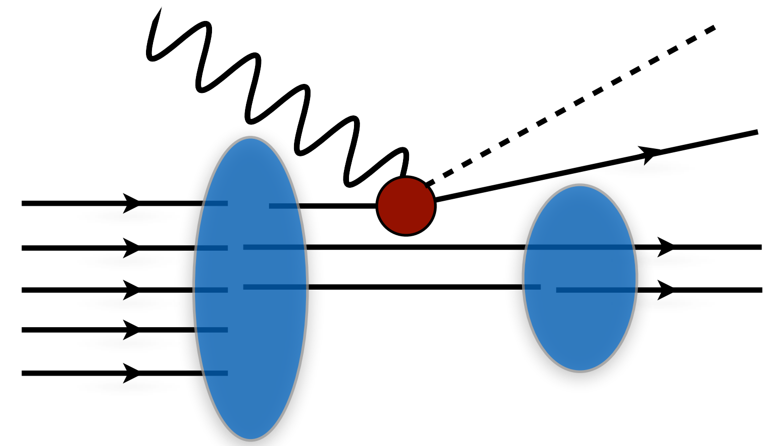


Dedicated code that **automatically** carries out the calculation of the **MEC spin-isospin matrix elements**, performing the integration using the Metropolis MC algorithm

Extended Factorization Scheme

- Production of real π in the final state

$$|f\rangle \to |p_\pi p\rangle \otimes |f_{A-1}\rangle$$

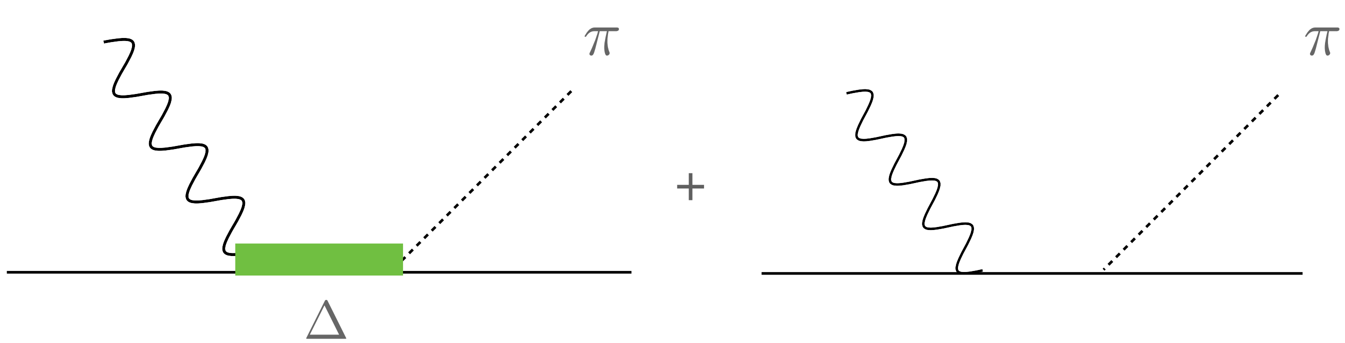


The hadronic tensor for two-body current processes reads

$$W^{\mu\nu}_{1b1\pi}(\mathbf{q},\omega) \propto \int \frac{d^3k}{(2\pi)^3} dE\, P_h(\mathbf{k},E) \frac{d^3p_\pi}{(2\pi)^3} \sum_i \langle k|j_i^{\mu\,\dagger}|p_\pi p\rangle \langle p_\pi p|j_i^{\nu}|k\rangle$$
$$\times\, \delta(\omega - E + m_N - e(\mathbf{p}) - e_\pi(\mathbf{p}_\pi))$$

Pion production elementary amplitudes derived within the extremely sophisticated **Dynamic Couple Chanel approach**; includes meson baryon channel and nucleon resonances up to W=2 GeV

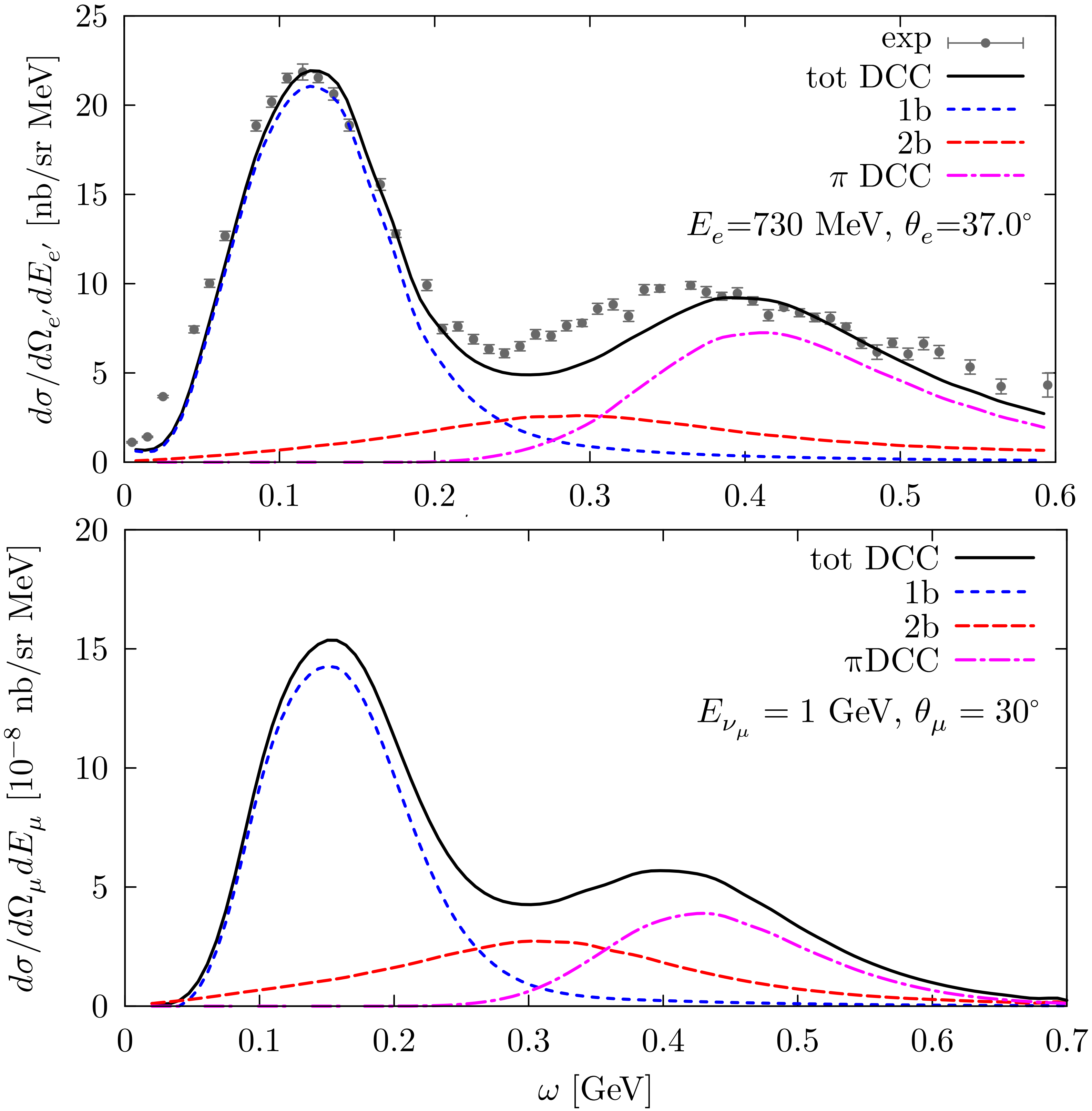
- The diagrams considered resonant and non resonant π production



- NR, et al, PRC100 (2019) no.4, 045503
- H. Kamano et al, PRC 88, 035209 (2013)
- S.X.Nakamura et al, PRD 92, 074024 (2015)

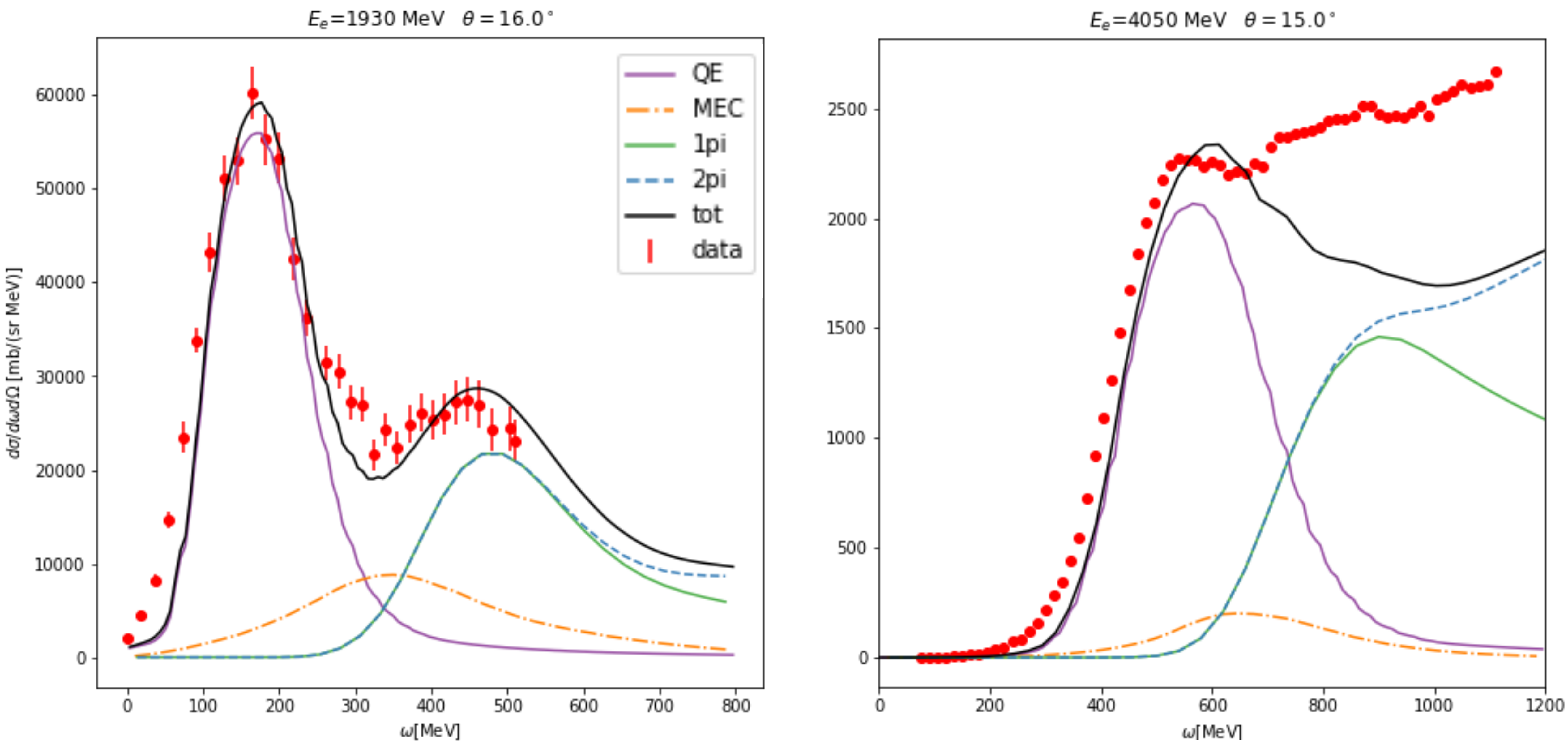
Extended Factorization Scheme: Results

✐ NR, S. Nakamura, T.S.H. Lee, A. Lovato, PRC100 (2019) no.4, 045503



- We included in the Extended Factorization Scheme the **one-** and **two-body current** contributions and the **pion production** amplitudes.
- Good agreement with electron scattering data when all reaction mechanisms are included
- Ongoing calculation of flux folded cross sections

Extended Factorization Scheme: Results



- We included the DCC predictions for two π production

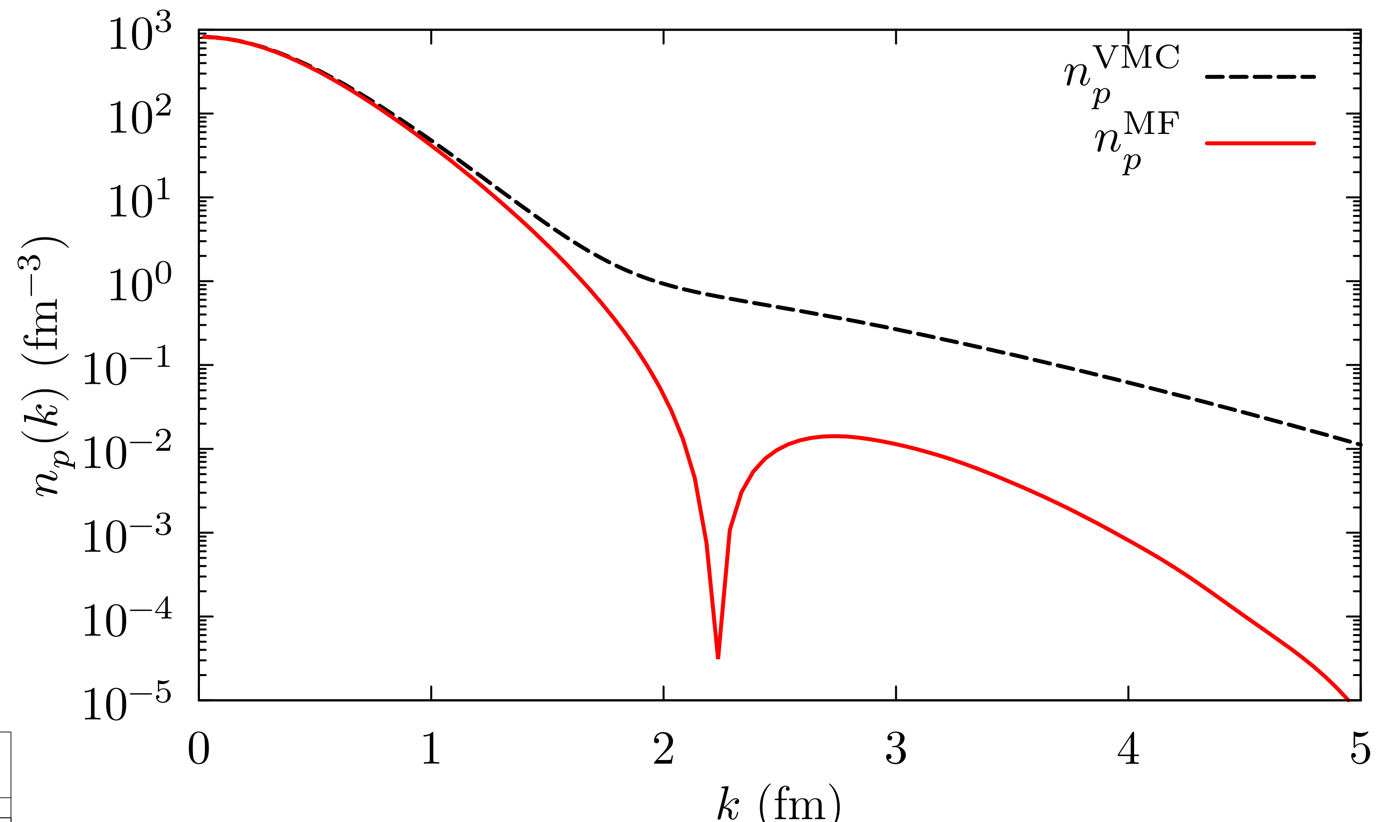
preliminary

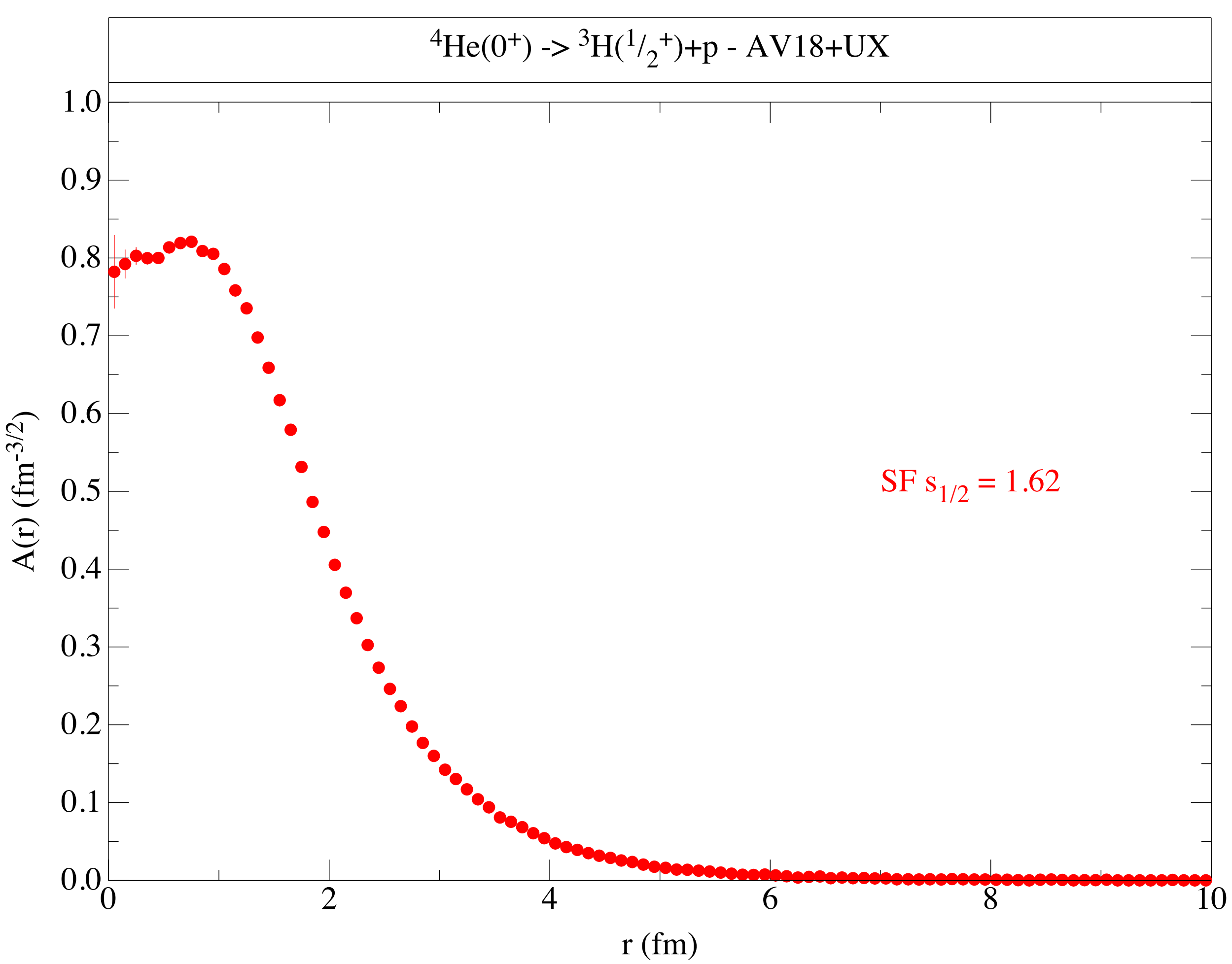
- We plan to tackle the DIS further extending the convolution approach: spectral function+nucleon pdf

QMC Spectral Function of light nuclei

- Single-nucleon spectral function:

$$P_{p,n}(\mathbf{k},E) = \sum_n \left| \langle \Psi_0^A | [|k\rangle |\Psi_n^{A-1}\rangle] \right|^2 \times \delta(E + E_0^A - E_n^{A-1})$$
$$= P^{MF}(\mathbf{k},E) + P^{corr}(\mathbf{k},E)$$





$$P_p^{\rm MF}(\mathbf{k},E) = n_p^{\rm MF}(\mathbf{k})\delta\left(E - B_{^4\rm He} + B_{^3\rm H} - \frac{k^2}{2m_{^3\rm H}}\right)$$

$$|\langle \Psi_0^{^4\rm He} | [|k\rangle \otimes |\Psi_0^{^3\rm H}\rangle]|^2$$

- The single-nucleon overlap has been computed within QMC (center of mass motion fully accounted for)

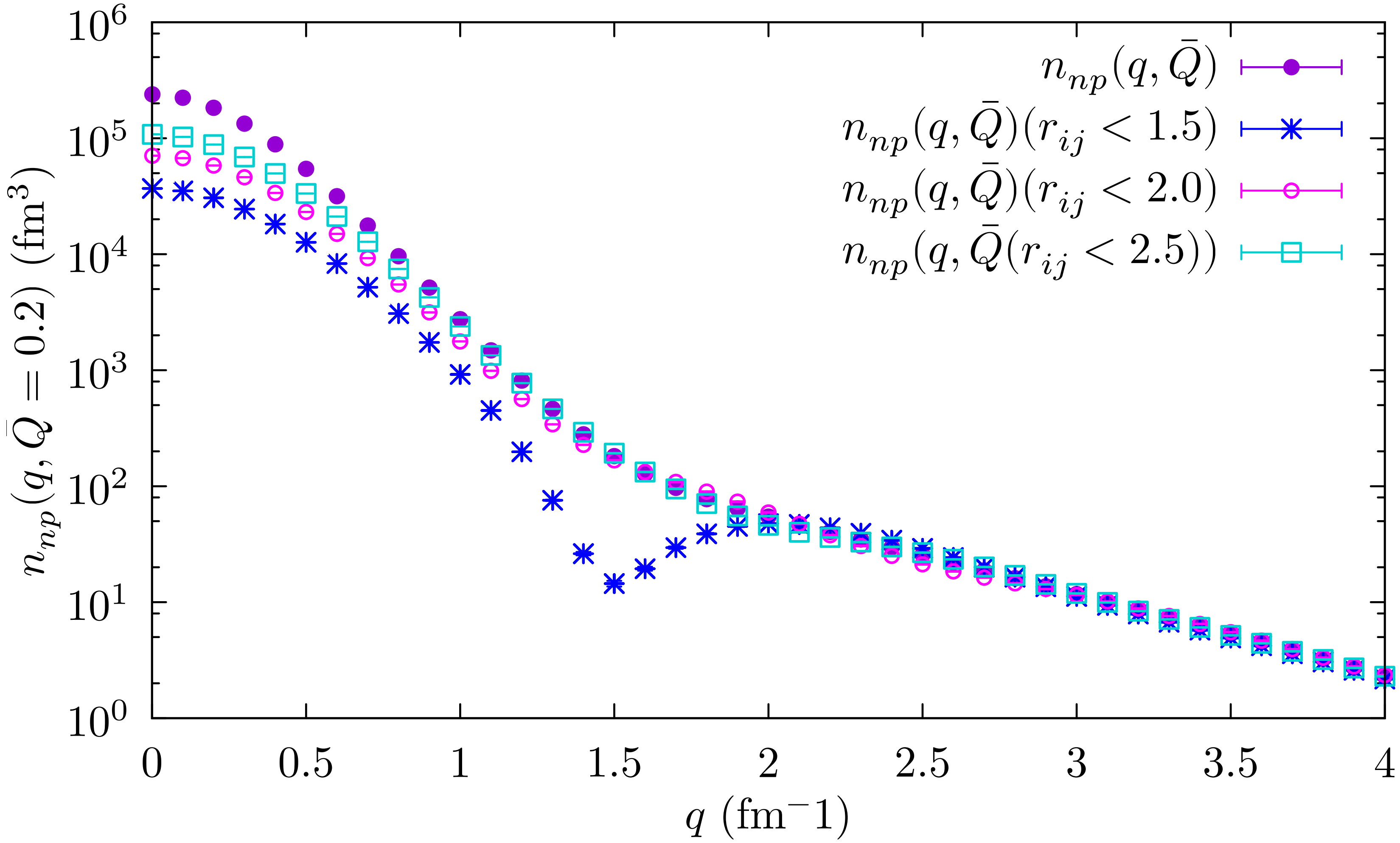
QMC Spectral Function of light nuclei

$$P_p^{\rm corr}(\mathbf{k}, E) = \sum_n \int \frac{d^3k'}{(2\pi)^3} |\langle \Psi_0^A | [|k\rangle |k'\rangle |\Psi_n^{A-2}\rangle]|^2 \delta(E + E_0^A - e(\mathbf{k}') - E_n^{A-2})$$

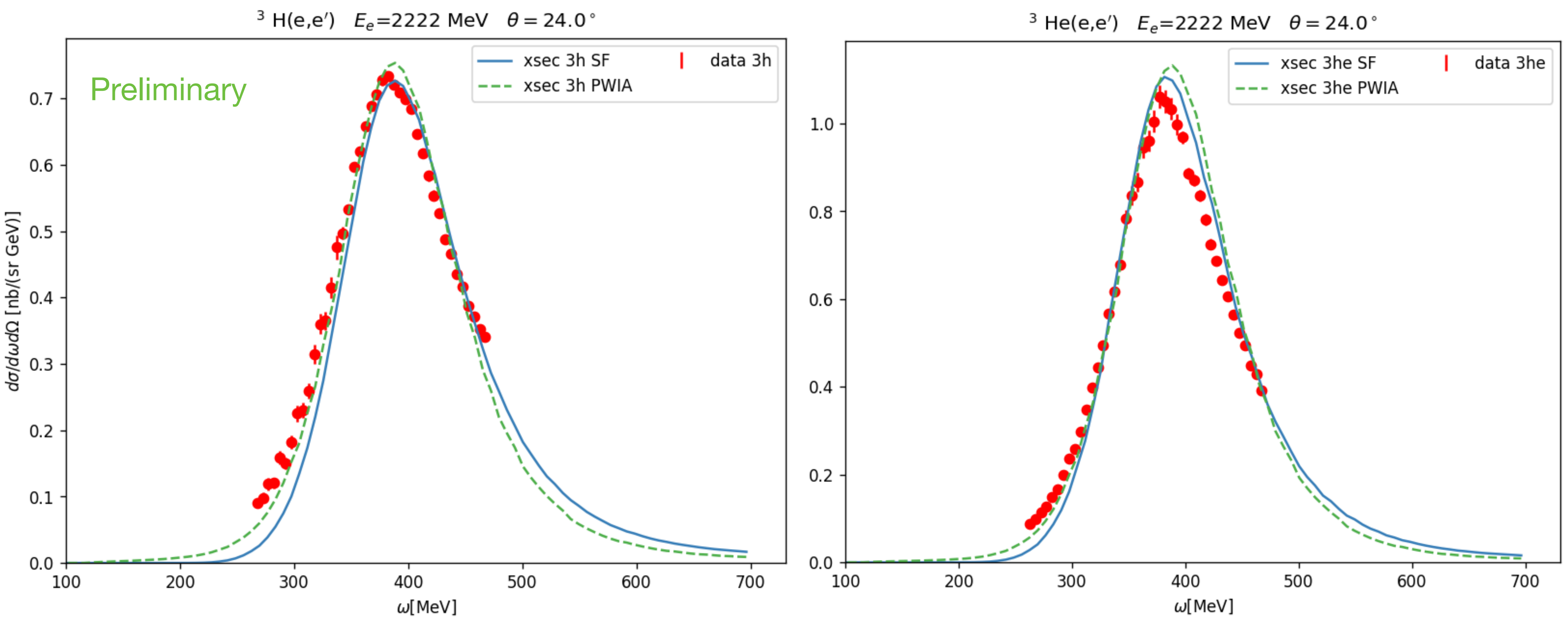
$$\sum_{\tau_{k'}=p,n} n_{p,\tau_{k'}}(\mathbf{k},\mathbf{k}')\delta\Big(E - B_{^4\mathrm{He}} - e(\mathbf{k}') + B_{A-2} - \frac{(\mathbf{k}+\mathbf{k}')^2}{2m_{A-2}}\Big)$$

Only SRC pairs should be considered: $|\psi_0^{^3H}\rangle$ and $|k'\rangle|\psi_n^{A-2}\rangle$ be orthogonalized

We introduce **cuts** on the **relative distance** between the particles in the two-body momentum distribution



QMC Spectral Function of light nuclei

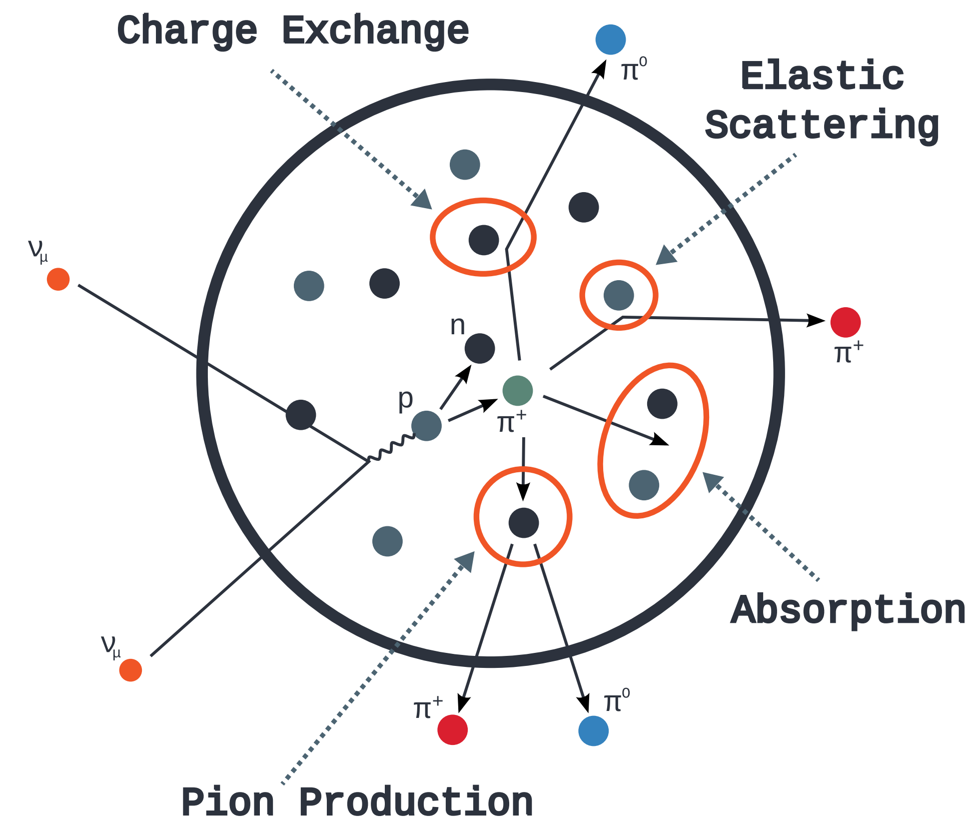


- Comparison with new sets of JLab data for electron scattering on ^{3}H and ^{3}He
- We are currently working on calculating the spectral function of ^{12}C and validation with other many-body approaches

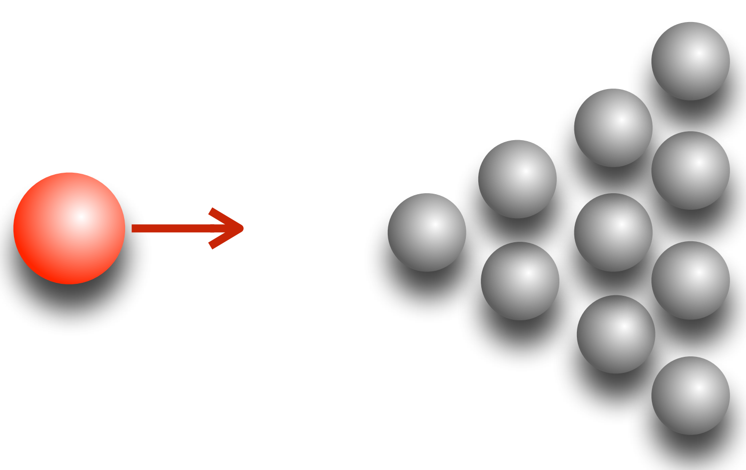
A QMC based approach to cascade

The propagation of **nucleons** through the **nuclear medium** is crucial in the analysis of electron-nucleus scattering and neutrino oscillation experiments.

Figure by T. Golan



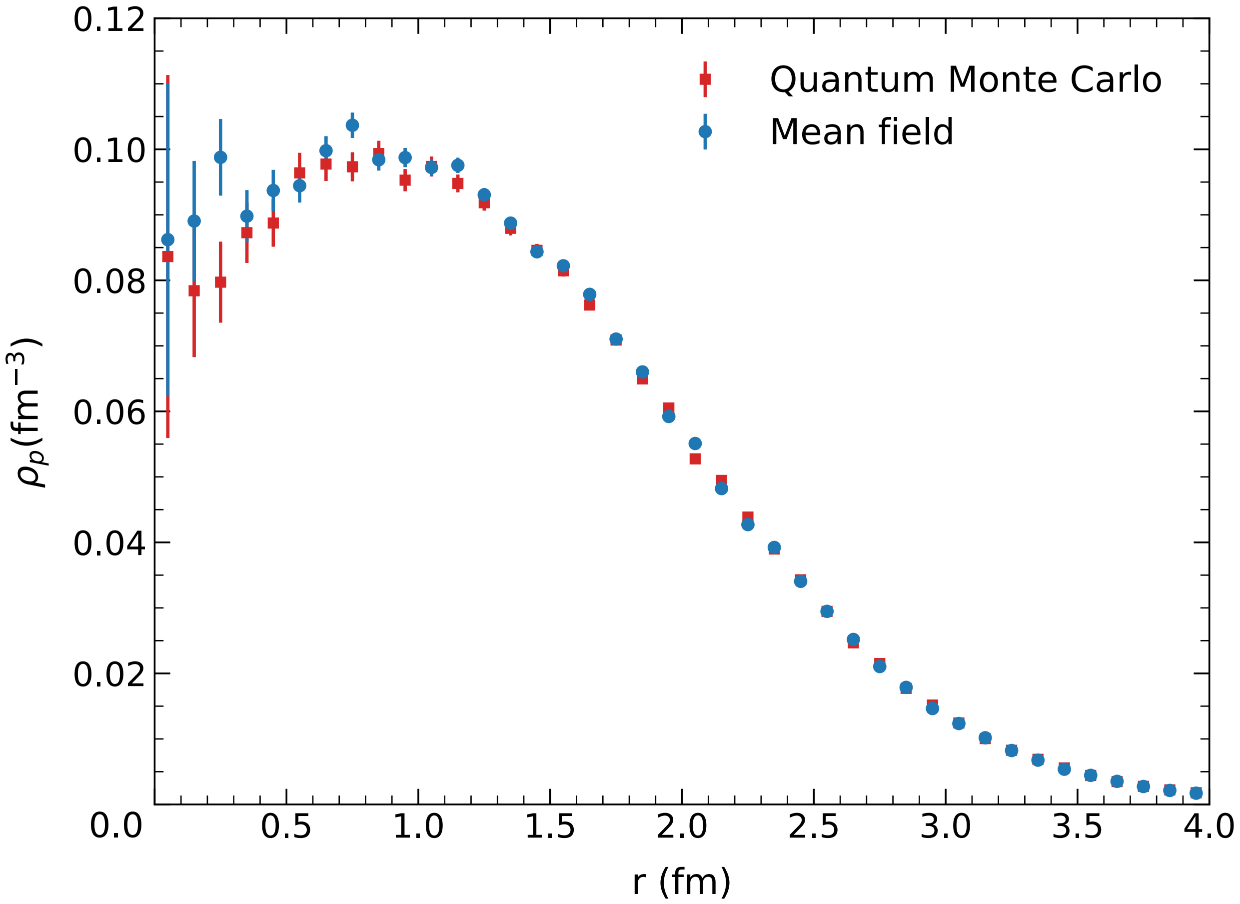
Describing nucleons' propagation in the nuclear medium would in principle require a fully quantum-mechanical description of the hadronic final state.

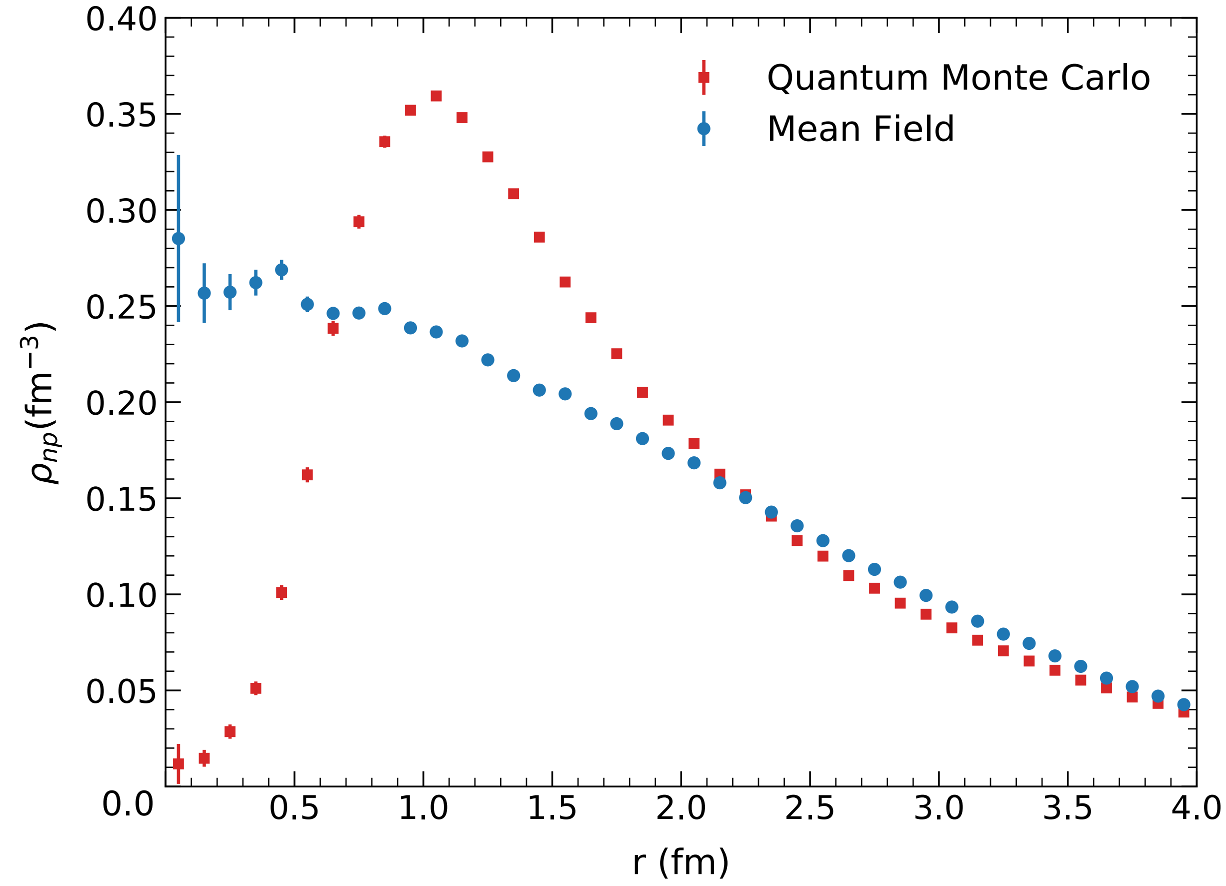


Due to its tremendous difficulty we follow a seminal work of Metropolis and develop a **semi-classical intranuclear cascade** (INC) that assume classical propagation between consecutive scatterings

J.Isaacson, W. Jay, P. Machado, A. Lovato, NR, arXiv:2007.15570

Sampling nucleon configurations



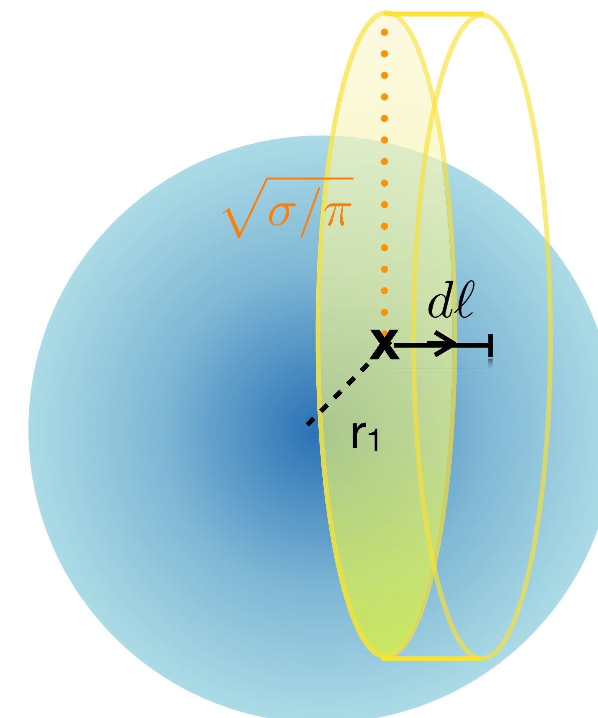


The nucleons' positions utilized in the INC are sampled from **36000 GFMC configurations**. For benchmark purposes we also sampled **36000 mean-field (MF) configurations** from the single-proton distribution.

The differences between GFMC and MF configurations are apparent when comparing the **two-body density distributions**: repulsive nature of two-body interactions reduced the probability of finding two particles close to each other

Probability of interaction

To check if an interaction between nucleons occurs an **accept-reject** test is performed on the **closest nucleon** according to a probability distribution.



We use a **cylinder probability distribution**, this mimics a more classical billiard ball like system where each billiard ball has a radius
In addition we consider a **gaussian probability distribution**

For benchmark purposes, we also implemented the **mean free path approach**, routinely used in event generators

$$P = \sigma \bar{\rho} d\ell$$

where a constant density is assumed $\rho(r_1) \sim \rho(r_1 + d\ell) \sim \bar{\rho}$

we sample a number $0 \leq x \leq 1$

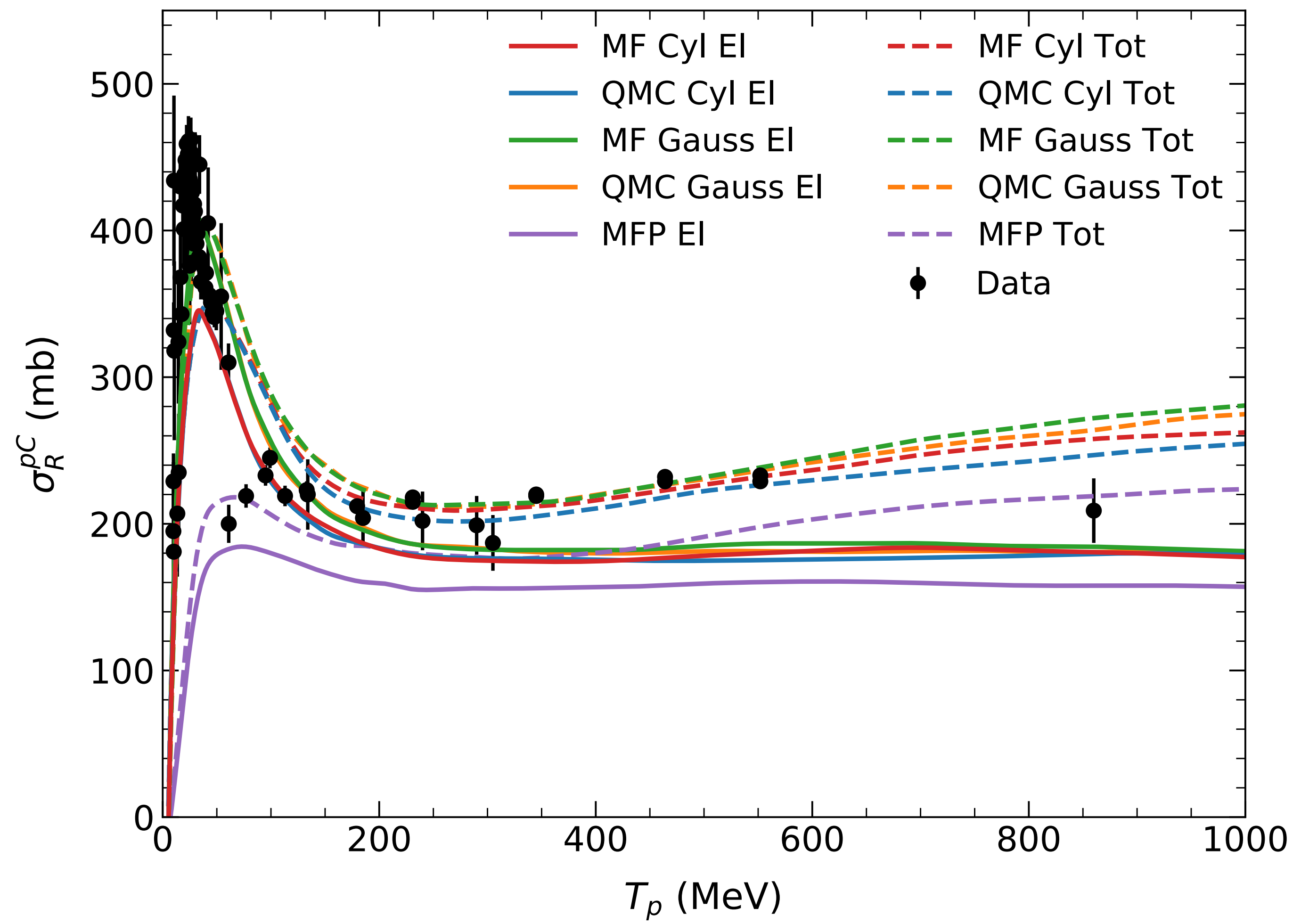
- $x < P$ ✔ the interaction occurred, check Pauli blocking
- $x > P$ ✘ the interaction DID NOT occur

Results: proton-Carbon cross section

Reproducing proton-nucleus cross section measurements is an important test of the accuracy of the INC model.

- We define a beam of protons with energy E, uniformly distributed over an area A.
- We propagate each proton in time and check for scattering at each step.
- The Monte Carlo cross section is defined as:

$$\sigma_{\mathrm{MC}} = A \frac{N_{\mathrm{scat}}}{N_{\mathrm{tot}}}$$



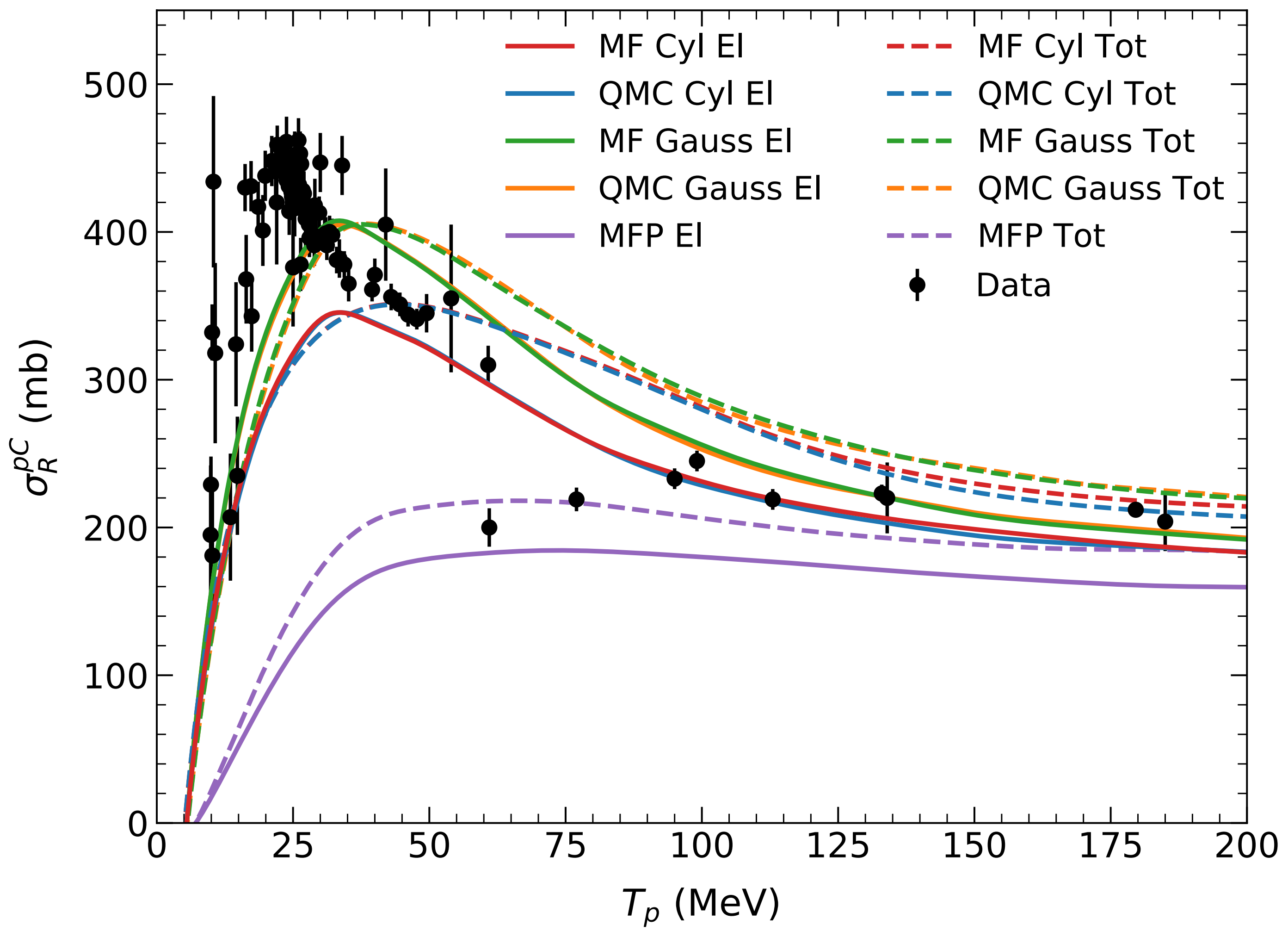
The **solid lines** have been obtained using the nucleon- nucleon cross sections from the **SAID database** in which only the **elastic contribution** is retained. The **dashed lines** used the **NASA parameterization** , which includes **inelasticities**.

Results: proton-Carbon cross section

The **Gauss** and **cylinder probability** distribution yield **similar results**

Large difference with the mean-free-path implementation: conceptual differences with respect to the previous cases

QMC and MF distribution lead to almost identical results: this observable does not depend strongly on correlations among the nucleons



The **solid lines** have been obtained using the nucleon- nucleon cross sections from the SAID database in which only the **elastic contribution** is retained. The **dashed lines** used the NASA parameterization , which includes **inelasticities**.

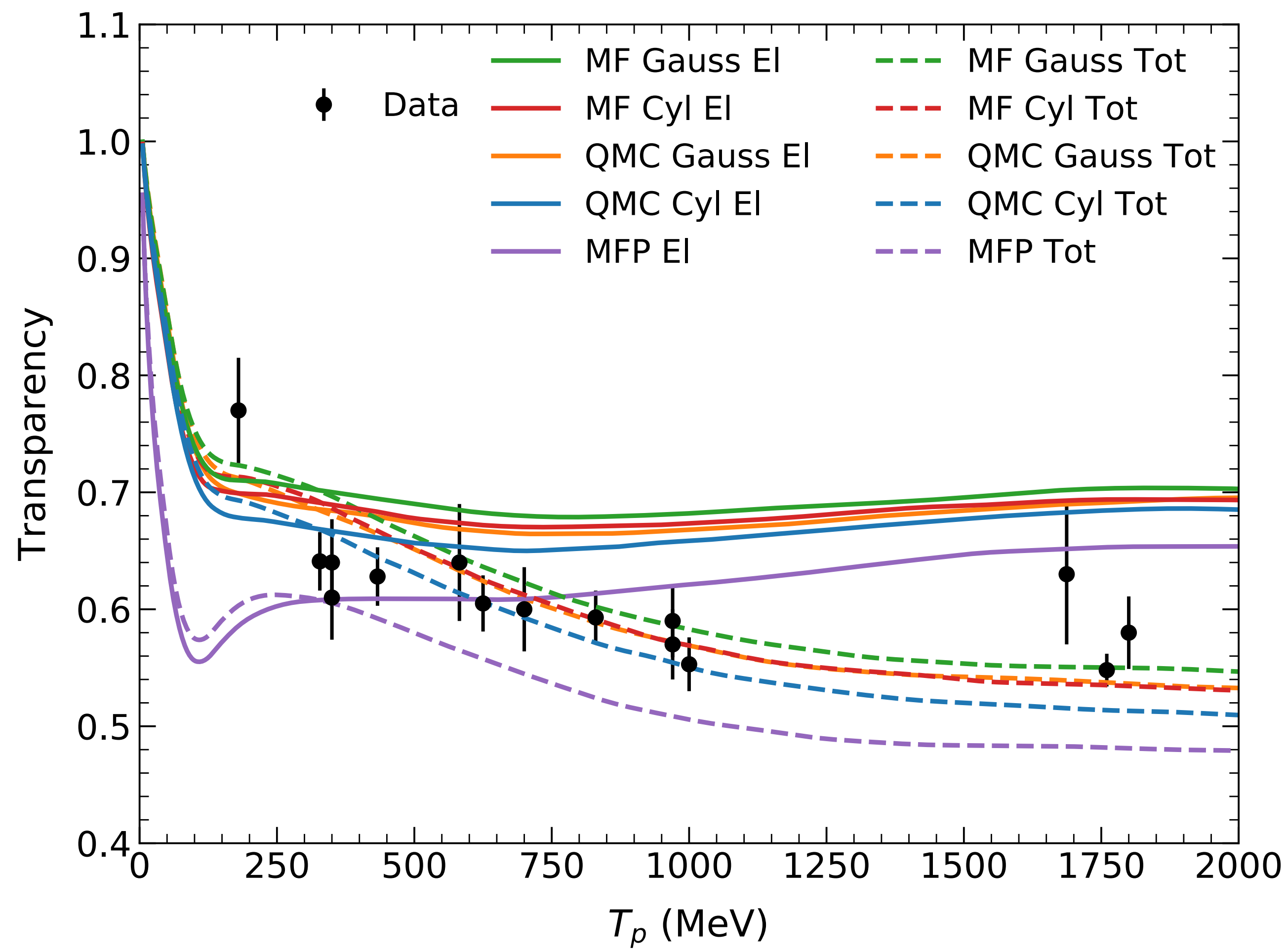
Results: nuclear transparency

The **nuclear transparency** yields the average probability that a struck nucleon leaves the nucleus without interacting with the spectator particles

Nuclear transparency is **measured in (e,e'p) scattering** experiments

Simulation: we randomly sample a nucleon with kinetic energy Tp and propagate it through the nuclear medium

$$T_{\rm MC} = 1 - \frac{N_{\rm hits}}{N_{\rm tot}}$$



Gaussian and cylinder curves are consistent and correctly reproduces the data. Correlations do not seem to play a big role.

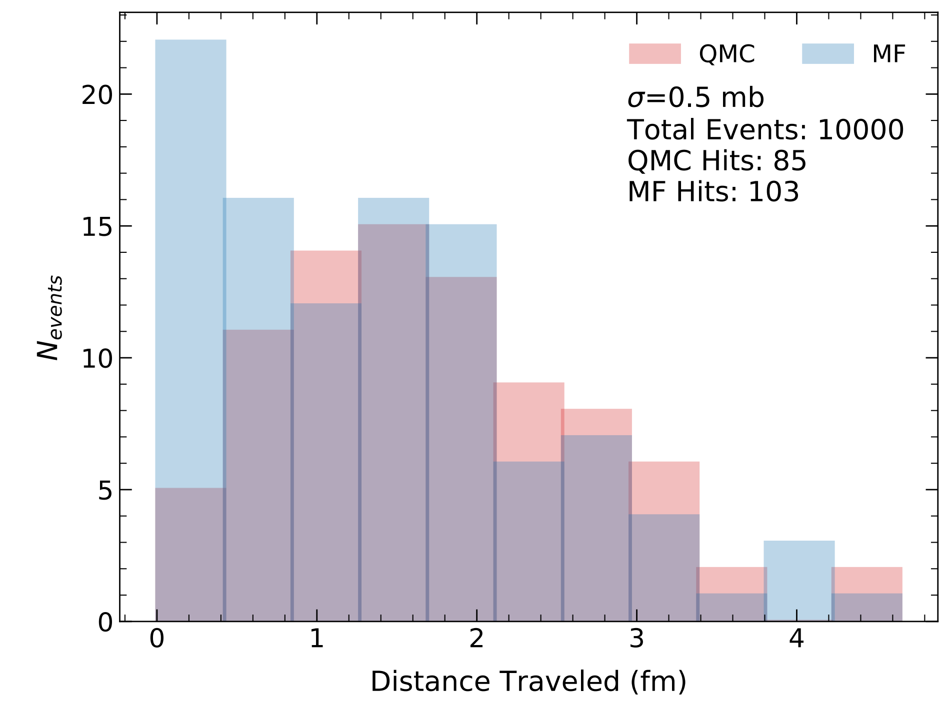
Results: correlation effects

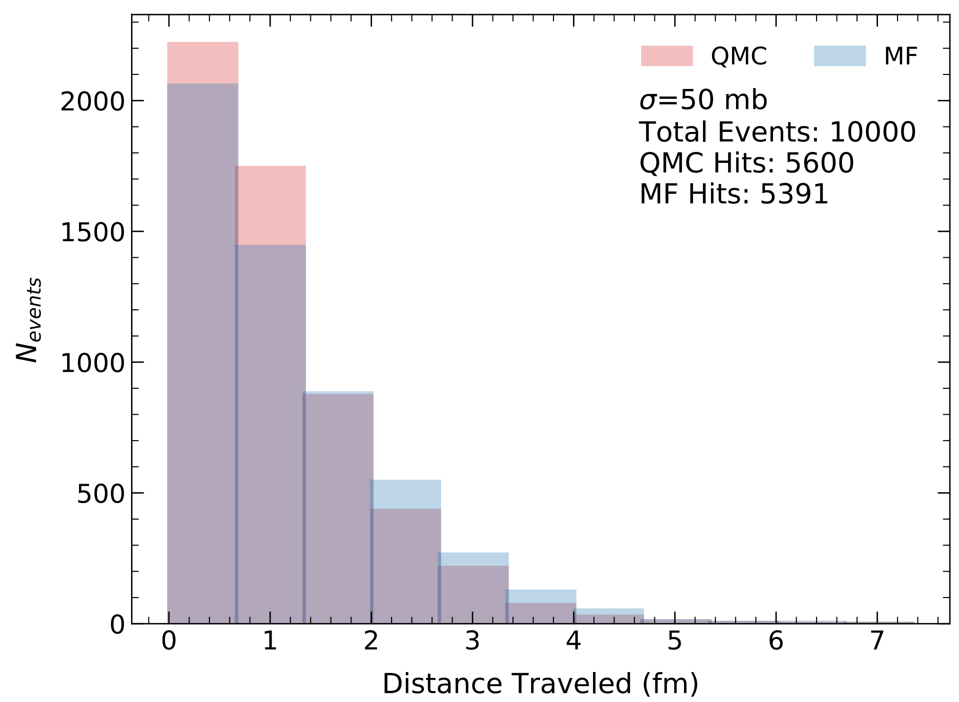
Histograms of the **distance traveled** by a struck particle **before the first interaction** takes place for different values of the interaction cross section

When using **QMC configurations**, the hit nucleon is surrounded by a short-distance **correlation hole**: expected to propagate freely for ~ 1 fm before interacting

For **σ=0.5 mb** the **MF distribution peaks toward smaller distances than the QMC one**: originates from the repulsive nature of the nucleon-nucleon potential

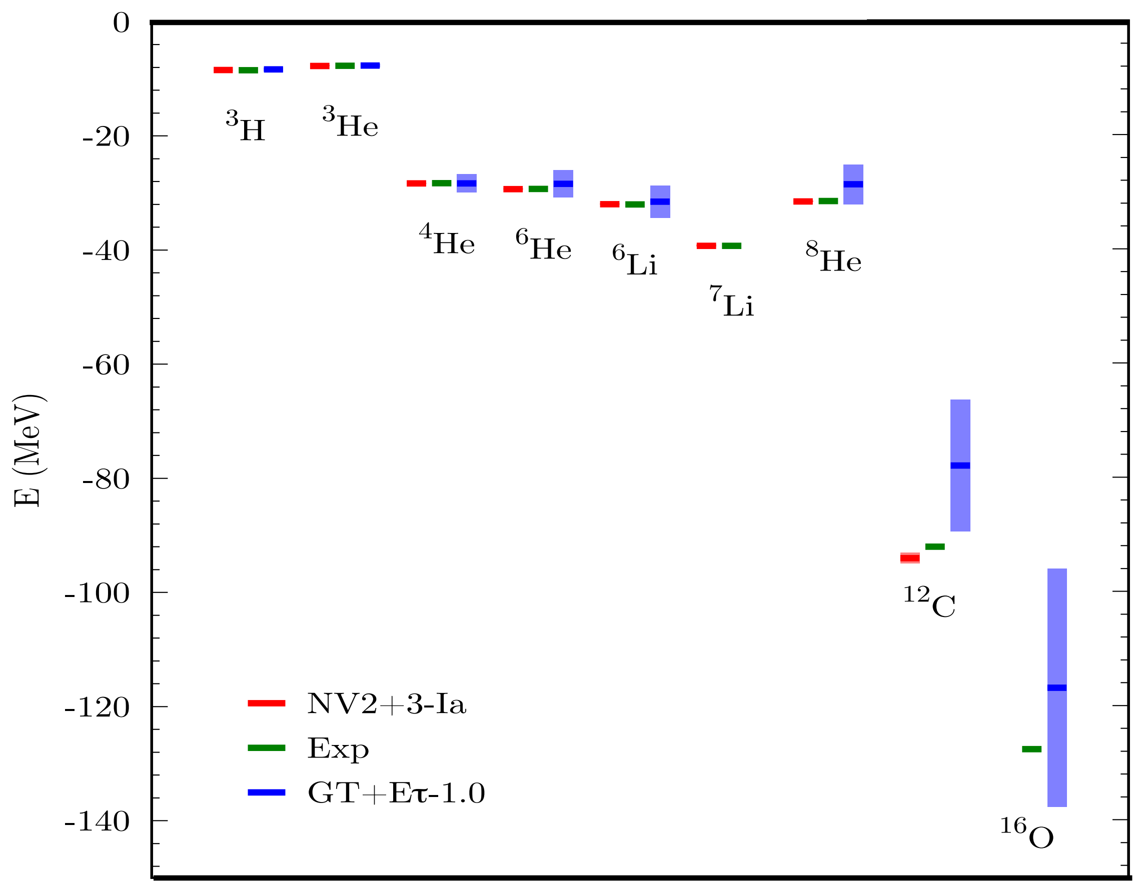
For **σ=50 mb** large cylinder, **MF and QMC distributions become similar.** The propagating particle is less sensitive to the local distribution of nucleons and more sensitive to the integrated density over a larger volume, reducing the effect of correlations





Future Prospects

✐ S.Gandolfi, D.Lonardoni, et al, *Front.Phys.* 8 (2020) 117



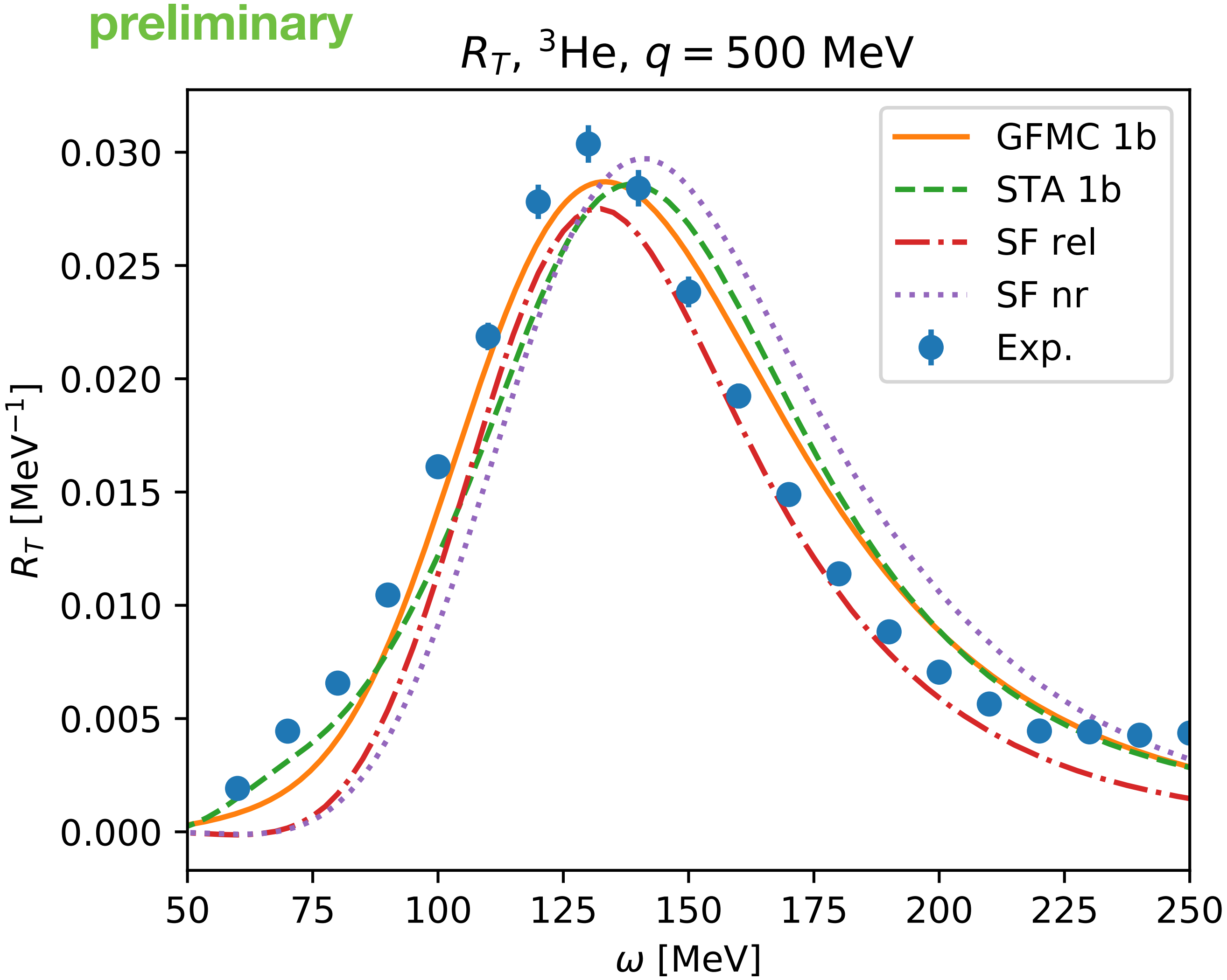
- Estimate the **uncertainty of the theoretical calculation**: can be achieved in QMC calculations. Work in this direction has been done for the energy spectra of light nuclei. The error band comes from the truncation of the chiral expansion and statistical uncertainty of the ab-initio method

- Further develop the machine-learning inversion of the nuclear responses to include full uncertainty quantification and propagation by leveraging the linearity of the Laplace transform.

- Extend the reach of QMC methods that can use highly-realistic nuclear interactions to medium-mass nuclei: A> 14 sampling the spin and isospin degrees of freedom to drastically reduce the computational cost.

Future Prospects

preliminary



- Comparisons among QMC, SF, and Short Time Approximation (STA) approaches to precisely quantify the **uncertainties inherent to the factorization of the final state**.

- Obtain a QMC spectral function for light and medium mass nuclei. Use it to compute inclusive and exclusive observables.

- Understand how to model the transition between RES and DIS region

- Utilize inputs from lattice-QCD to describe nucleons' properties and couplings to further constrain two-body dynamics.

- **Intranuclear cascade:** include **π degrees of freedom**: π production, absorption and elastic scattering as well as **in medium corrections**

Thank you for your attention!