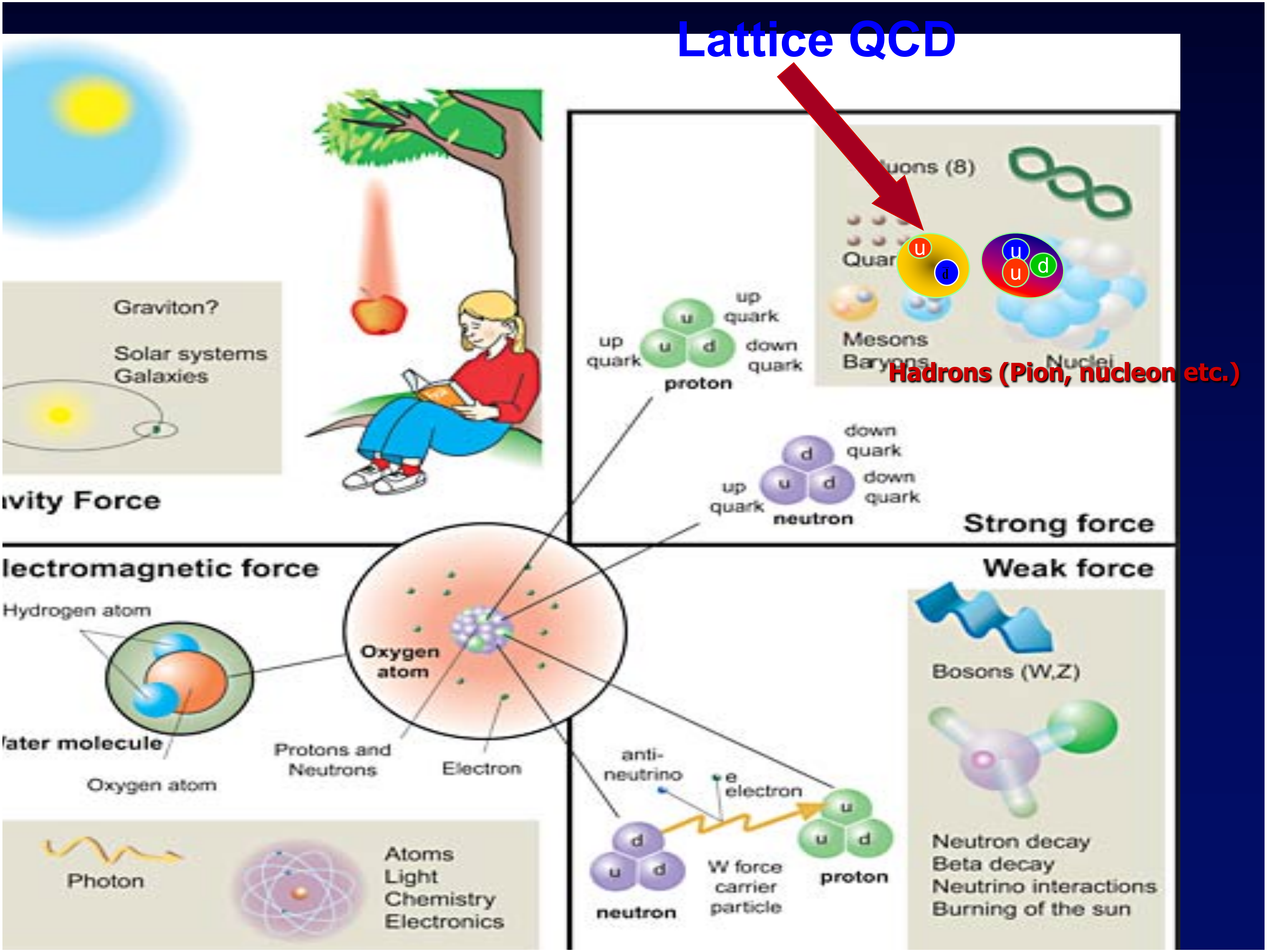
Proton Spin and Mass Decompositions and Neutrino-Nucleon Scattering from Lattice QCD

- Synopsis of Lattice QCD
- Proton Spin Decomposition -- Quark and Glue Spin, Orbital Angular Momentum
- Proton Mass Decomposition – Quark Condensate, Quark and Glue energies, and Trace Anomaly
- Neutrino-Nucleon Scattering -- Hadronic Tensor
  - Inverse Problem

χQCD Collaboration

Michigan State University,

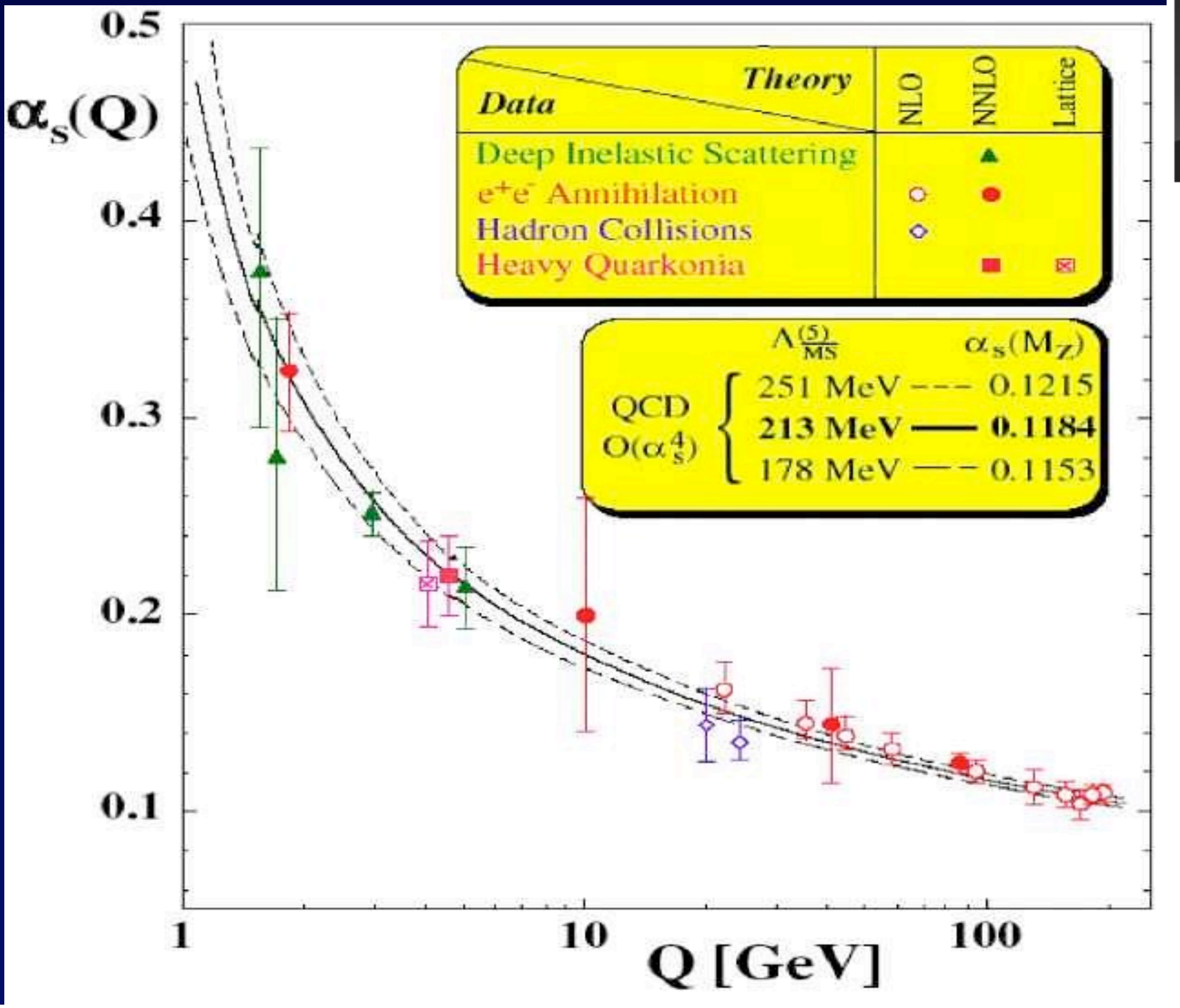
Feb. 11, 2020



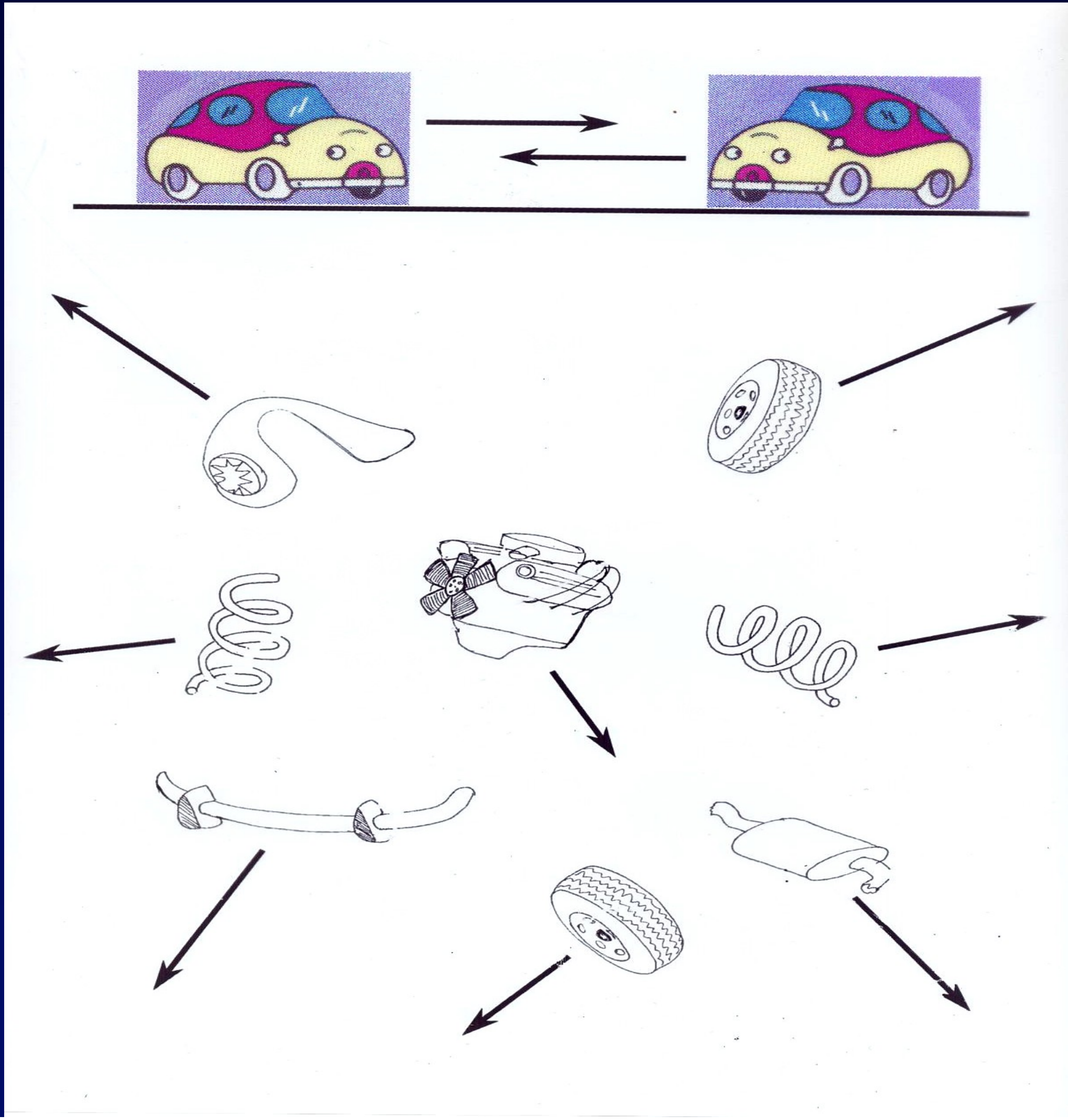
Lattice QCD
Hadrons (Pion, nucleon etc.)
Graviton?
Solar systems
Galaxies
vity Force
uons (8)
Quar
Mesons
Baryons
Nuclei
up quark
up quark
down quark
proton
down quark
up quark
down quark
neutron
Strong force
lectromagnetic force
Hydrogen atom
ater molecule
Oxygen atom
Oxygen atom
Protons and Neutrons
Electron
Photon
Atoms
Light
Chemistry
Electronics
Weak force
Bosons (W,Z)
anti-neutrino
e electron
neutron
W force carrier particle
proton
Neutron decay
Beta decay
Neutrino interactions
Burning of the sun

Theory of Strong Interactions : QCD

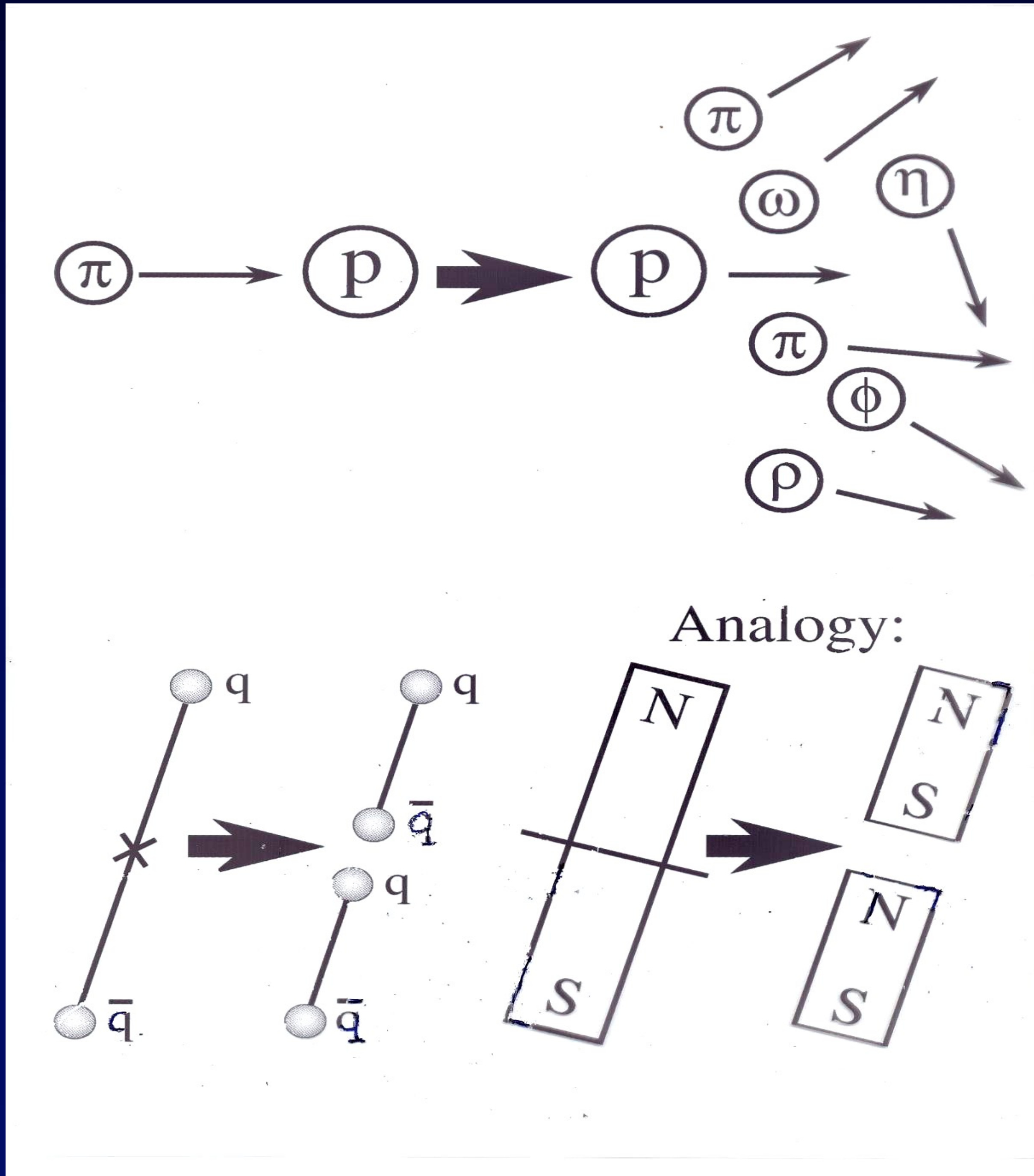
The force between quarks becomes weak for small quark separations--- a phenomenon known as Asymptotic Freedom. ……. *Nobel Prize* 2004

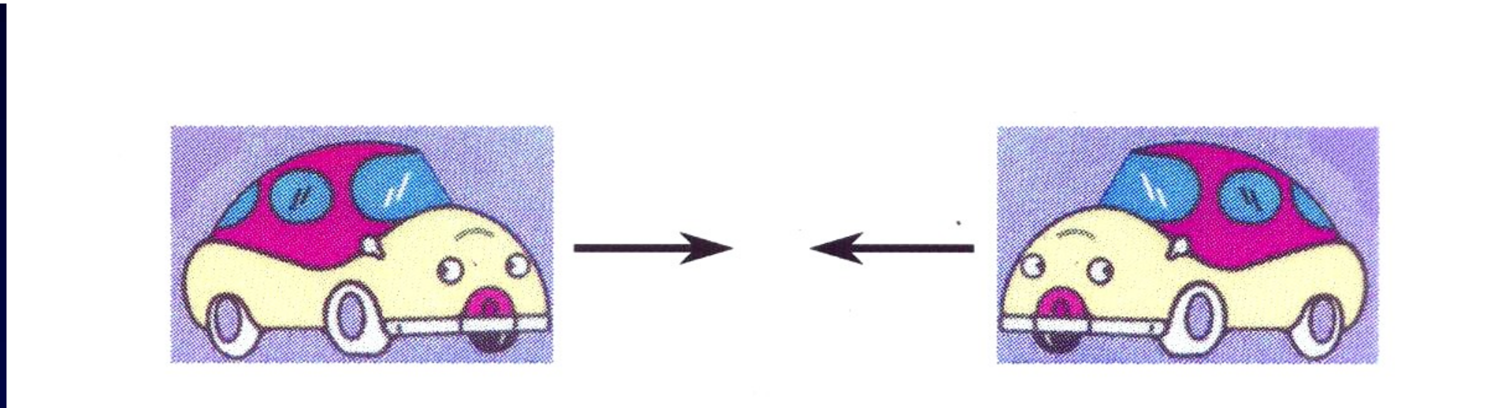


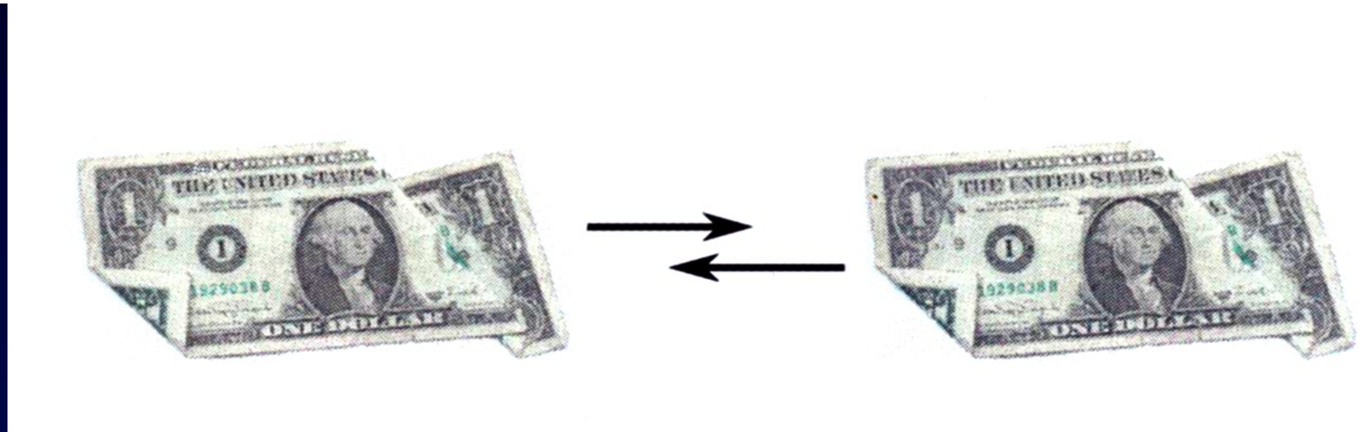
Confinement

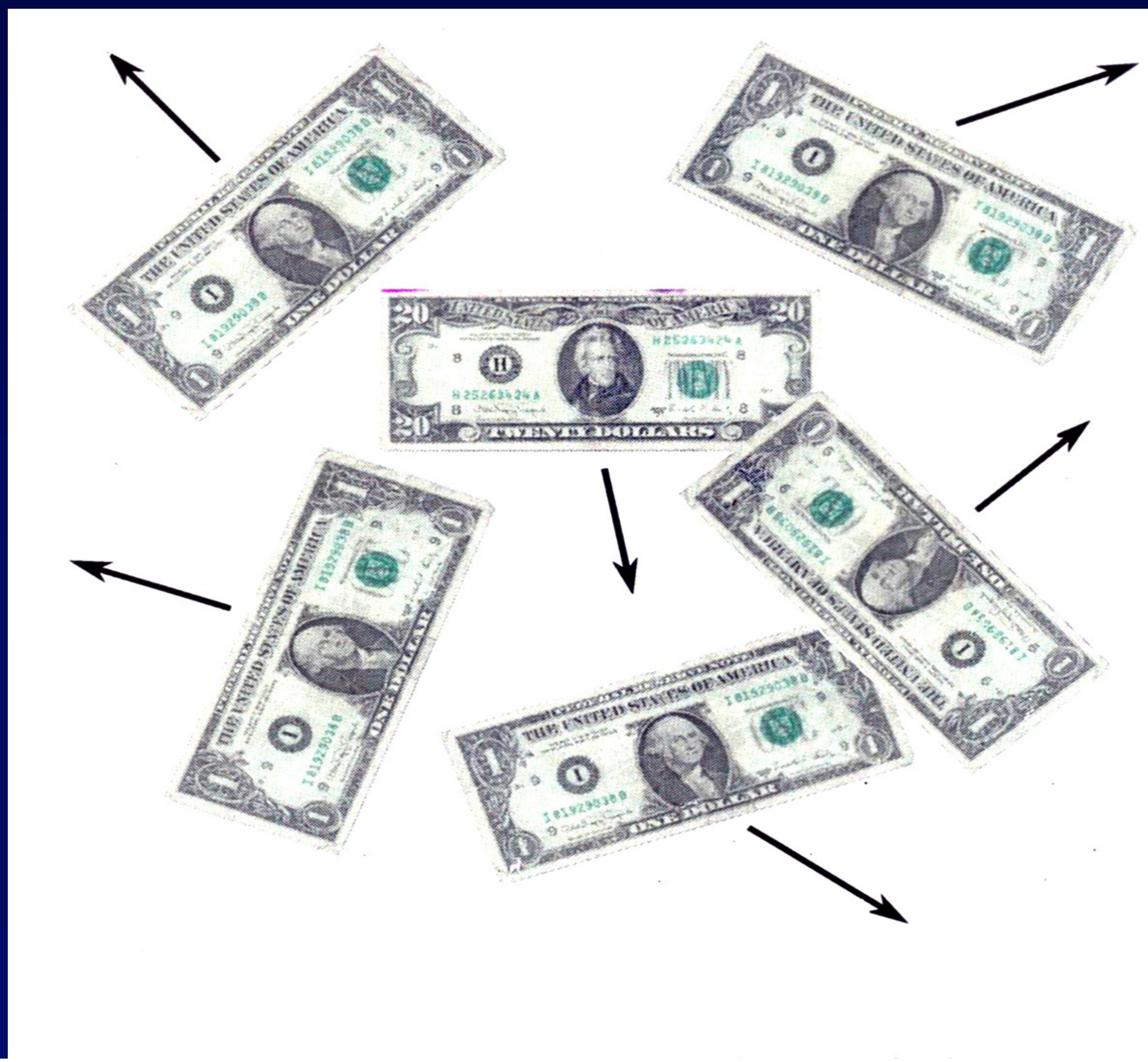


Quark Confinement






THE UNITED STATES
ONE DOLLAR

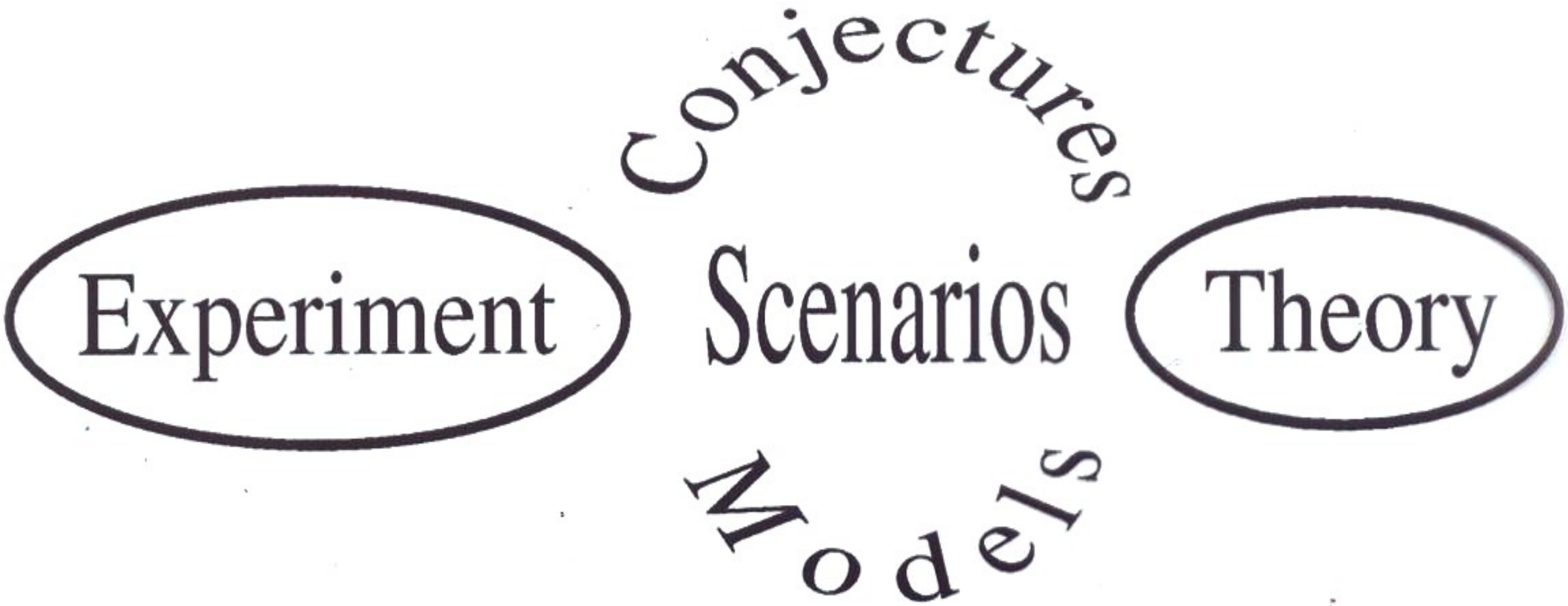

THE UNITED STATES OF AMERICA
ONE DOLLAR
TWENTY DOLLARS

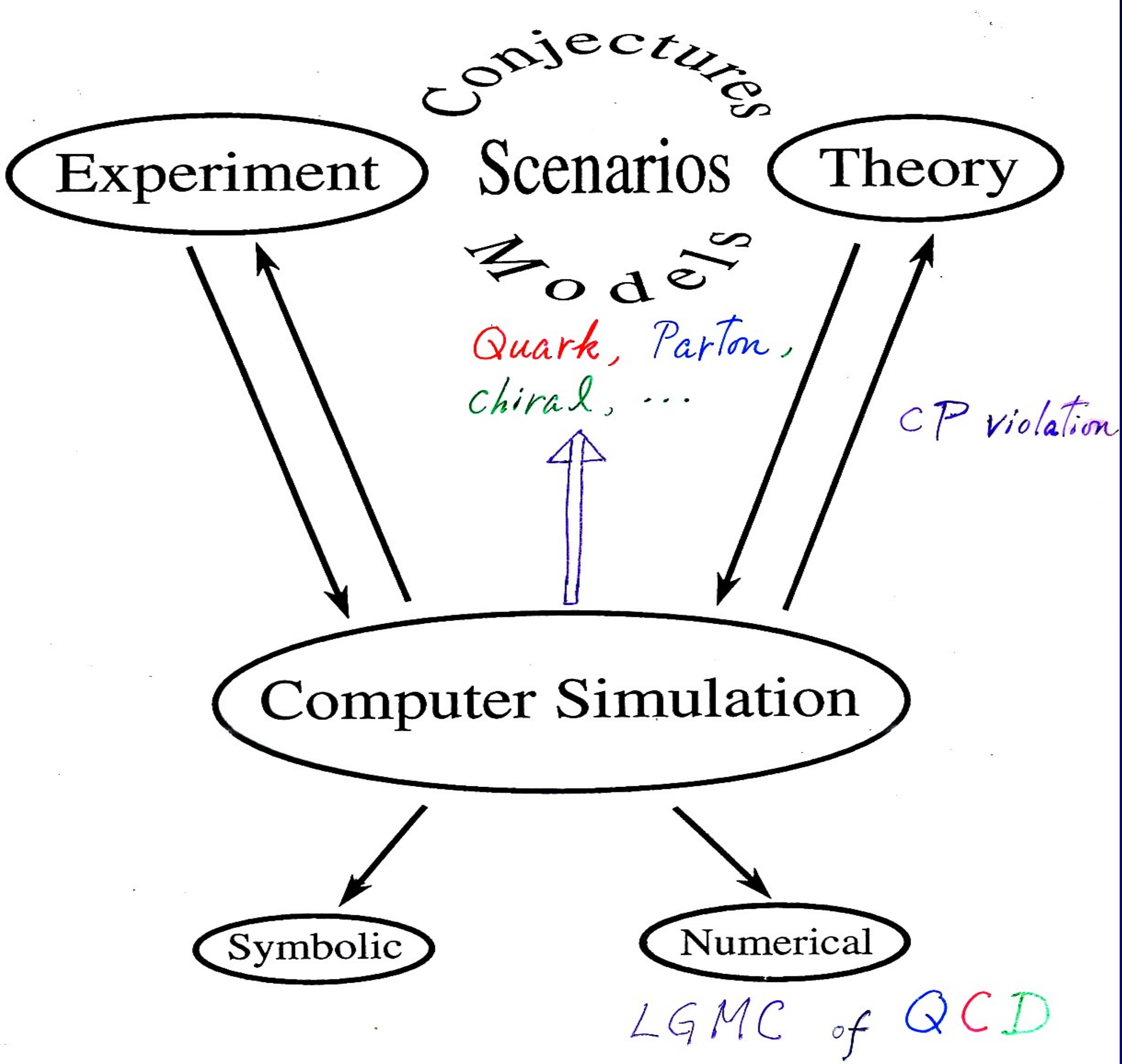
"The Understanding cannot *See*.
The Senses cannot *Think*.
By their union only can *Knowledge*
be produced."

— Immanuel Kant

"*Imagination* is more
important than *Knowledge*."

— Albert Einstein

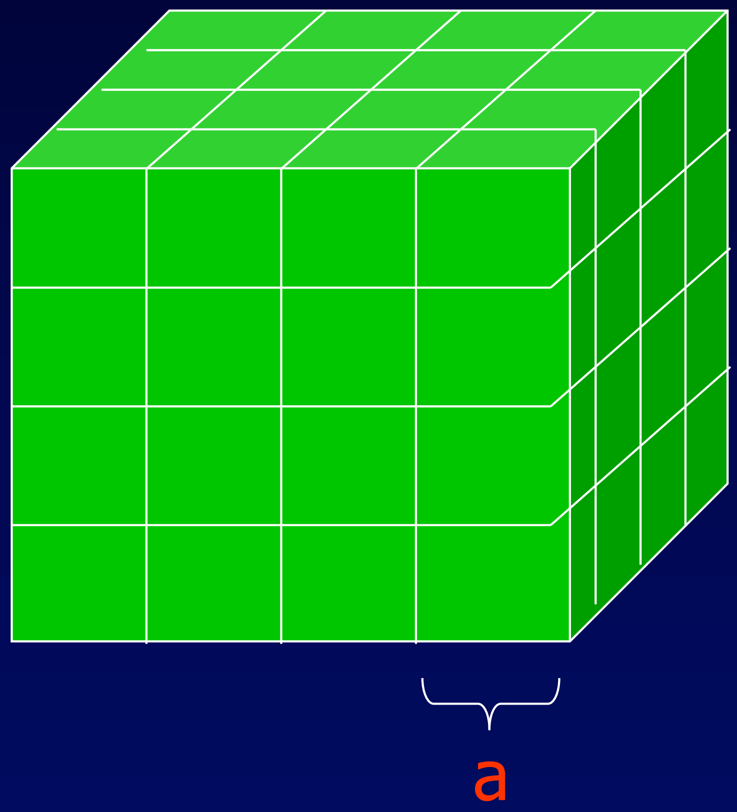

Conjectures
Experiment
Scenarios
Theory
Models


Conjectures
Scenarios
Models
Experiment
Theory
Quark, Parton,
chiral, ...
CP violation
Computer Simulation
Symbolic
Numerical
LGMC of QCD

Lattice QCD

Why Lattice?

- Regularization
  - Lattice spacing a
  - Hard cutoff, p ≤ π/a
  - Scale introduced (dimensional transmutation)



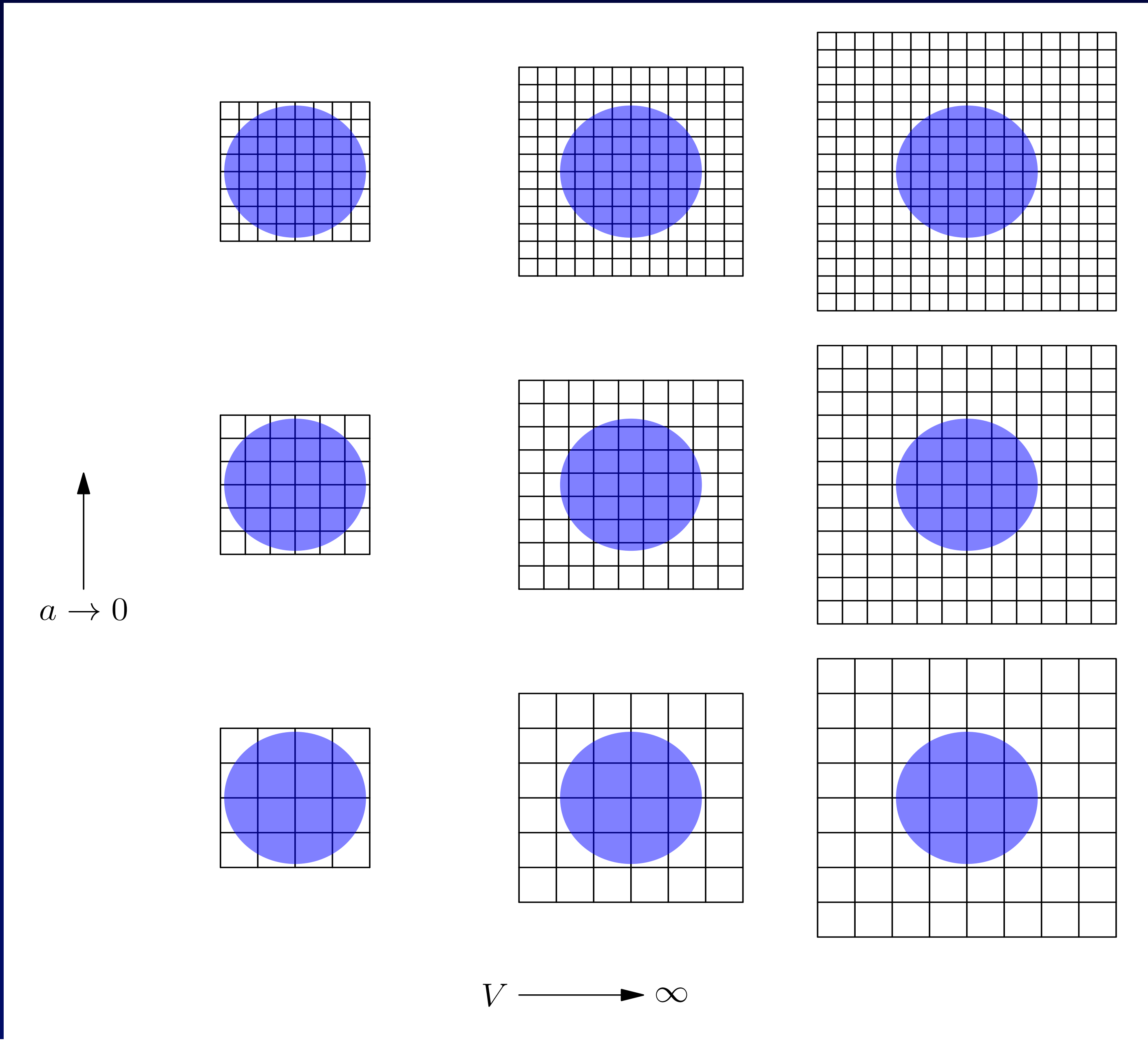
- Renormalization
  - Perturbative
  - Non-perturbative

    Regularization independent Scheme
    Schroedinger functional
    Current algebra relations

- Numerical Simulation
  - Quantum field theory → classical statistical mechanics
  - Monte Carlo simulation in Euclidean space (importance sampling)

$$e^{-S_G}\,\det M \geq 0$$

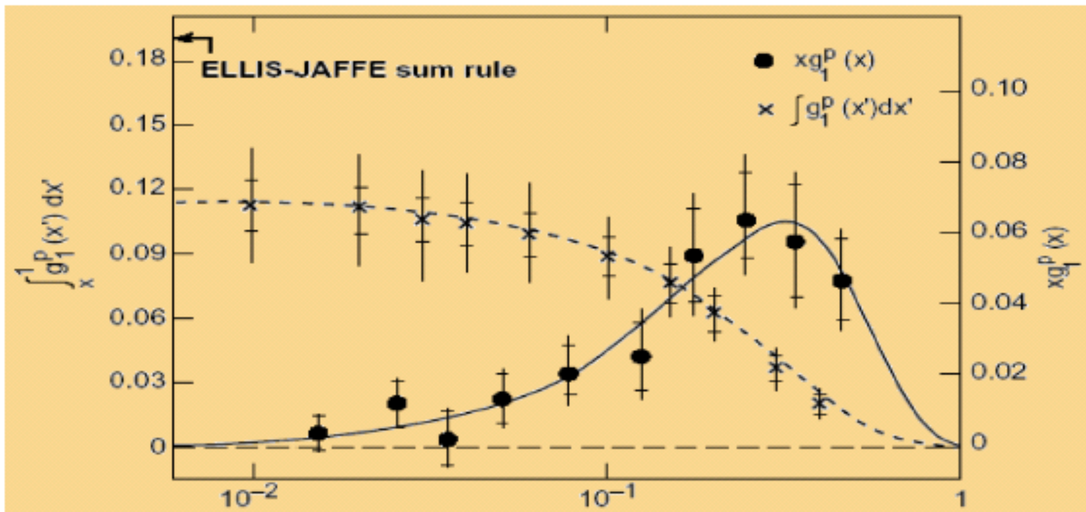
Continuum and Infinite Volume Limits at Physical Pion Mass (Systemic Errors)



Where does the spin of the proton come from?

Thirty years since the "spin crisis"

- **EMC experiment in 1988/1989 – "the plot":**

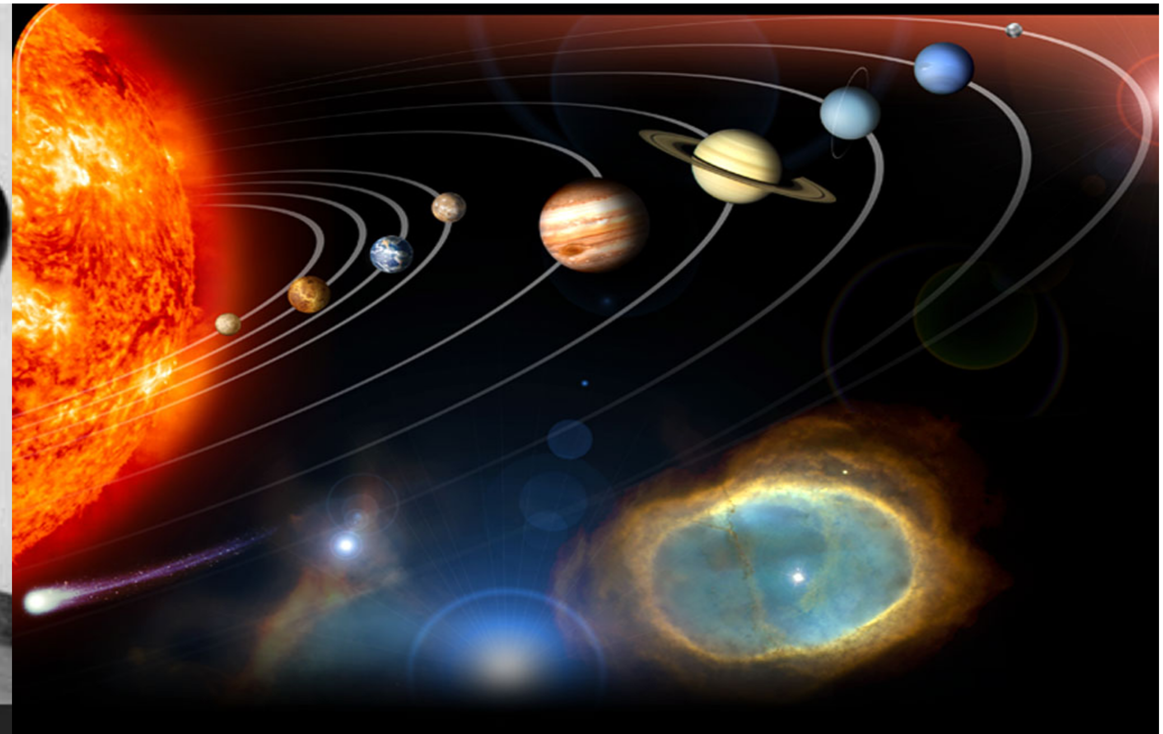


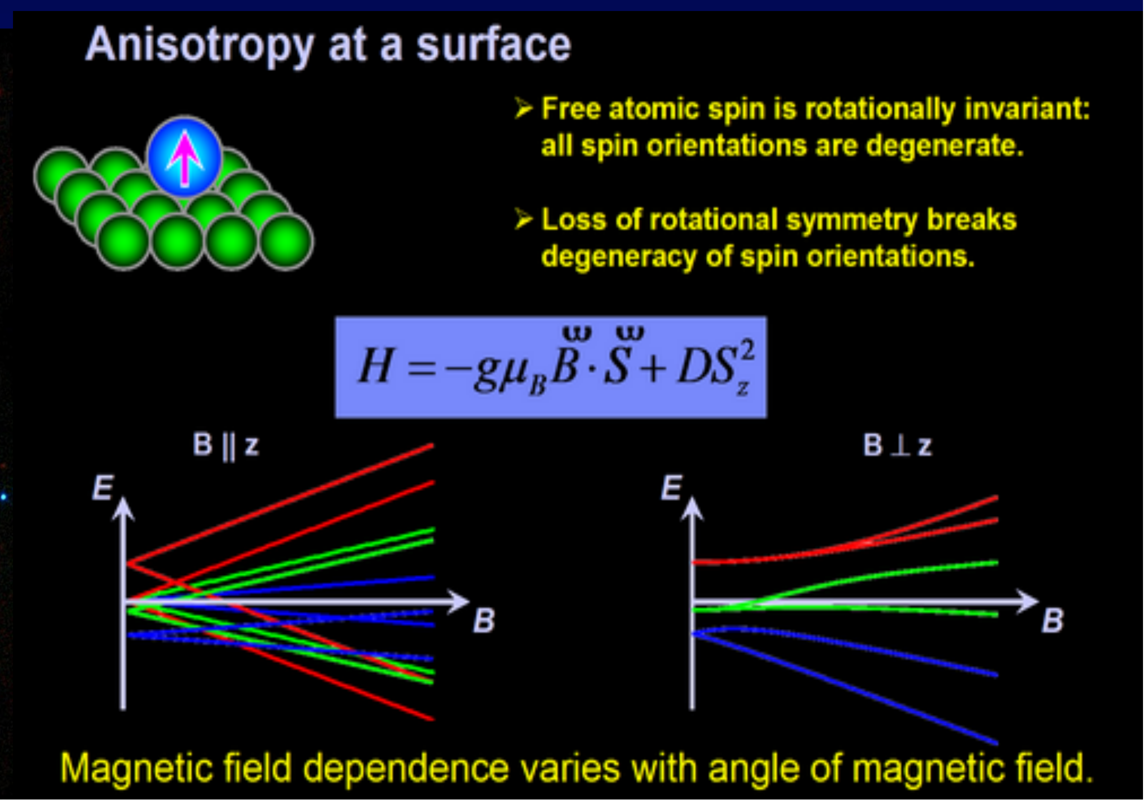
$$g_1(x) = \frac{1}{2}\sum_q e_q^2\left[\Delta q(x) + \Delta\bar{q}(x)\right] + \mathcal{O}(\alpha_s) + \mathcal{O}(1/Q)$$

$$\Delta q = \int_0^1 dx \Delta q(x) = \langle P, s_\parallel | \overline{\psi}_q(0)\gamma^+\gamma_5\psi_q(0) | P, s_\parallel\rangle$$

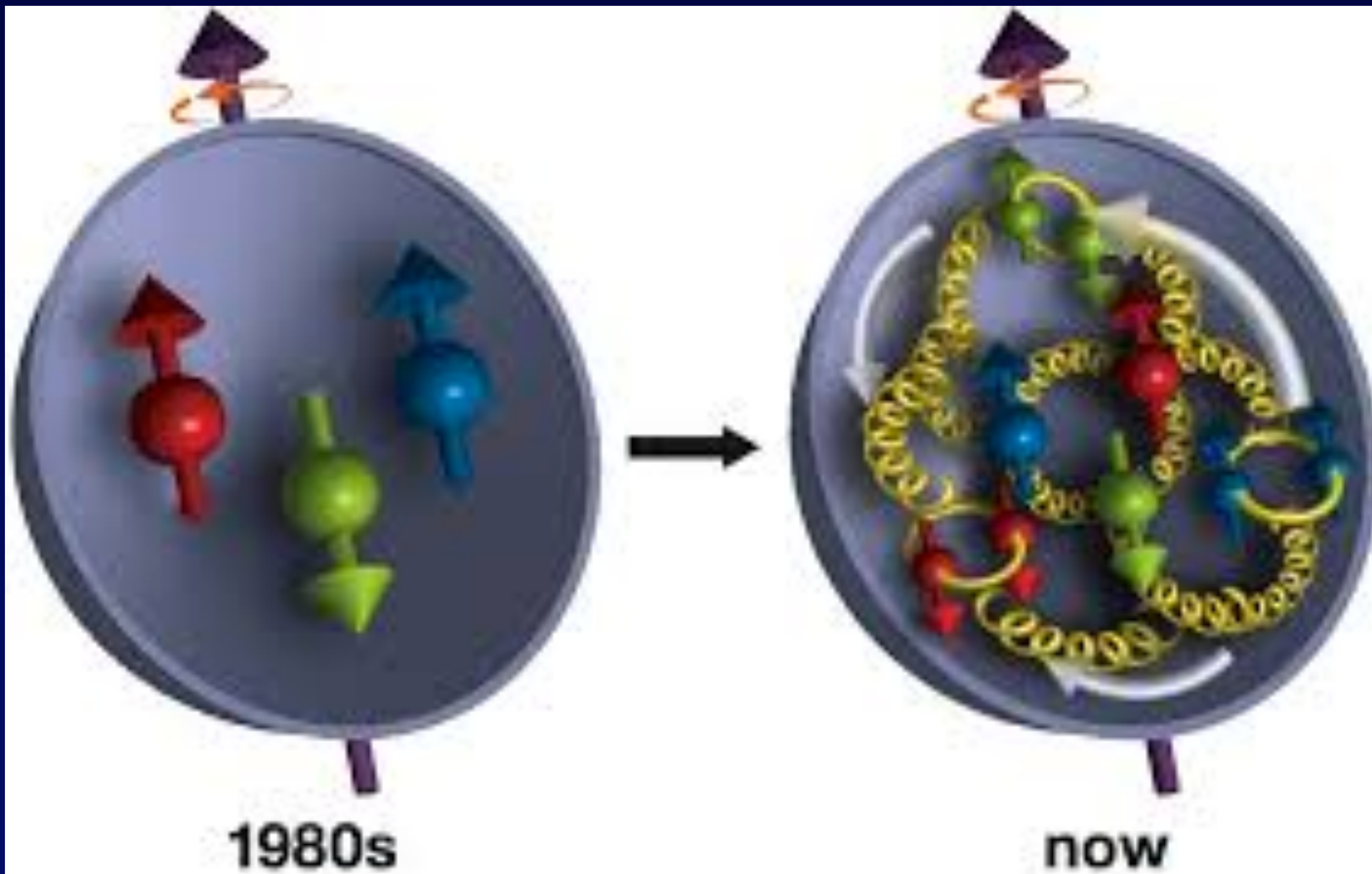
- **"Spin crisis" or puzzle:** $\Delta\Sigma = \sum_q \Delta q + \Delta\bar{q} \sim 0.3$

Scanned at the American Institute of Physics





Picture from quark model to QCD

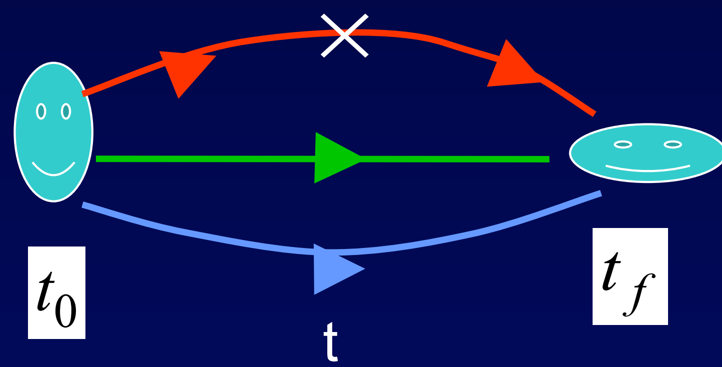


Lattice Calculations of Quark and Glue Spins

- Quark and Glue Momentum and Angular Momentum in the Nucleon

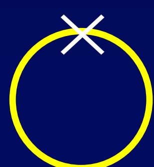
$\sum_i w_i e^{-E_i t}$

$(\bar{u}\gamma_\mu D_\nu u + \bar{d}\gamma_\mu D_\nu d)(t)$

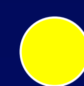


Connected insertion (CI)

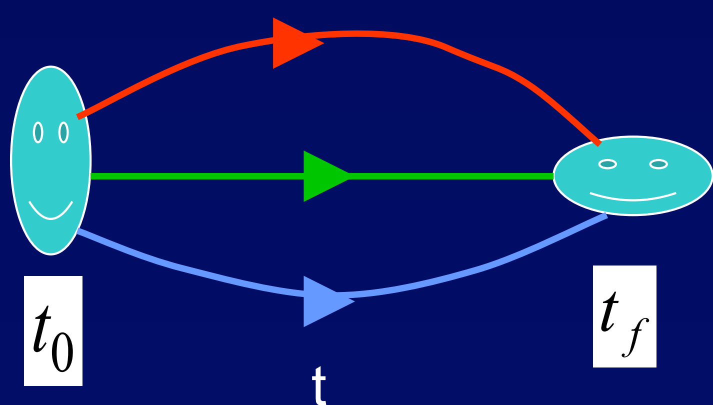
Disconnected insertion (DI)

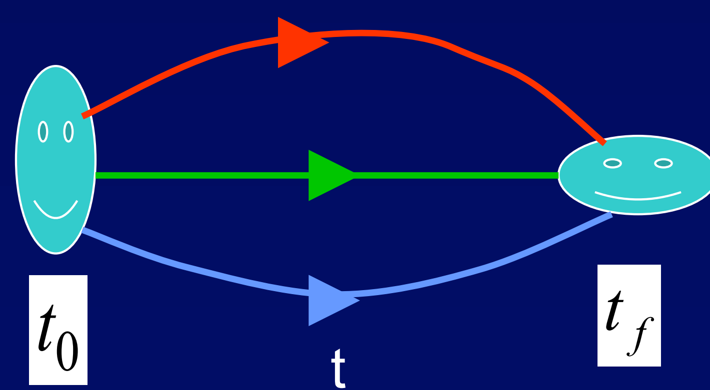


$\bar{\Psi}\gamma_\mu D_\nu \Psi(t)(u,d,s)$



$F_{\mu\alpha}F_{\nu\alpha} - \frac{1}{4}\delta_{\mu\nu}F^2$





Quark Spin Compoments $\overline{\text{MS}}$ (2 GeV)

g_A	Δ(u+d) CI	Δ(u/d) DI	Δs	Δu	Δd	Δu-Δd (g_A^3)	ΔΣ
PNDME			-0.053 (8)	0.777 (25)(30)	-0.438 (18)(30)	1.128 (27)(30)	0.286 (62)(72)
C. Alexandrou	0.598 (24)(6)	-0.077 (15)(5)	-0.042 (10)(2)	0.830 (26)(4)	-0.386 (16)(6)	1.216 (31)(7)	0.402 (34)(10)
χ QCD	0.580 (16)(30)	-0.070 (12)(15)	-0.035 (6)(7)	0.847 (18)(32)	-0.407 (16)(18)	1.254 (16)(30)	0.405 (25)(37)
NPPDFpol1.1 (Q^2=10 GeV^2)			-0.10 (8)	0.76 (4)	-0.41 (4)	1.17 (6)	0.25 (10)
DSSV (Q^2=10 GeV^2)			-0.012 +(56)-(62)	0.793 +(28)-(34)	-0.416 +(35)-(25)	1.209 +(45)-(42)	0.366 +(62)-(42)

PNDME, N_F=2+1, Clover fermion, multiple ensembles, m_π = 315 - 135 MeV

C. Alexandrou et al., N_F=2, twisted mass fermion, , m_π = 131 MeV, one lattice

χ QCD, N_F=2+1, Overlap fermion, , m_π = 170, 290, 330 MeV, 5 - 6 valence quarks for each of the three lattices → non-perturbative renormalization and normalization with anomalous Ward identity

Expt. g_A^3 = 1.2723(23); CalLat: g_A^3 = 1.271(13)

Momenta and Angular Momenta of Quarks and Glue

- Energy momentum tensor operators decomposed in quark and glue parts gauge invariantly --- Xiangdong Ji (1997)

$$T^q_{\mu\nu} = \frac{i}{4}\left[\bar{\psi}\gamma_\mu \overleftrightarrow{D}_\nu \psi + (\mu \leftrightarrow \nu)\right] \rightarrow \vec{\mathrm{J}}_q = \int d^3x \left[\frac{1}{2}\bar{\psi}\vec{\gamma}\gamma_5\psi + \vec{x}\times\bar{\psi}\gamma_4(-i\vec{D})\psi\right]$$

$$T^g_{\mu\nu} = F_{\mu\lambda}F_{\lambda\nu} - \frac{1}{4}\delta_{\mu\nu}F^2 \rightarrow \vec{\mathrm{J}}_g = \int d^3x\left[\vec{x}\times(\vec{E}\times\vec{B})\right]$$

- Nucleon form factors

$$\langle p,s\,|\,T_{\mu\nu}\,|\,p's'\rangle = \bar{u}(p,s)[T_1(q^2)\gamma_\mu\bar{p}_\nu - T_2(q^2)\bar{p}_\mu\sigma_{\nu\alpha}q_\alpha/2m$$

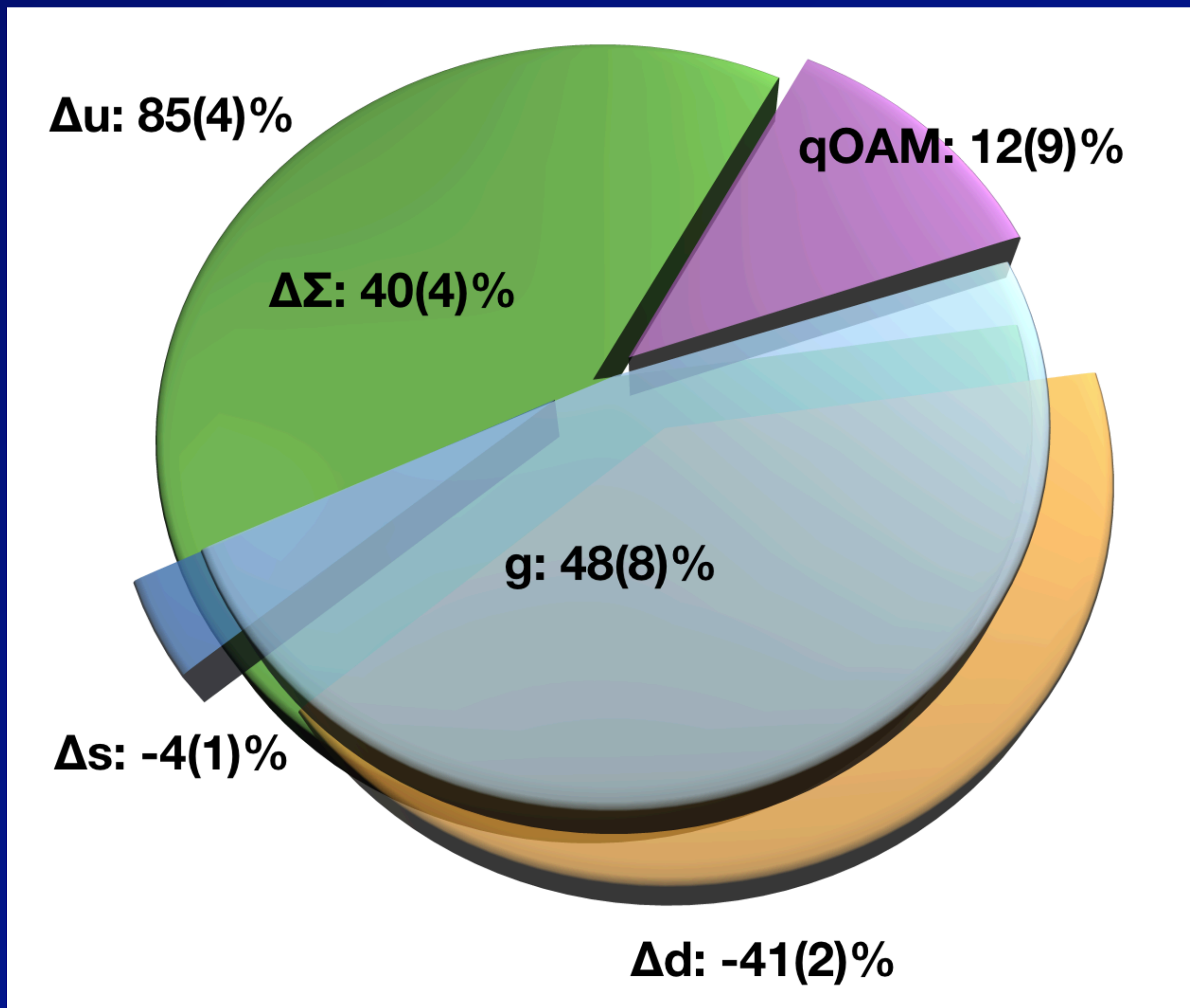
$$\text{-i}\mathrm{T}_3(q^2)(q_\mu q_\nu - \delta_{\mu\nu}q^2)/m + T_4(q^2)\delta_{\mu\nu}m/2]u(p's')$$

- Momentum and Angular Momentum

$$Z_{q,g}T_1(0)_{q,g}\ [\mathrm{OPE}] \rightarrow \langle x\rangle_{q/g}(\mu,\overline{\mathrm{MS}}),\quad Z_{q,g}\left[\frac{T_1(0)+T_2(0)}{2}\right]_{q,g} \rightarrow J_{q/g}(\mu,\overline{\mathrm{MS}})$$

Proton Spin Decomposition (2+1 Flavor)

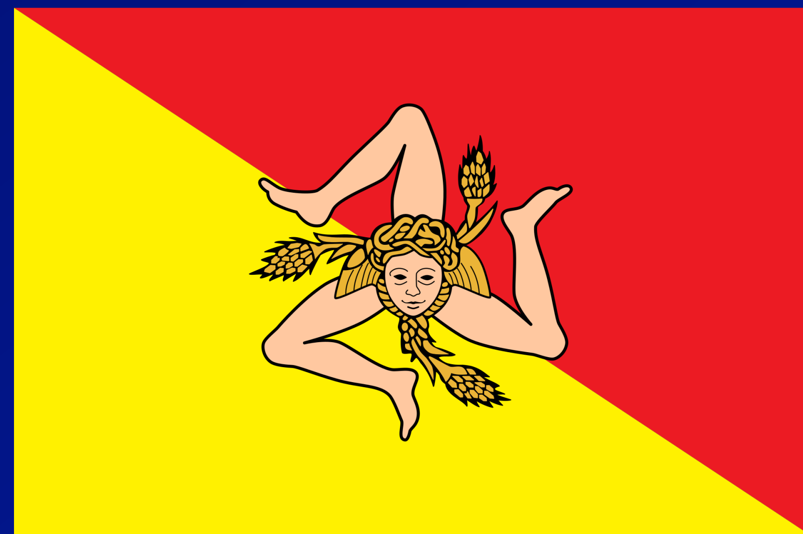
χQCD Preliminary



Approximate by setting $T_2 = 0$

Symbols for Proton Spin Components

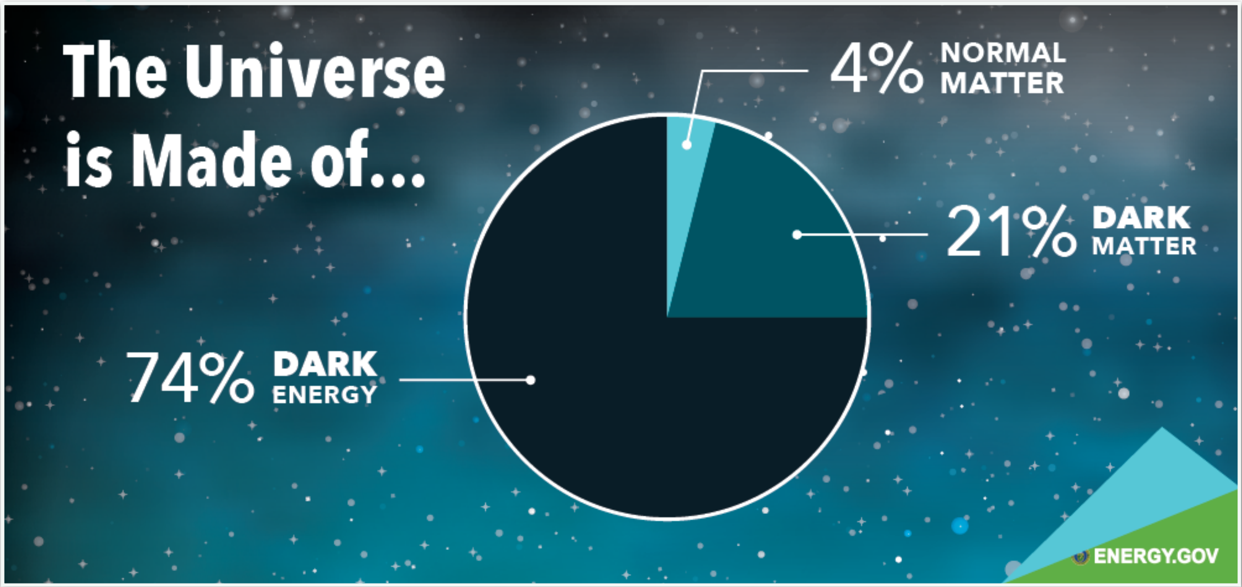
Skyrmion, a topological soliton

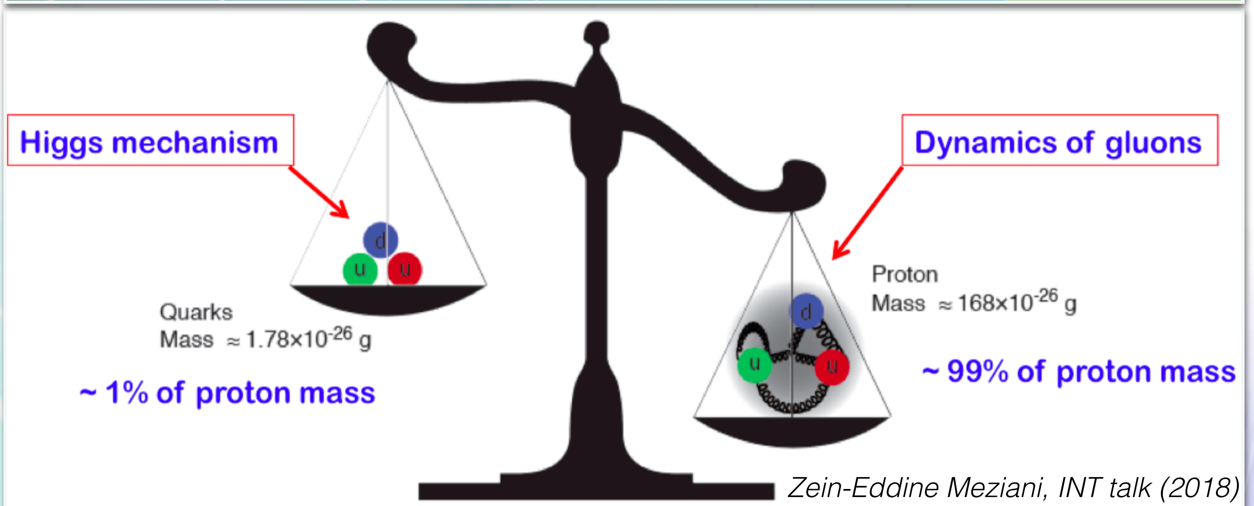


Trinacria – symbol of Sicily

Where does the proton mass come cfrom?

Proton mass





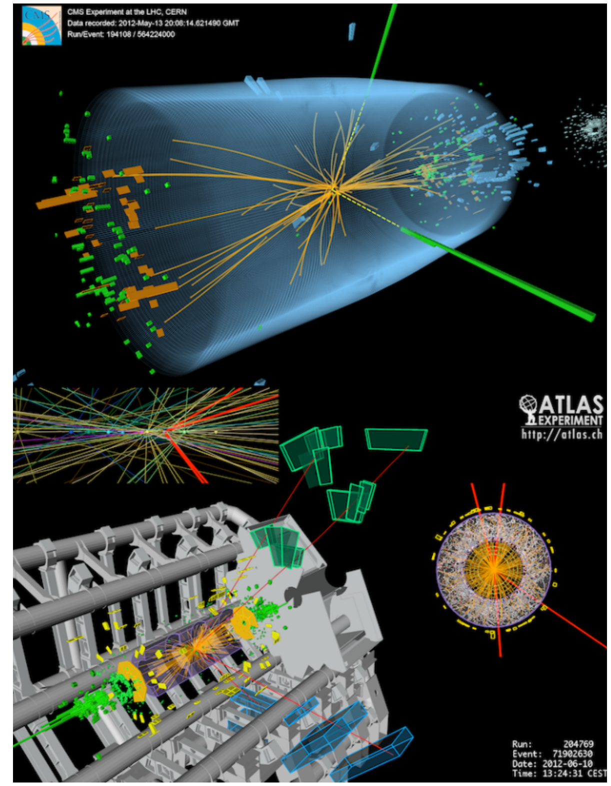
Zein-Eddine Meziani, INT talk (2018)

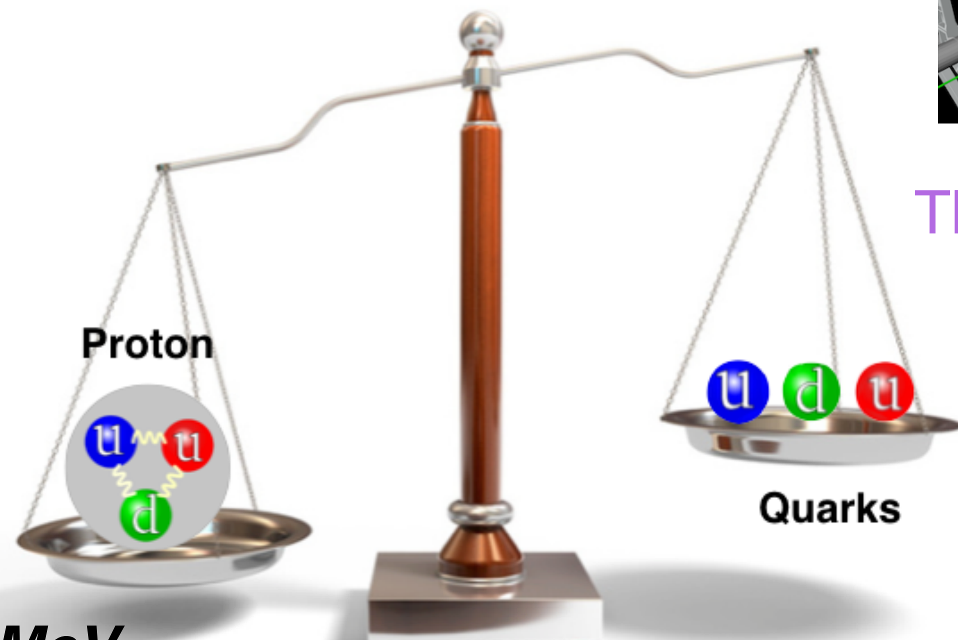
X

#

Motivation

Where does the proton mass come from, and how ?





The Higgs boson make the u/d quark having masses (2GeV MS-bar):

$m_u = 2.08(9)$ *MeV*
$m_d = 4.73(12)$ *MeV*

Laiho, Lunghi, & Van de Water, Phys.Rev.D81:034503,2010

But the mass of the proton is ***938.272046(21) MeV.***
~100 times of the sum of the quark masses!

Quark and Glue Components of Hadron Mass

- Energy momentum tensor

$$T_{\mu\nu} = \frac{1}{4}\bar{\psi}\gamma_{(\mu}\overleftrightarrow{D}_{\nu)}\psi + G_{\mu\alpha}G_{\nu\alpha} - \frac{1}{4}\delta_{\mu\nu}G^2 \qquad \langle P | T_{\mu\nu} | P\rangle = P_\mu P_\nu / M$$

- Trace anomaly

$$T_{\mu\mu} = -m(1+\gamma_m)\bar{\psi}\psi + \frac{\beta(g)}{2g}G^2$$

- Separate into traceless part $\bar{T}_{\mu\nu}$ and trace part $\hat{T}_{\mu\nu}$

$$\langle P | \bar{T}^{q,g}_{\mu\nu} | P\rangle = \langle x\rangle_{q,g}(\mu^2)(P_\mu P_\nu - \frac{1}{4}\delta_{\mu\nu}P^2)/M, \quad \langle x\rangle_q(\mu^2) + \langle x\rangle_g(\mu^2) = 1$$

$$\langle \bar{T}_{44}\rangle = -3/4\,M; \qquad \langle \hat{T}_{\mu\mu}\rangle = -M$$

Proton mass decomposition

$$M = -\langle T_{44}\rangle = \langle H_m\rangle + \langle H_E\rangle(\mu) + \langle H_g\rangle(\mu) + \frac{1}{4}\langle H_a\rangle$$

$$M = -\langle \hat{T}_{\mu\mu}\rangle = \langle H_m\rangle + \langle H_a\rangle$$

X. Ji, PRL74:1071 (1995)

quark mass $$\langle H_m\rangle = \sum_{u,d,s,\ldots}\int d^3x m\bar{\psi}\psi$$

quark energy $$\langle H_E\rangle = \frac{3}{4}\left(\langle x\rangle_q M - \langle H_m\rangle\right)$$

glue energy $$\langle H_g\rangle = \frac{3}{4}\langle x\rangle_g M$$

anomaly $$\langle H_a\rangle = \langle H_g^a\rangle + \langle H_m^\gamma\rangle$$

$$\langle x\rangle_{q,g} = \int_0^1 dx x f_{q,g}(x) = -\frac{\langle N|\frac{4}{3}\bar{T}^{q,g}_{44}|N\rangle}{M\langle N|N\rangle}$$

$$\bar{T}^q_{44} = \int d^3x\bar{\psi}\frac{1}{2}\left(\gamma_4\overleftrightarrow{D}_4 - \frac{1}{4}\sum_{i=0,1,2,3}\gamma_i\overleftrightarrow{D}_i\right)\psi$$

$$\bar{T}^g_{44} = \int d^3x\frac{1}{2}\left(E^2 - B^2\right)$$

$$\langle H_g^a\rangle = \int d^3x\frac{-\beta(g)}{g}\left(E^2 + B^2\right)$$

$$\langle H_m^\gamma\rangle = \sum_{u,d,s,\ldots}\int d^3x\gamma_m m\bar{\psi}\psi$$

Ingredients

- proton mass
- scalar charge
- momentum fractions (both quark and glue)
- renormalization of momentum fractions including mixing

Non-perturbative Renormalization and Mixing

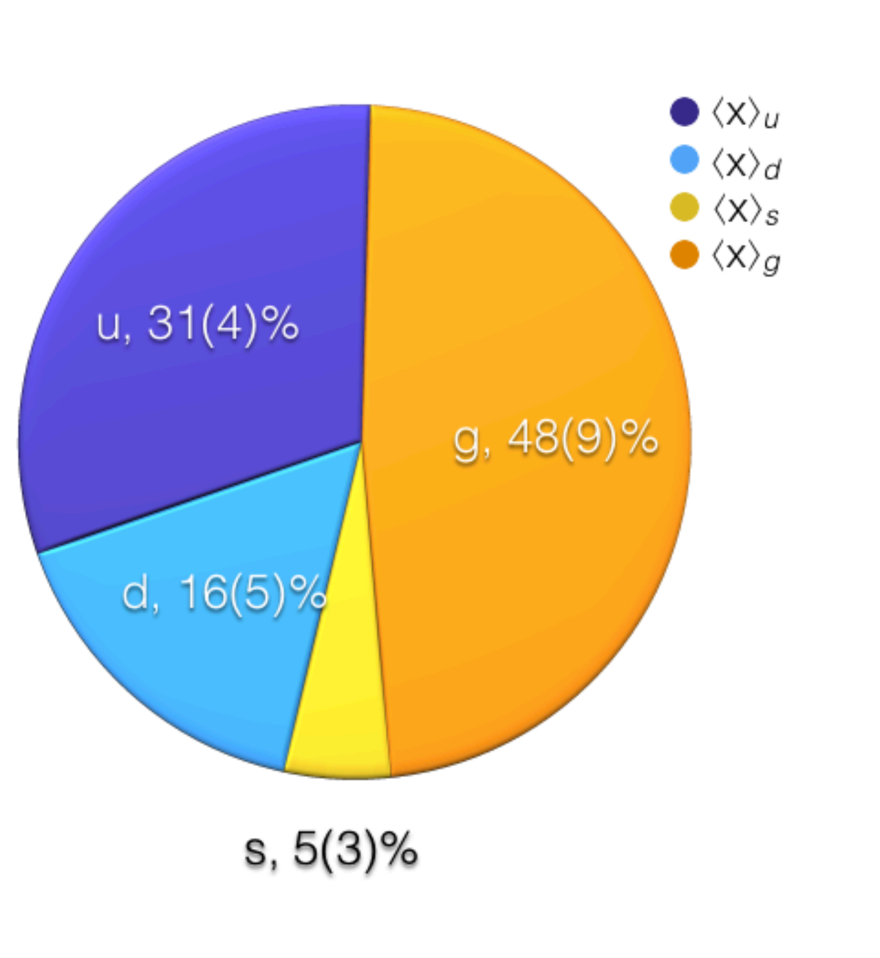
- Renormalized $<x>_q$ and $<x>_g$ in MS-bar at μ

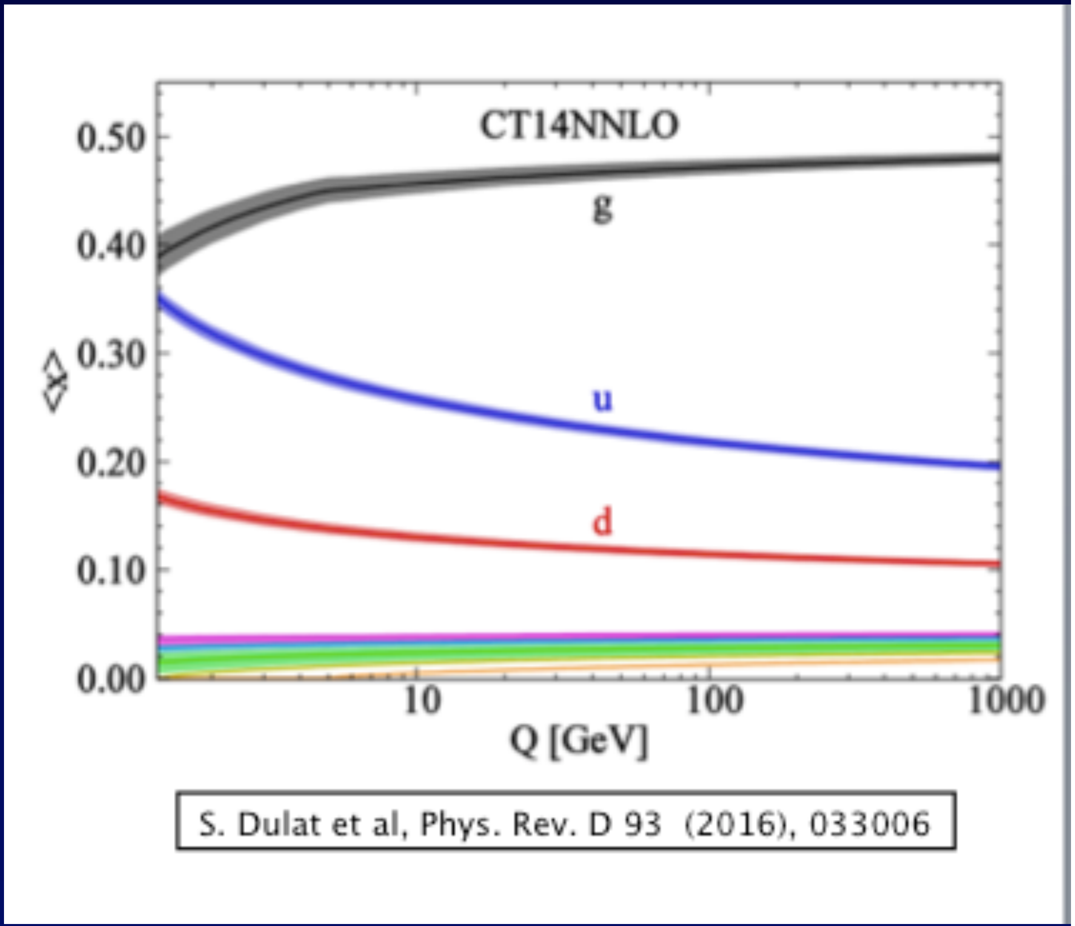
$$\langle x\rangle^R_{u,d,s} = Z^{\overline{MS}}_{QQ}(\mu)\langle x\rangle_{u,d,s} + \delta Z^{\overline{MS}}_{QQ}(\mu)\sum_{q=u,d,s}\langle x\rangle_q + Z^{\overline{MS}}_{QG}(\mu)\langle x\rangle_g,\quad \langle x\rangle^R_g = Z^{\overline{MS}}_{GQ}(\mu)\sum_{q=u,d,s}\langle x\rangle_q + Z^{\overline{MS}}_{GG}\langle x\rangle_g,$$

$$\begin{pmatrix} Z^{\overline{MS}}_{QQ}(\mu)+N_f\delta Z^{\overline{MS}}_{QQ}(\mu) & N_f Z^{\overline{MS}}_{QG}(\mu) \\ Z^{\overline{MS}}_{GQ}(\mu) & Z^{\overline{MS}}_{GG}(\mu)\end{pmatrix} = \left\{\left[\begin{pmatrix} Z_{QQ}(\mu_R)+N_f\delta Z_{QQ} & N_f Z_{QG}(\mu_R) \\ Z_{GQ}(\mu_R) & Z_{GG}(\mu_R)\end{pmatrix}\right.\right.$$
$$\left.\left.\begin{pmatrix} R_{QQ}(\frac{\mu}{\mu_R})+\mathcal{O}(N_f\alpha_s^2) & N_f R_{QG}(\frac{\mu}{\mu_R}) \\ R_{GQ}(\frac{\mu}{\mu_R}) & R_{GG}(\frac{\mu}{\mu_R})\end{pmatrix}\right]\Big|_{a^2\mu_R^2\to 0}\right\}^{-1}$$

- Renormalization of glue operator in gluon propagator is very noisy → Cluster Decomposition Error Reduction (CDER)

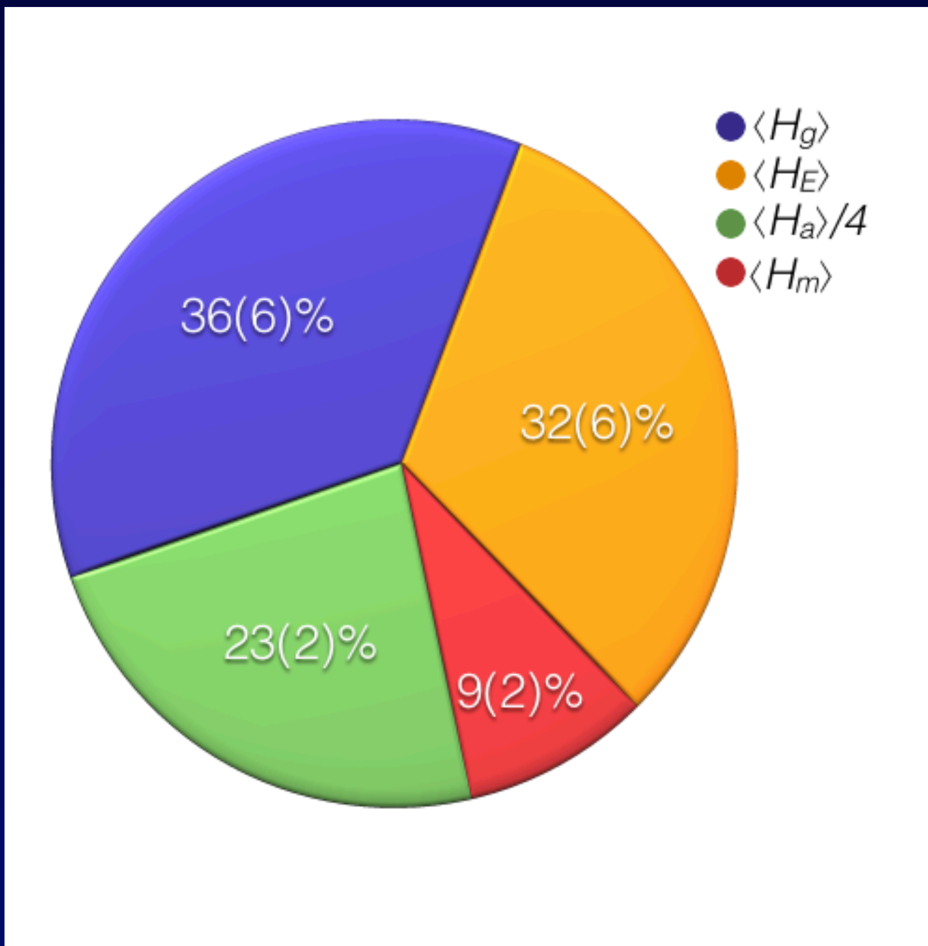
Comparison with Global Fitting of <x> MS-bar at 2 GeV

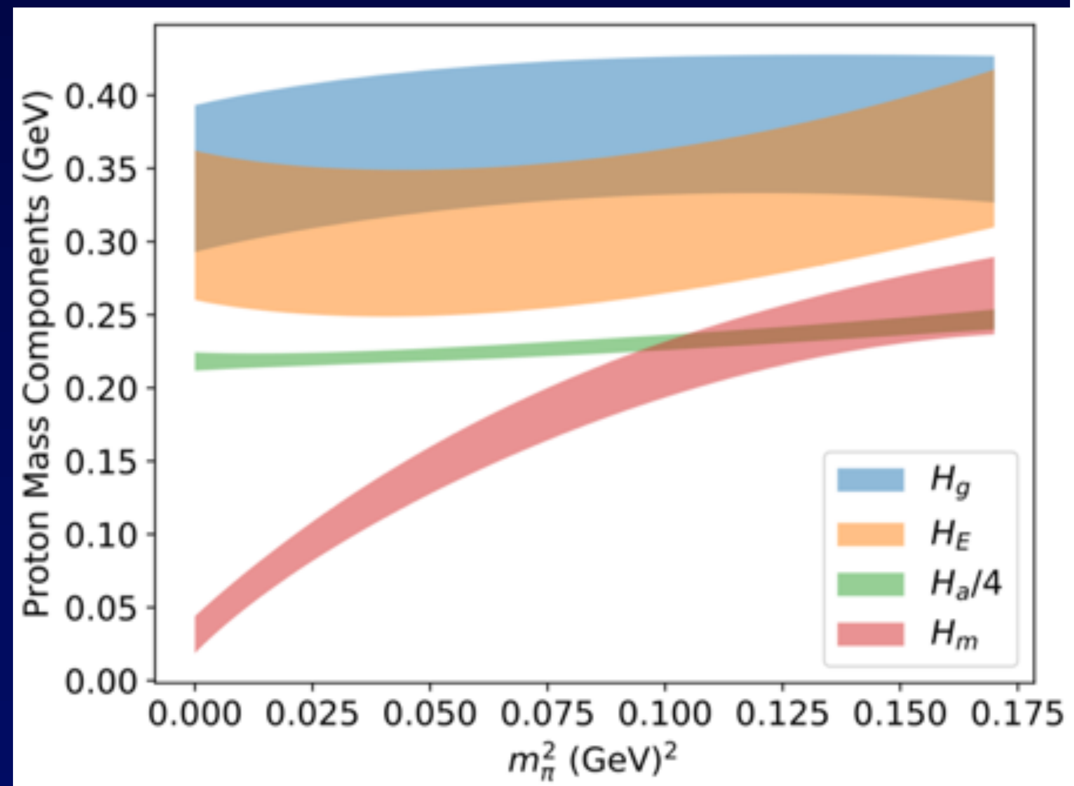




χQCD, preliminary

Proton Mass Decomposition





Y.B. Yang et al (χQCD), PRL 121, 212001 (2018)

PRL as Editor's Suggestion

(Y.B. Yang, PRL 121, 212001 (2018))

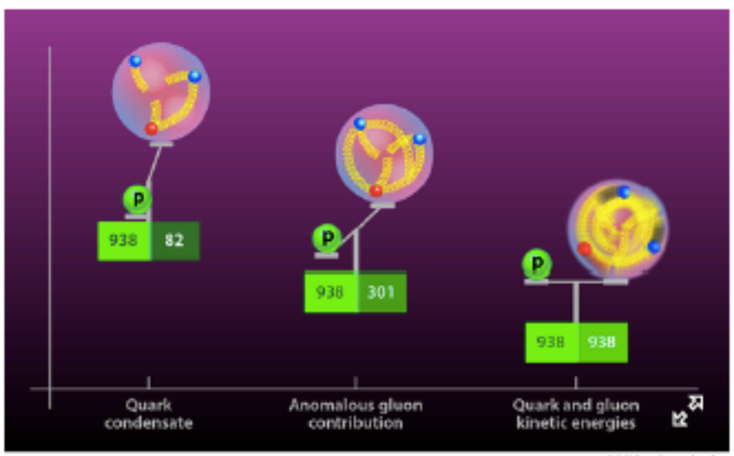
APS physics | Physics

Viewpoint: Dissecting the Mass of the Proton

André Walker-Loud, Nuclear Science Division, Lawrence Berkeley National Laboratory, Berkeley, CA, USA

November 19, 2018 • *Physics* 11, 118

A calculation determines four distinct contributions to the proton mass, more than 90% of which arises entirely from the dynamics of quarks and gluons.



APS/Alan Stonebraker

Figure 1: The proton is comprised of two up quarks and one down quark, but the sum of these quark masses is a mere 1% of the proton mass. Using lattice QCD, Yang and colleagues determined the relative contributions of the four sources of the proton mass [1]. ... Show more

ScienceNews

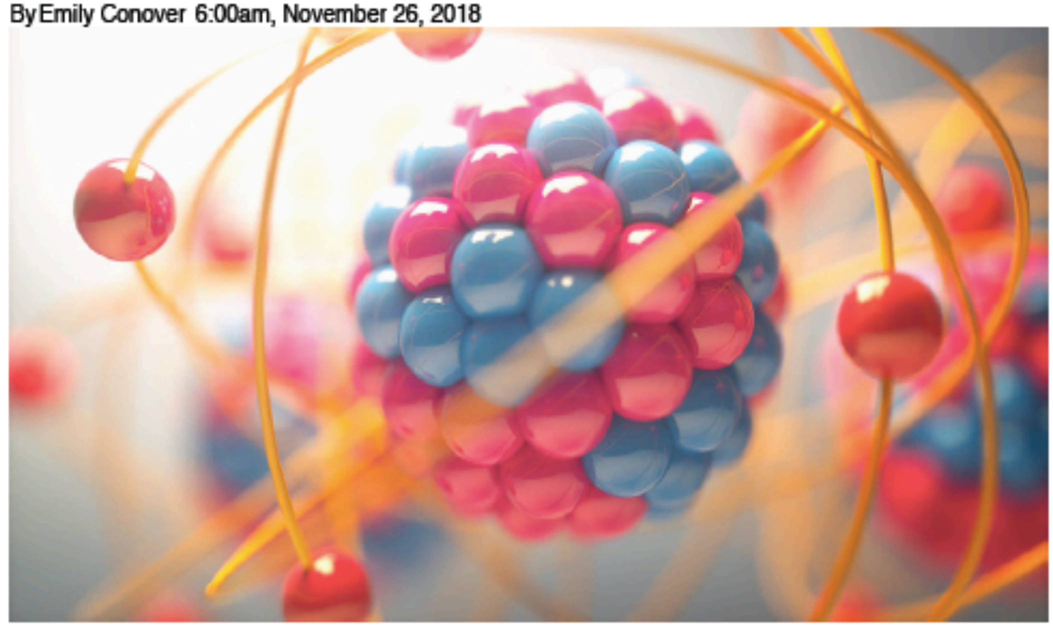
MAGAZINE OF THE SOCIETY FOR SCIENCE & THE PUBLIC

News: Particle Physics

Physicists finally calculated where the proton's mass comes from

Only 9 percent of the subatomic particle's bulk comes from the mass of its quarks

By Emily Conover 6:00am, November 26, 2018



MASSIVE UNDERTAKING Using a technique called lattice QCD, scientists figured out how protons (illustrated here in the nucleus of an atom) get their mass.

ktsdesign/Shutterstock

Hadronic Tensor in Euclidean Path-Integral Formalism

- Deep inelastic scattering
  In Minkowski space

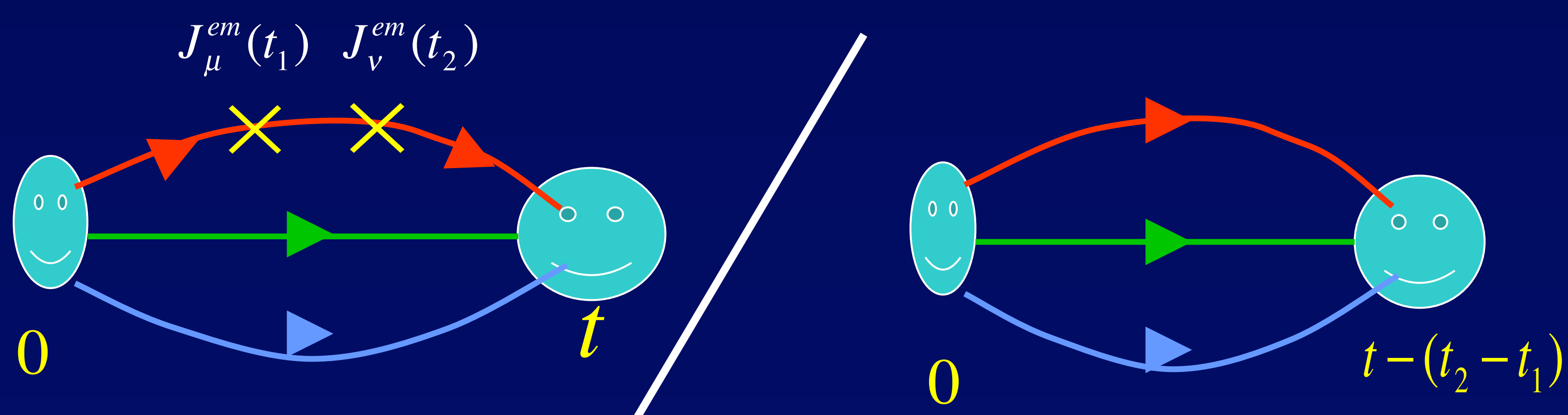
$$\frac{d^2\sigma}{dE'\,d\Omega} = \frac{\alpha^2}{q^4}\left(\frac{E'}{E}\right) l^{\mu\nu} W_{\mu\nu}$$

$$W_{\mu\nu}(\vec{q},\vec{p},\nu) = \frac{1}{\pi}\mathrm{Im}\, T_{\mu\nu} = \left\langle N(\vec{p})\right| \int \frac{d^4x}{4\pi}\, e^{iq\cdot x} J_\mu(x) J_\nu(0) \left| N(\vec{p})\right\rangle_{\text{spin avg}}$$

$$= \frac{1}{2}\sum_n \int \prod_{i=1}^{n} \left[\frac{d^3 p_i}{(2\pi)^3 2E_{pi}}\right] (2\pi)^3 \delta^4(p_n - p - q) < N(\vec{p}) | J_\mu | n >< n | J_\nu | N(\vec{p}) >_{\text{spin avg}}$$

- Euclidean path-integral

KFL and S.J. Dong, PRL 72, 1790 (1994)
KFL, PRD 62, 074501 (2000)



Inverse problem with Laplace transform

Hadronic tensor on the lattice

four-point function with **3-dimensional Fourier transform**

$$C_4 = \sum_{\vec{x}_f} e^{-i\vec{p}\cdot\vec{x}_f} \sum_{\vec{x}_2 \vec{x}_1} e^{-i\vec{q}\cdot(\vec{x}_2-\vec{x}_1)} \left\langle \chi_N(\vec{x}_f, t_f) J_\mu^\dagger(\vec{x}_2, t_2) J_\nu(\vec{x}_1, t_1) \bar{\chi}_N(\vec{0}, t_0) \right\rangle$$

Euclidean hadronic tensor defined as a function of time difference between the currents

$$\tilde{W}_{\mu\nu}(p, \vec{q}, \tau) = \frac{E_p}{m_N} \frac{\mathrm{Tr}[\Gamma_e C_4]}{\mathrm{Tr}[\Gamma_e C_2]} \rightarrow \sum_{\vec{x}_2 \vec{x}_1} e^{-i\vec{q}\cdot(\vec{x}_2-\vec{x}_1)} \langle p, s \,|\, J_\mu(\vec{x}_2, t_2) J_\nu(\vec{x}_1, t_1) \,|\, p, s \rangle$$

$$= \sum_n A_n e^{-(E_n - E_p)\tau}, \quad \tau \equiv t_2 - t_1$$

Solving the **inverse problem** of a Laplace transform to get back to Minkowski space

$$\tilde{W}_{\mu\nu}\left(p, \vec{q}, \tau\right) = \int d\nu W_{\mu\nu}\left(p, \vec{q}, \nu\right) e^{-\nu\tau}$$

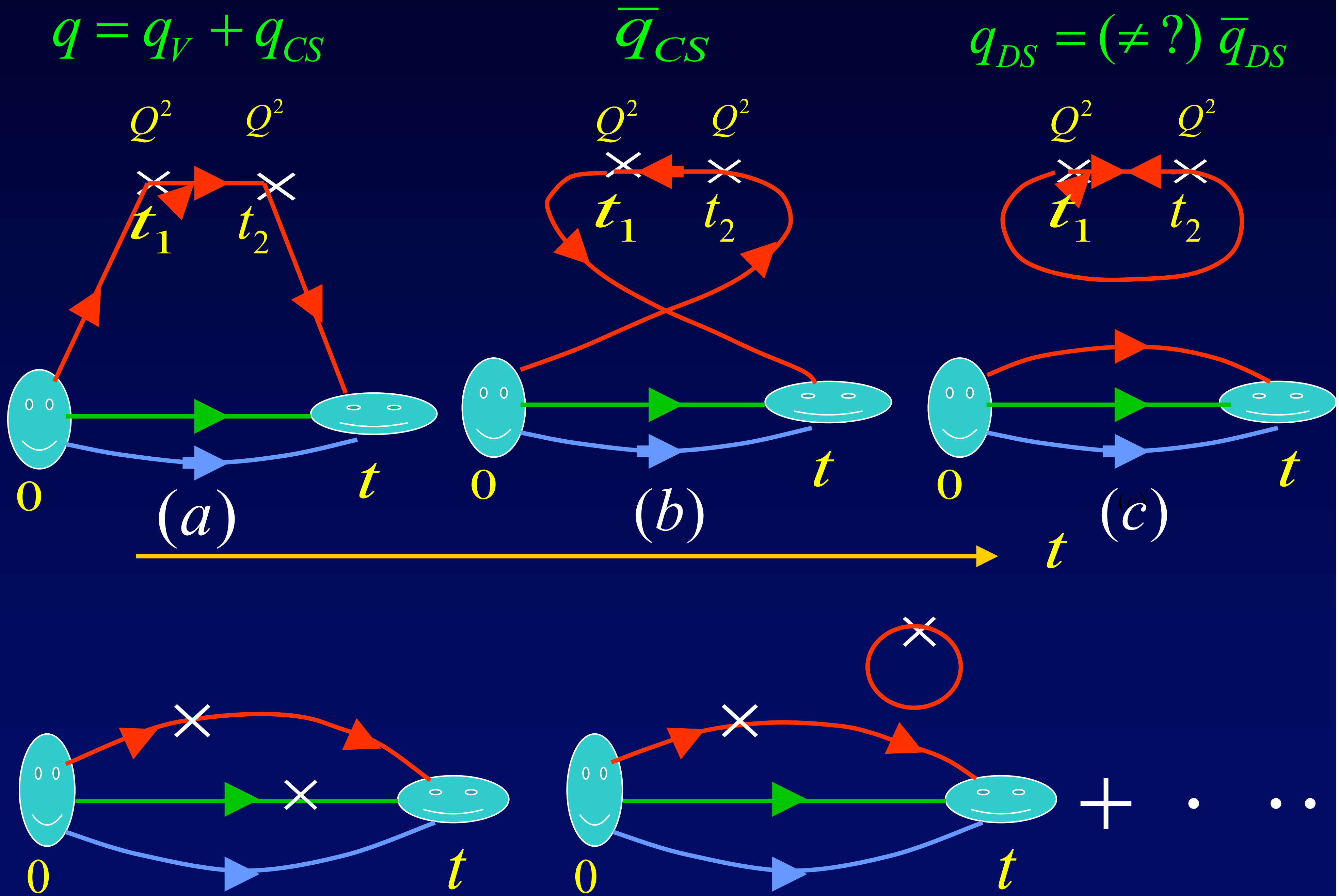
K.F. Liu and S. J. Dong, PRL 72, 1790 (1994)

K.-F. Liu, PRD 62, 074501 (2000)

J. Liang et. al., EPJ Web Conf. 175, 14014 (2018)

J. Liang et. al., arXiv:1906.05312

X



Cat's ears diagrams are suppressed by O(1/Q²).

Inverse problems are ubiquitous

- Extracting spectral functions from lattice data: $c_2(t) = \int d\omega e^{-\omega t} \rho(\omega)$

Y. Burnier and A. Rothkopf, PRL 111, 182003 (2013)

- Global fittings of PDFs: $F_i = \sum_a C_i^a \otimes f_a$

- Lattice calculation of Quasi-PDFs: $\tilde{q}(x, P_3) = \frac{2P_3}{4\pi} \sum_{z=-z_{\max}}^{z_{\max}} e^{-ixP_3 z} h_\Gamma(P_3, z)$

J. Karpie et. al., JHEP04, 057 (2019)

$$\tilde{q}(x, \mu^2, P^z) = \int_0^1 \frac{dy}{y} Z\left(\frac{x}{y}, \frac{\mu}{P^z}\right) q(y, \mu^2) + \mathcal{O}\left(\Lambda^2/(P^z)^2, M^2/(P^z)^2\right)$$

X. Xiong et. al., PRD90:014051 (2014)

- Lattice cross sections:

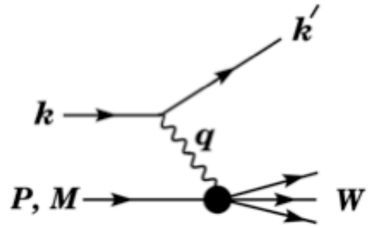
$$\sigma_n(\omega, \xi^2, P^2) = \sum_a \int_{-1}^1 \frac{dx}{x} f_a(x, \mu^2) \times K_n^a(x\omega, \xi^2, x^2 P^2, \mu^2) + O(\xi^2 \Lambda_{\mathrm{QCD}}^2)$$

Y.-Q. Ma and J.-W. Qiu, PRL120, 022003 (2018)

- Lattice calculation of Pseudo-PDFs: $\mathfrak{M}_R(\nu, \mu^2) \equiv \int_0^1 dx \, \cos(\nu x) \, q_v(x, \mu^2)$

K. Orginos et al., PRD96, 094503 (2017)

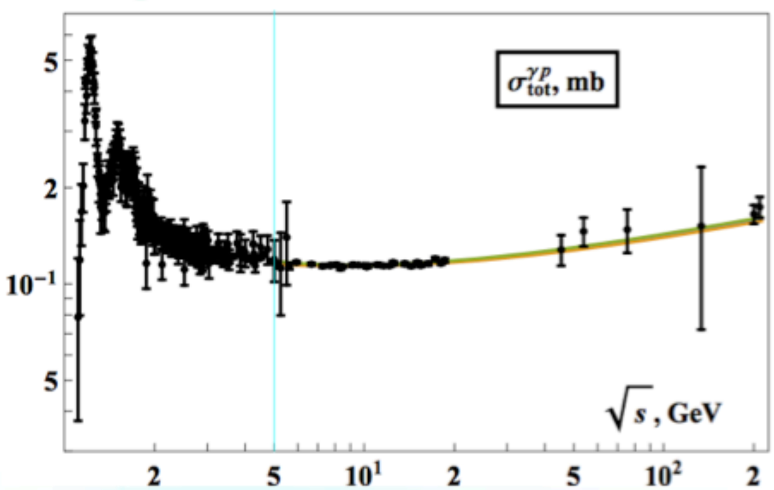
Sketch the hadronic tensor

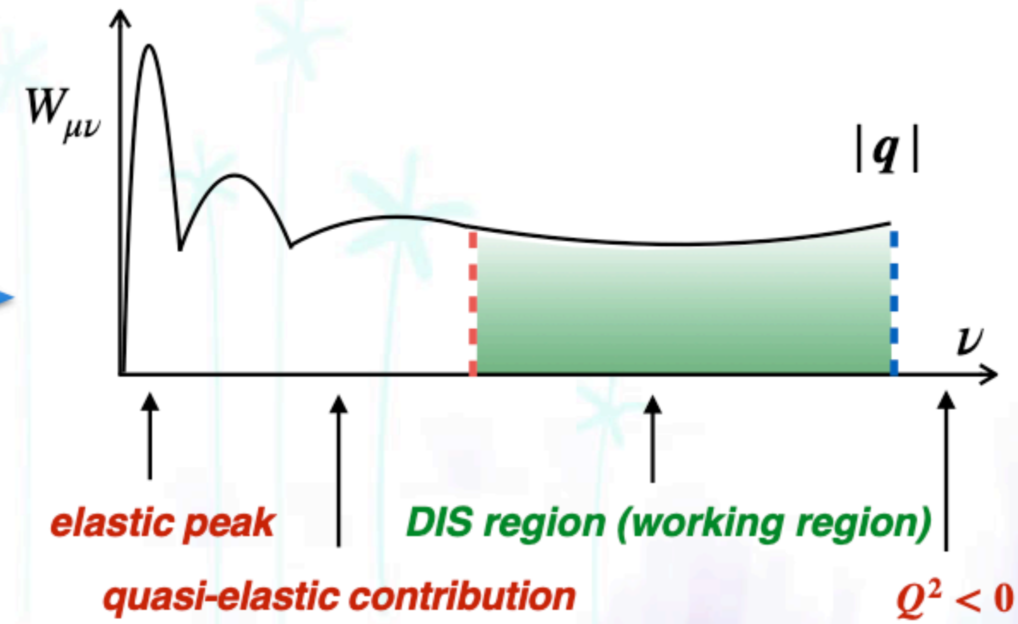


$$Q^2 = -q^2 = \boldsymbol{q}^2 - \nu^2 \qquad x = \frac{Q^2}{2m\nu}$$

fixed by momentum setup

comes from solving the inverse problem





$\nu > (E_{n=0} - E_p) + \Delta E$ (away from the elastic peak)

$\nu < |\boldsymbol{q}|$ (physical x and Q^2)

x

Hadronic tensor and neutrino-nucleus scattering

- New long-baseline neutrino experiments are in preparation: T2K, NOvA, PINGU, ORCA, Hyper-Kamiokande, DUNE...
- Be
  nu
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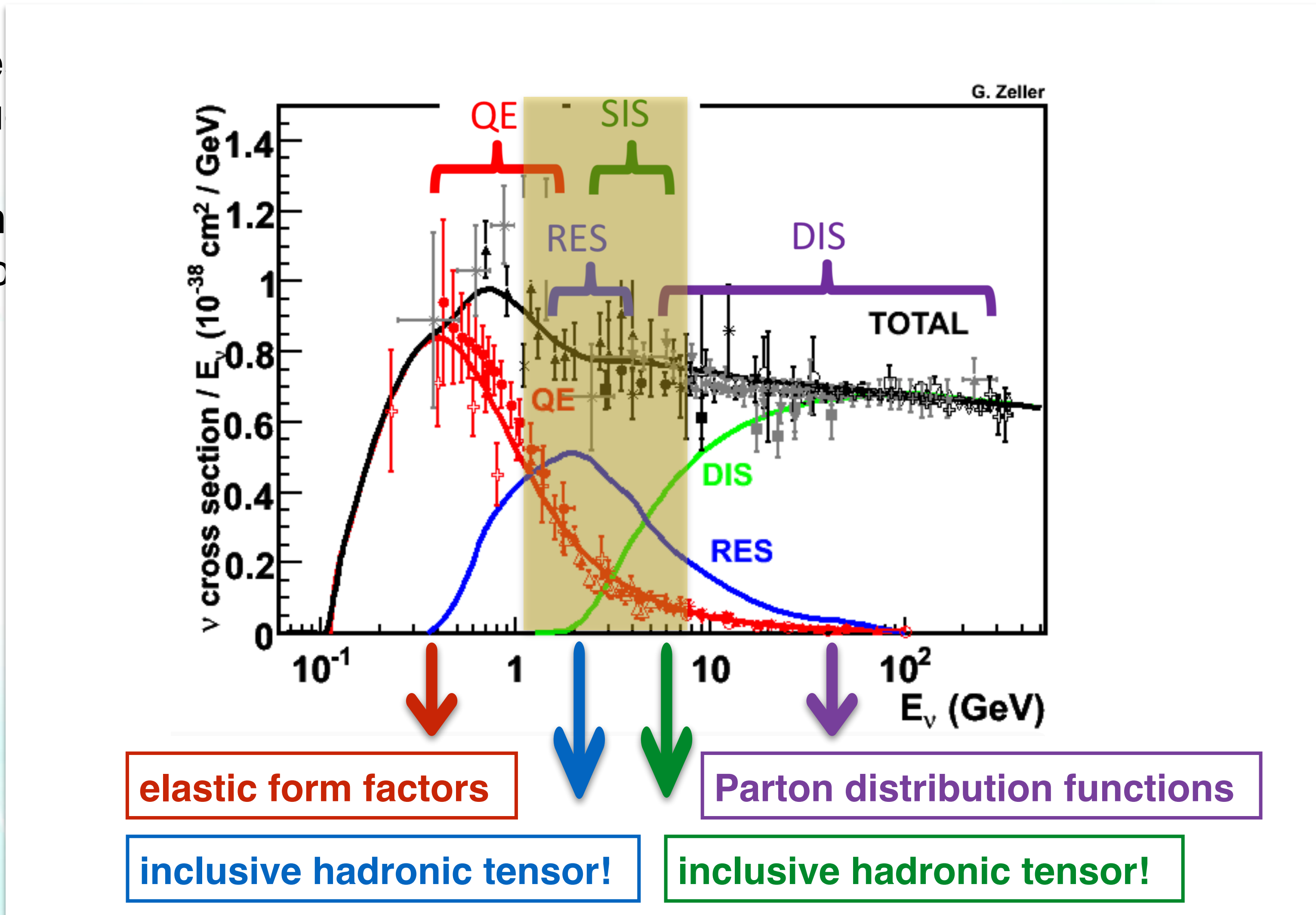
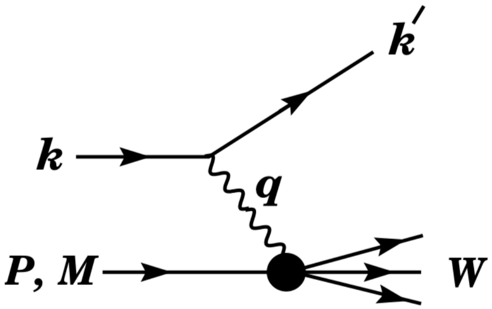


J.A. Formaggio and G.P. Zeller, RMP84, 1307 (2012)

X

Hadronic tensor



for lepton-nucleon scatterings $\frac{d^2\sigma}{dxdy} = \frac{2\pi y \alpha^2}{Q^4} \sum_j \eta_j L_j^{\mu\nu} W_{\mu\nu}^j$

$$W_{\mu\nu}\left(p, \vec{q}, \nu\right) = \frac{1}{4\pi} \int d^4 z e^{iq\cdot z} \left\langle p, s \left| \left[J_\mu^\dagger(z) J_\nu(0) \right] \right| p, s \right\rangle \sim \mathrm{Im}\left[T_{\mu\nu}\right]$$

$$W_{\mu\nu} = \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right) F_1(x, Q^2) + \frac{\hat{P}_\mu \hat{P}_\nu}{P \cdot q} F_2(x, Q^2)$$

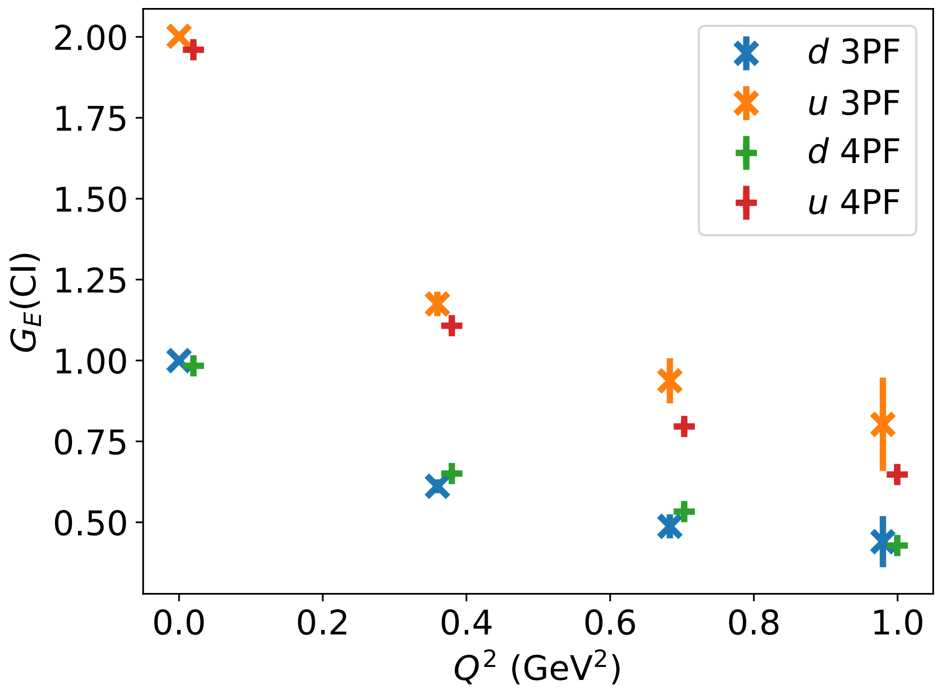
The hadronic tensor and structure functions encode the non-perturbative nature of the nucleon.

- ✦ Elastic form factors: $F_2^{\mathrm{el}} = \delta(q^2 + 2m_N \nu) \frac{2m_N}{1 - q^2/4m_N{}^2} \left(G_E^2(q^2) - \frac{q^2}{4M_N^2} G_M^2(q^2) \right)$
- ✦ Neutrino-nucleon scattering
- ✦ PDFs from DIS: $F_i = \sum_a C_i^a \otimes f_a$

Elastic FFs

$$\tilde{W}_{44}(\vec{p},\vec{q},\tau) = \sum_{\vec{x}_2\vec{x}_1} e^{-i\vec{q}\cdot(\vec{x}_2-\vec{x}_1)}\langle p,s\,|\,J_\mu(\vec{x}_2,t_2)J_\nu(\vec{x}_1,t_1)\,|\,p,s\rangle = \sum_n A_n e^{-\nu_n\tau}$$

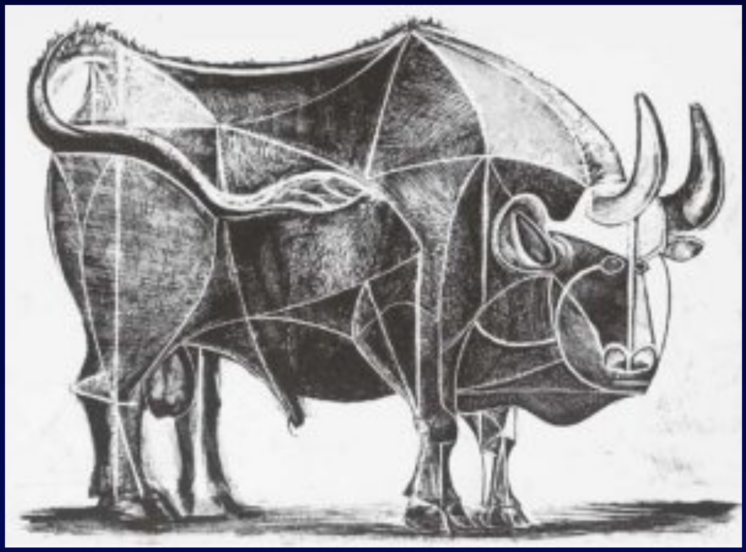
$$A_0 = \langle p,s\,|\,J_4(\vec{q})\,|\,n=0\rangle\langle n=0\,|\,J_4(-\vec{q})\,|\,p,s\rangle = G_E^2(Q^2)$$

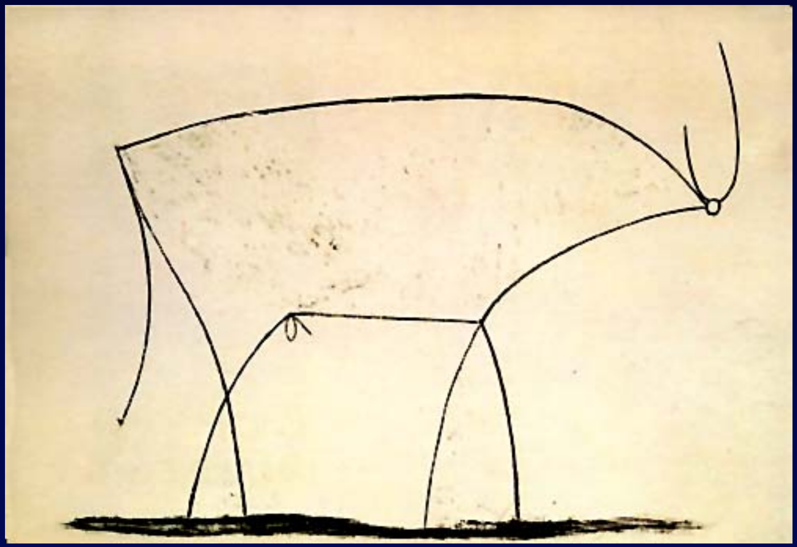


FFs extracted from 3-point functions and 4-point functions show consistency.

(32IF lattice, a ~0.063 fm, pion mass ~370 MeV)

Le Taureau of Pablo Picasso (1945)





5th stage

11th stage

Dynamical chiral fermion
Physical pion mass
Continuum limit
Infinite volume limit

Quenched approximation

Summary and Challenges

- Together with experiments on LHC and EIC and global fitting of PDF, lattice QCD calculations of hadron structure (proton spin and mass decomposition, moments of PDFs, form factors, etc.) can advance our understanding of the nucleon properties in more detail.

- Hadronic tensor calculation from lattice QCD involves a numerically challenging inverse problem. It can address the low-energy neutrino-nucleon elastic scattering, the inelastic resonance region, the shallow inelastic as well as the deep inelastic region. The latter will require large lattices with lattice spacing as small as 0.02 fm. Together with experiments from DUNE, they can help us understand the salient properties of the neutrinos.

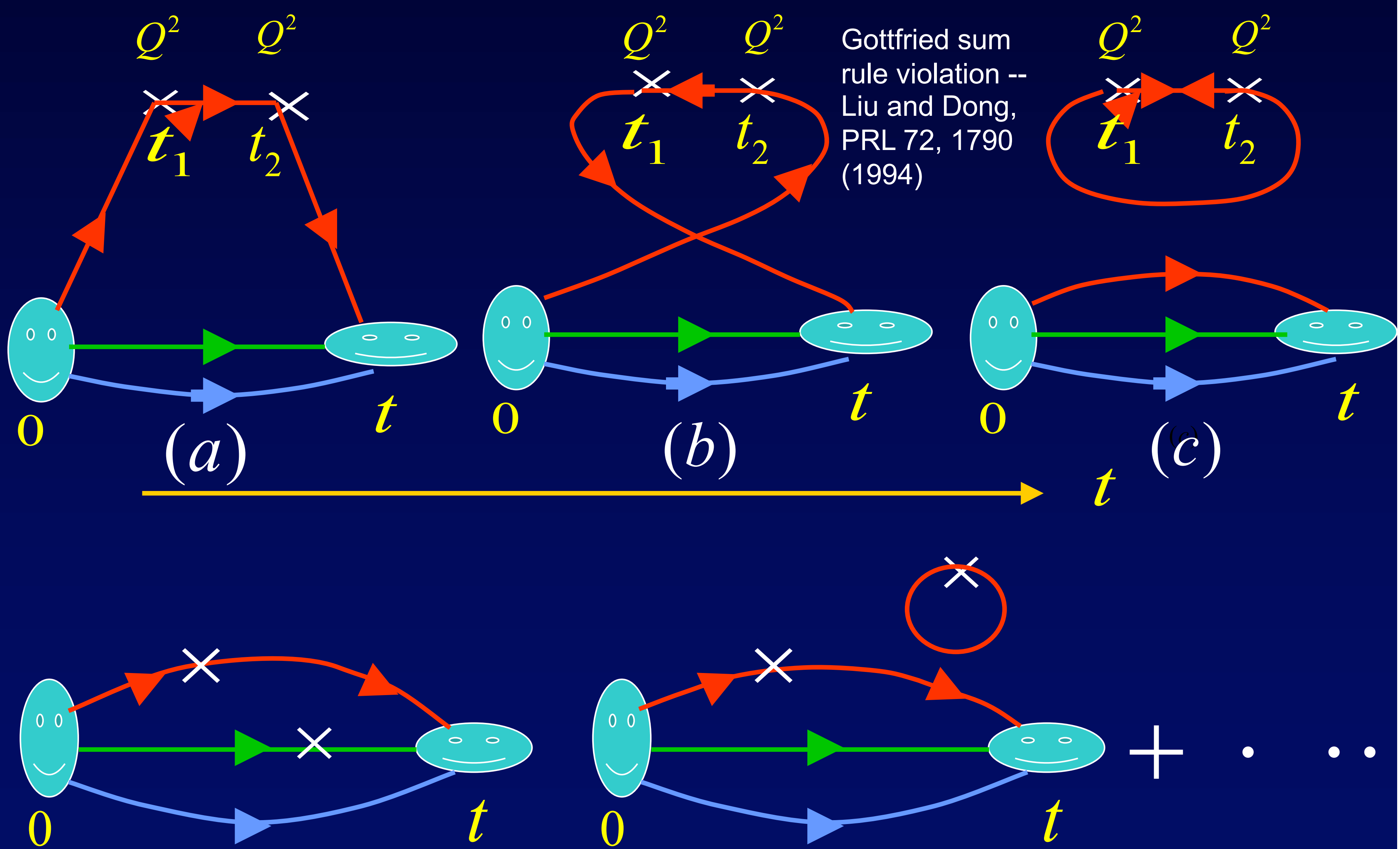
Imagination **may not be relevant.**

Knowledge **cannot predict.**

By simulating ***imagination*** **and testing against** ***knowledge*** **only can** ***reality*** **be re-created.**

Hadronic Tensor

$q = q_V + q_{CS}$ $\quad \bar{q}_{CS}$ $\quad q_{DS} = (\neq ?)\ \bar{q}_{DS}$



Cat's ears diagrams are suppressed by $O(1/Q^2)$.

$$q_i^- = q_i^{v+cs} - \bar{q}_i^{cs} + q_i^{ds} - \bar{q}_i^{ds} \equiv q_i^v + q_i^{ds} - \bar{q}_i^{ds}$$

Correlators

- One point collerators → $\langle \bar{\psi}\psi \rangle, \langle G_{\mu\nu} G_{\mu\nu} \rangle$ …
- Two point correlators → hadron masses, decay constants …
- Three point correlators → parton moments form factors, nucleon matrix elements …
- Four point correlators → hadronic tensor, PDF, GPD, TMD, nEDM…

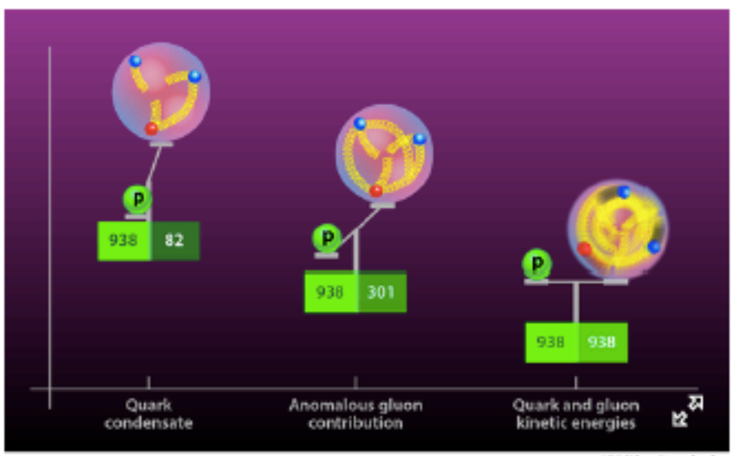
APS physics | Physics

Viewpoint: Dissecting the Mass of the Proton

André Walker-Loud, Nuclear Science Division, Lawrence Berkeley National Laboratory, Berkeley, CA, USA

November 19, 2018 • *Physics* 11, 118

A calculation determines four distinct contributions to the proton mass, more than 90% of which arises entirely from the dynamics of quarks and gluons.



APS/Alan Stonebraker

Figure 1: The proton is comprised of two up quarks and one down quark, but the sum of these quark masses is a mere 1% of the proton mass. Using lattice QCD, Yang and colleagues determined the relative contributions of the four sources of the proton mass [1]. ... **Show more**

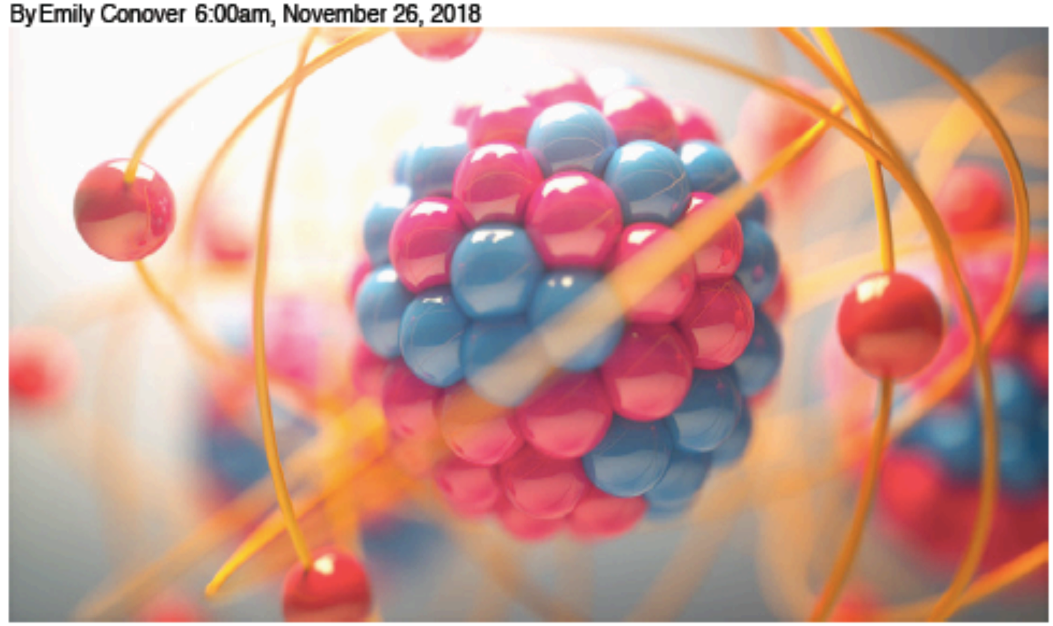
ScienceNews
MAGAZINE OF THE SOCIETY FOR SCIENCE & THE PUBLIC

News: Particle Physics

Physicists finally calculated where the proton's mass comes from

Only 9 percent of the subatomic particle's bulk comes from the mass of its quarks

By Emily Conover 6:00am, November 26, 2018



MASSIVE UNDERTAKING Using a technique called lattice QCD, scientists figured out how protons (illustrated here in the nucleus of an atom) get their mass.

ktsdesign/Shutterstock