

Some questions from Lecture 1 (Berg):

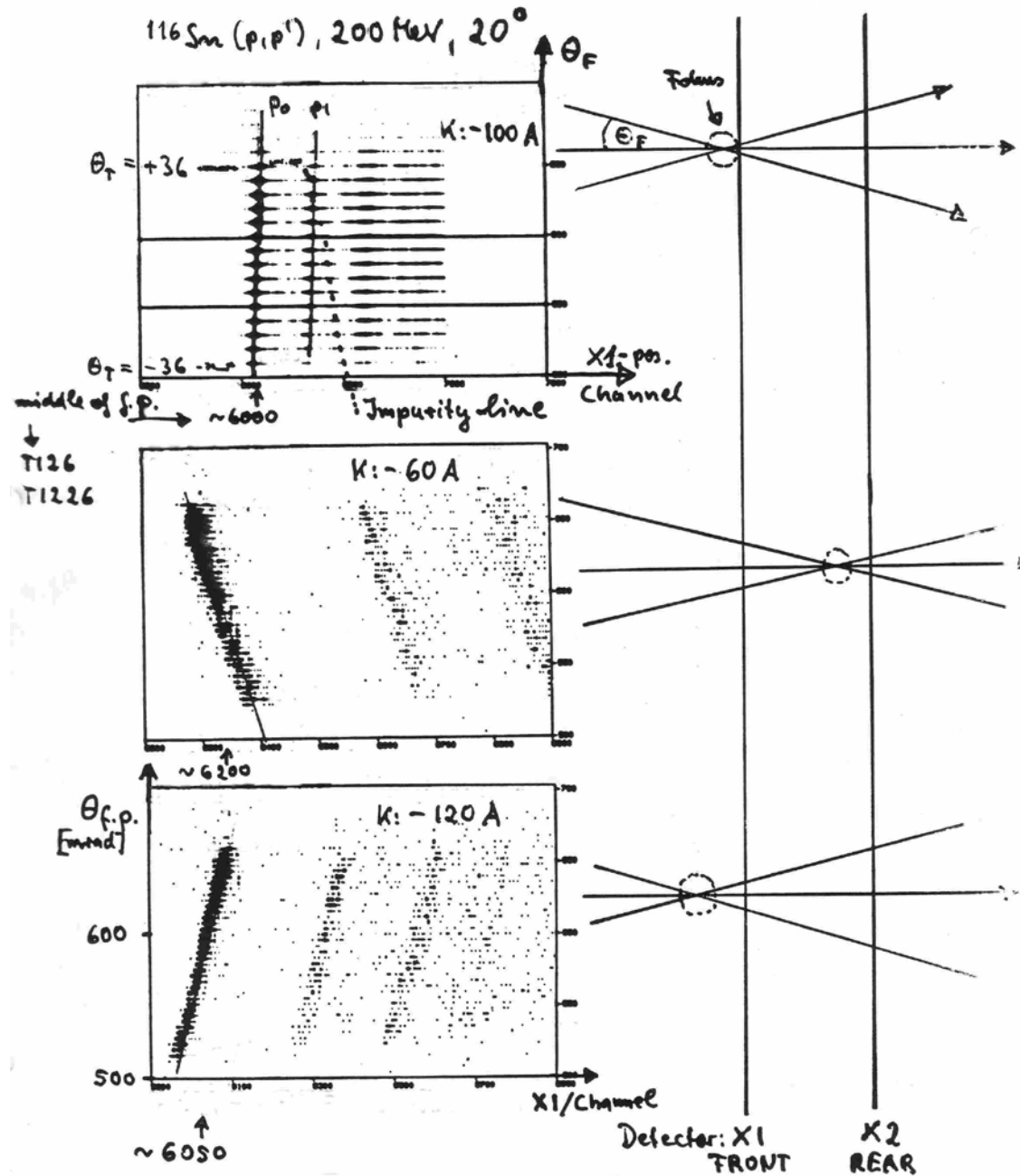
- 1) Why are the dots on the “Diagnostics in focal plane of Spectrometer” so evenly spaced?
- 2) What is the real world example of measuring emittance?
- 3) Describe the focal plane diagnostics of the spectrometer in more detail
- 4) I doesn't appear to have a dependence on anything. Can you clarify how $I(t)$ transforms? How exactly is I define?
- 5) Can you show a $(x|a) \neq 0$ case
- 6) Why is sigma matrix symmetric? What is definition of σ_{21} ? How do we create the 6x6 ellipse matrix?
- 7) How does the x-Theta measurement setup look like (measure orientation of ellipse)
- 8) Definition of Louisville Theorem
- 9) How do we define the central ray?
- 10) R_{11} , R_{12} , R_{21} , R_{22} - can more than one be zero?

Question 1)

Why are the dots on the “Diagnostics in focal plane of Spectrometer” so evenly spaced?

Answer: This is an artifact of poor binning, see next slide upper scatter plot Theta vs. x1 position. This was corrected in the lower scatter plot on the next slide

Focussing with triangular K-coil in Dipole D2



Diagnostics in focal plane of spectrometer

Typical in focal plane of Modern Spectrometers:

Two position sensitive Detectors:

Horizontal: X1, X2

Vertical: Y1, Y2

Fast plastic scintillators:

Particle identification

Time-of-Flight

Measurement with IUCF K600 Spectrometer illustrates from top to bottom: focus near, downstream and upstream of X1 detector, respectively

Question 2)

What is the real world example of measuring emittance?

Answer: A real world measurement would have been done in an existing device. Since we do not have such we use “calculated data” for the exercise.

Question 3)

Describe the focal plane diagnostics of the spectrometer in more detail

Answer: See next slide of K600 with focal plane detector system. The see slide 3 with x-Theta plot for detailed explanation.

K600 Spectrometer

Bending radius $\rho_0 = 2.0$ m

$B_{\max} = 1.7$ T

Gap = 5 cm (D1), 6cm (D2)

Weight = ~ 30 tons (D1)

~ 45 tons (D2)

Medium Dispersion: $B(D1) = B(D2)$

Resolving power: $p/\Delta p = 20000$

Dispersion = 12 cm/% (= 12 m)

Magnification $M_x = 0.41$

Large range: $E_{\min}/E_{\max} = 1.14$

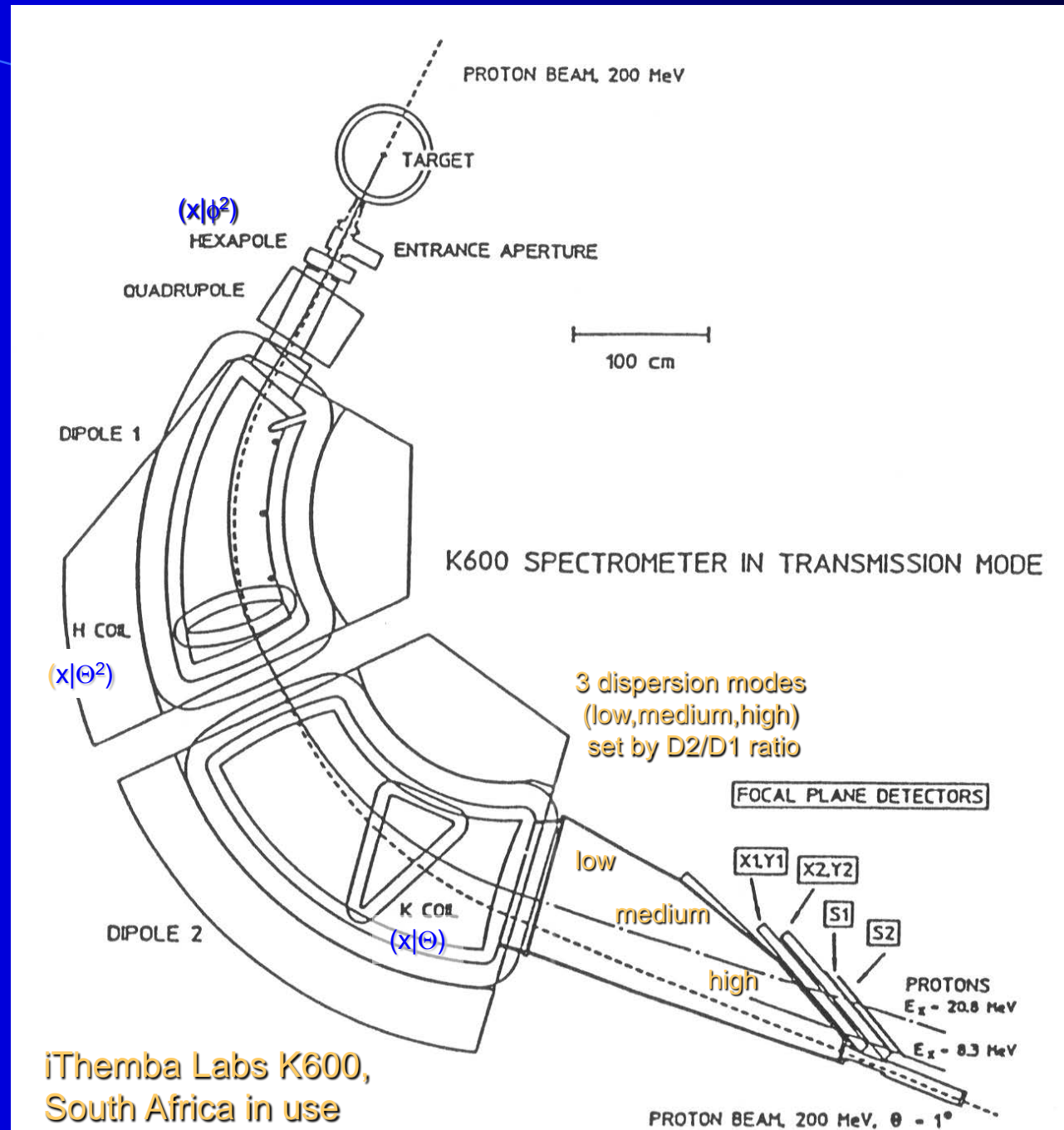
Kinematic correction: K coil

Hexapole correction: H coil

The K600 is shown in 0° Transmission mode

High Dispersion Plane
 $B(D1) > B(D2)$

IUCF K600, decommissioned
In 1999, now in WS line RCNP



iThemba Labs K600,
South Africa in use

Question 4)

I doesn't appear to have a dependence on anything. Can you clarify how $l(t)$ transforms? How exactly is I define?

Linear (1st order) TRANSPORT Matrix R_{nm}

$$\begin{array}{c}
 \begin{array}{l}
 \text{Magnification } M_x \\
 \downarrow \\
 \begin{array}{l}
 x(t) \\
 \theta(t) \\
 y(t) \\
 \varphi(t) \\
 l(t) \\
 \delta(t)
 \end{array} \\
 \xrightarrow{X_2}
 \end{array}
 =
 \begin{array}{c}
 \begin{array}{l}
 \text{Focusing fct} \\
 \downarrow \\
 \begin{array}{l}
 R_{11} \quad R_{12} \\
 R_{21} \quad R_{22} \\
 0 \quad 0 \\
 0 \quad 0 \\
 R_{51} \quad R_{52} \\
 0 \quad 0
 \end{array}
 \end{array}
 \begin{array}{l}
 0 \quad 0 \\
 0 \quad 0 \\
 R_{33} \quad R_{34} \\
 R_{43} \quad R_{44} \\
 0 \quad 0 \\
 0 \quad 0
 \end{array}
 \begin{array}{l}
 0 \\
 0 \\
 0 \\
 0 \\
 1 \\
 0
 \end{array}
 \begin{array}{l}
 \text{Lateral Dispersion} \\
 \downarrow \\
 \begin{array}{l}
 R_{16} \\
 R_{26} \\
 0 \\
 0 \\
 R_{56} \\
 1
 \end{array}
 \end{array}
 \begin{array}{l}
 x_0 \\
 \theta_0 \\
 y_0 \\
 \varphi_0 \\
 l_0 \\
 \delta_0
 \end{array} \\
 \xrightarrow{X_1}
 \end{array}
 \end{array}
 \end{array}$$

= TRANSPORT-R-Matrix

Angular Disp

Answer: $l(t) = R_{51} * x_0 + R_{52} * \theta_0 + l_0 + R_{56} * \delta_0$

Question 5)

Can you show a $(x|a) \neq 0$ case?

Answer: See slide 3

Question 6)

Why is sigma matrix symmetric? What is definition of sigma_21? How do we create the 6x6 ellipse matrix?

Answers: See this and next 2 slides

Ellipse in matrix form, see TRANSPORT MANUAL p. 169

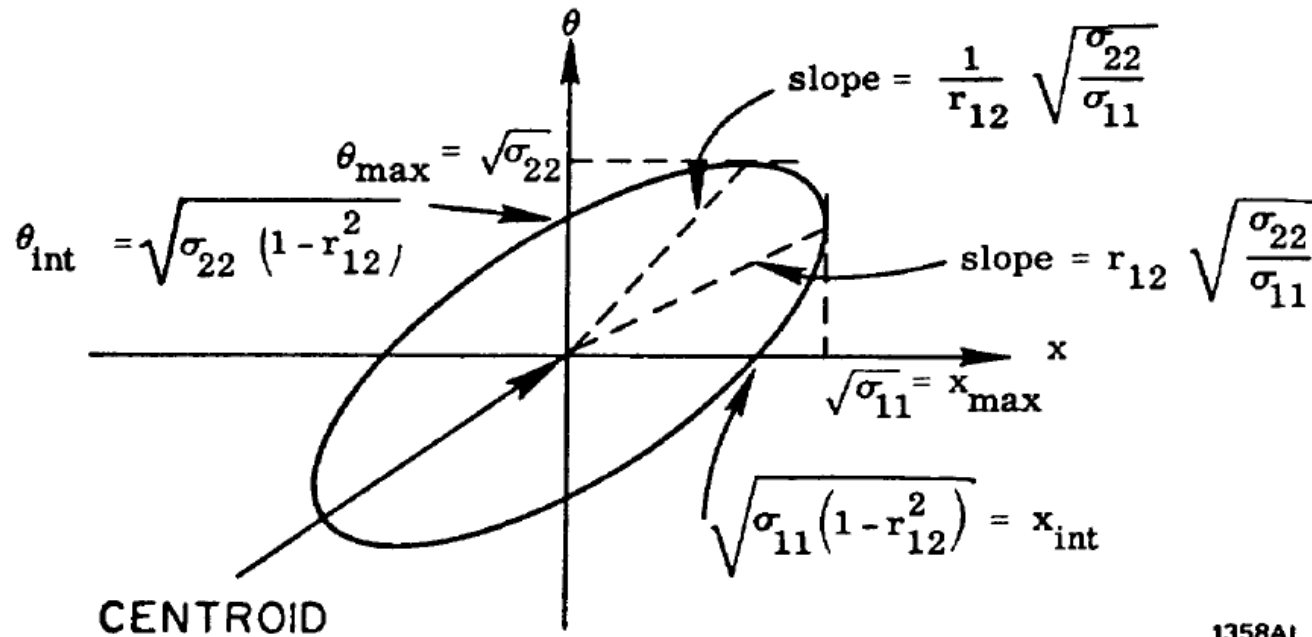
The equation of an n-dimensional ellipsoid may be written in matrix form as follows:

$$X(0)^T \sigma(0)^{-1} X(0) = 1 \quad (27)$$

where $X(0)^T$ is the transpose of the coordinate vector $X(0)$, and $\sigma(0)$ is a real, positive definite, symmetric matrix.

To make an ellipse the sigma matrix has to fulfill the following conditions

- a) Real
- b) Positive definite: $\det(\sigma) > 0$ (Note: Area of ellipse is $A = \text{Pi} * \epsilon$)
- c) Symmetric ($\sigma(n) = \sigma(m)$ for m not equal n)



$\sigma_{21} = \sigma_{12}$ represents the tilt of the ellipse and is in the ellipse drawing via r_{12}

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The correlation between x and θ (the orientation of the ellipse) depends upon the off-diagonal term σ_{21} . This correlation is defined as

$$r_{21} = r_{12} = \frac{\sigma_{21}}{\sqrt{\sigma_{11} \sigma_{22}}}$$

Symmetric sigma of 6 dimensional ellipsoid

and to allow for the possibility of more general systems, the sigma BEAM matrix used in a TRANSPORT calculation has the following general 6-dimensional construction.

	x	θ	y	φ	l	δ
x	$\sigma(11)$					
θ	$\sigma(21)$	$\sigma(22)$				
y	$\sigma(31)$	$\sigma(32)$	$\sigma(33)$			
φ	$\sigma(41)$	$\sigma(42)$	$\sigma(43)$	$\sigma(44)$		
l	$\sigma(51)$	$\sigma(52)$	$\sigma(53)$	$\sigma(54)$	$\sigma(55)$	
δ	$\sigma(61)$	$\sigma(62)$	$\sigma(63)$	$\sigma(64)$	$\sigma(65)$	$\sigma(66)$

Question 7)

How does the x-Theta measurement setup look like (measure orientation of ellipse)

Answer: See slide 6 of K600 with focal plane detector system. The see slide 3 with x-Theta plot for detailed explanation. Caveat. Beam intensity may be too high for detector system. Use attenuated (faint) beam.

Question 8)

Definition of Liouville Theorem

Answer: Liouville's Theorem states that "The density of states in an ensemble of many identical states with different initial conditions is constant along every trajectory in phase space".

Applied to our case it means that the emittance epsilon (area/volume of the ellipse) does not change under conservative forces. Conservative forces F are forces where the work ($\int F \cdot dl$) is independent of the path.

Question 9)

How do we define the central ray?

Answer: The Central Ray is a fictitious line through the magnetic center of quadrupoles. For passage through dipoles see drawing below, TRANSPORT manual p. 136

Consider two particles of momentum p_0 and $p_0 + \Delta p$ passing through the midplane of a static magnetic field, as illustrated in Figure 11.

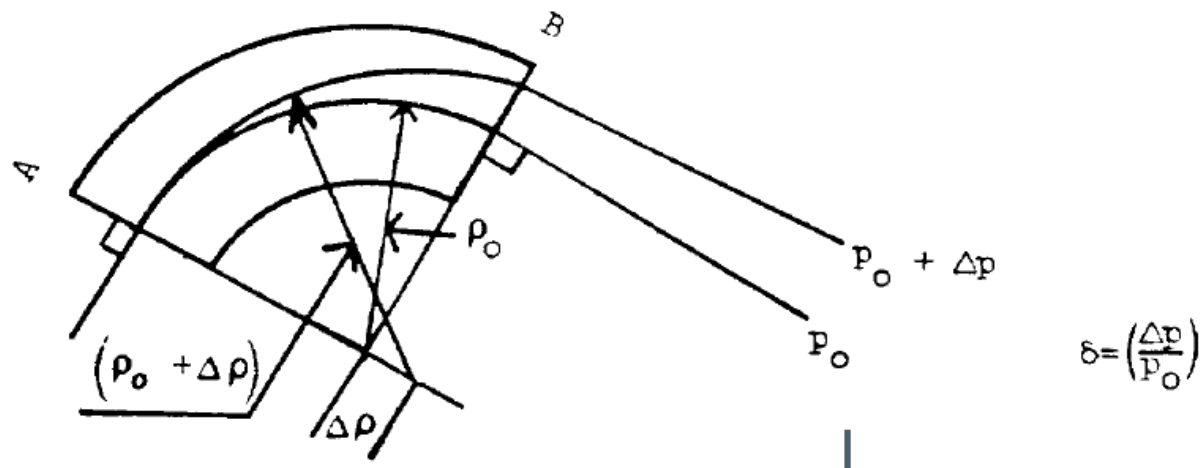


Figure 11

Question 10)

R11, R12, R21, R22 - can more than one be zero?

Answer: If $R_{12}=0$ it follows that $R_{11} = 1/R_{22}$. I do not think $R_{21} = 0$ has a physical meaning. But I do not have a proof. Note that this is the Transfer matrix, not the symmetric matrix that defines an ellipse. So R is not necessarily symmetric.