JINA–CEE

Separator Ion Optics School

NSCL, Michigan State University

Series of Four Lectures plus COSY Tutorials
September 10-14, 2018

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University of Notre Dame
JINA Center for the Evolution of the Elements

The Lecture Series

An Introduction to Ion-Optics

1st Lecture: 9/10/18: Formalism and ion-optical elements

2nd Lecture: 9/12/18: Ion-optical systems and spectrometers

3rd Lecture: 9/12/18: Recoil separators for nuclear astrophysics, St. GEORGE

4rd Lecture: 9/13/18: The recoil separator SECAR for FRIB

Hands-on sessions in the afternoon: 9/10/18 – 9/14/18: COSY Infinity

Review 1st Lecture
3 slides

Lorentz Force: $\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$ **(1)**

$\underbrace{q\vec{E}}_{\text{Electric force}}$, $\underbrace{q\vec{v} \times \vec{B}}_{\text{Magnetic force}}$

TRANSPORT of Ray X_0 $\quad X_n = R\, X_0$ **(3)**

using Matrix R $\quad R = R_n\, R_{n-1} \ldots R_0$ **(4)**

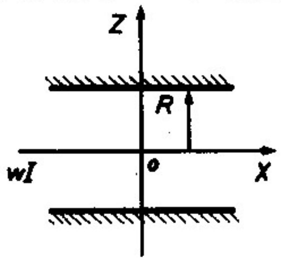
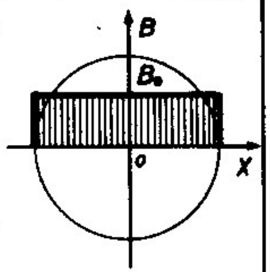
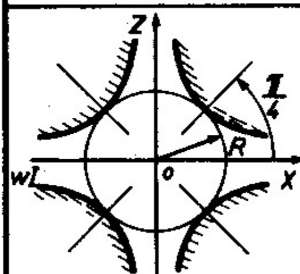
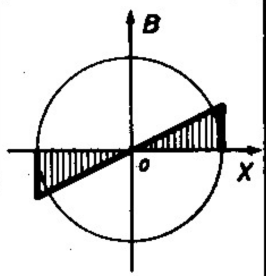
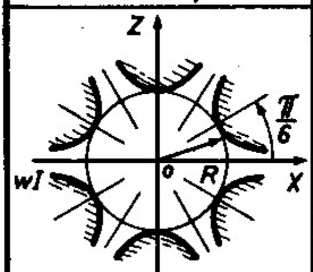
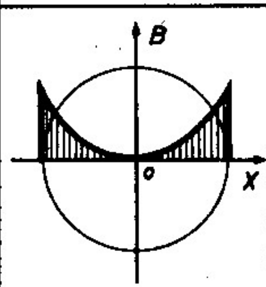
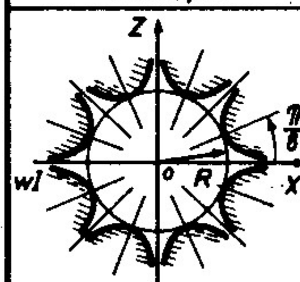
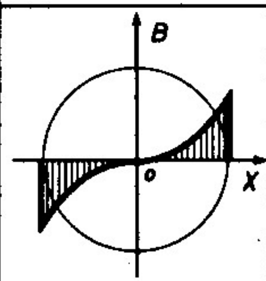
TRANSPORT of σ Matrix (Phase space ellipsoid) $\quad \sigma_1 = R\sigma_0 R^T$ **(10)**

Beam emittance: $\quad \varepsilon = \sqrt{\sigma_{11}\sigma_{22} - (\sigma_{12})^2}$ **(5)**

Taylor expansion, higher orders, solving the equation of motion, some applications

Ion-optical elements, dipole, quadrupole magnets

Schematic Overview of Magnetic Elements (Iron dominated)

Pole shape	Field	Pole,analyt.	B_x	wI
 Dipol		$z = \pm R$	$a_1 = B_0$	$\frac{2}{\mu_0} B_0 R$
 Quadrupol		$xz = \pm \frac{R^2}{2}$ Power	$a_2 x = g x$ ~ I^2 ~	$\frac{1}{\mu_0} g R^2$ R^4
 Sextupol		$z(x^2 - \frac{z^2}{3}) = \pm \frac{R^3}{3}$	$a_3(x^2 - z^2)$	$\frac{2}{3\mu_0} a_3 R^3$
 Oktupol		$xz(x^2 - z^2) = \pm \frac{R^4}{4}$	$a_4 x(x^2 - 3z^2)$	$\frac{1}{2\mu_0} a_4 R^4$

Iron dominated:

B field is determined by properties & shape of iron pole pieces

Required wI = Ampere-turns for desired magnet strength B_0, g, a_3, a_4 can be calculated formula in last column.

Coils are not shown in drawing in 1st column

G. Schnell, Magnete, Verlag K. Thiemig, Muenchen 1973

Forces on ions (quadrupole)

Quadrupole Hexapole Octopole

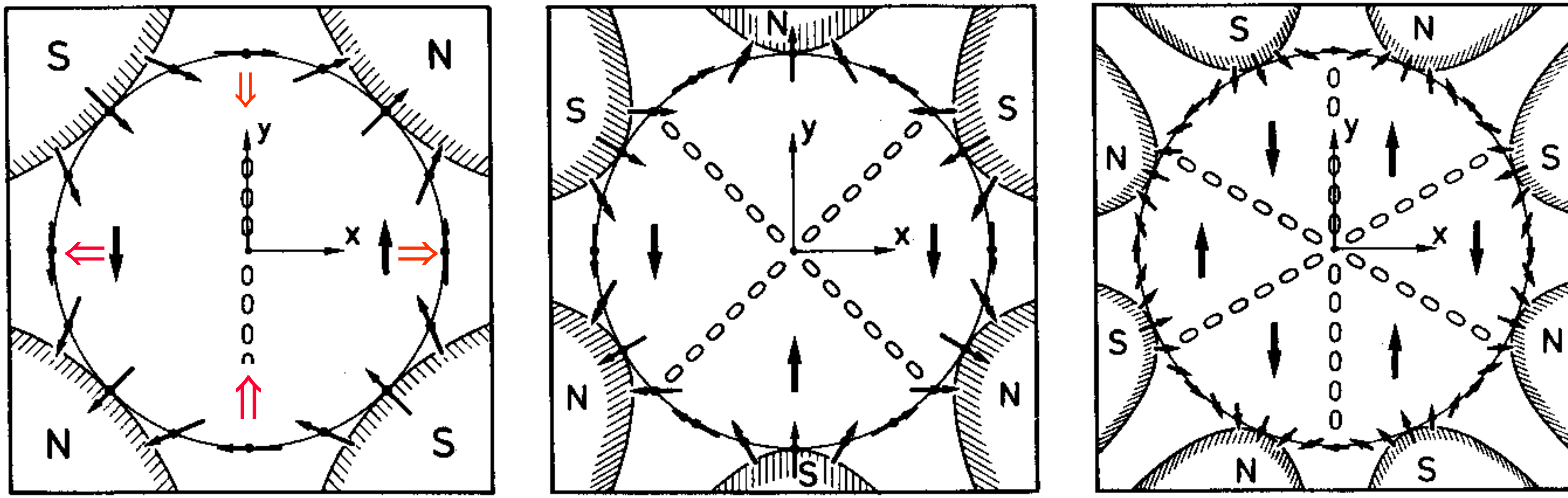


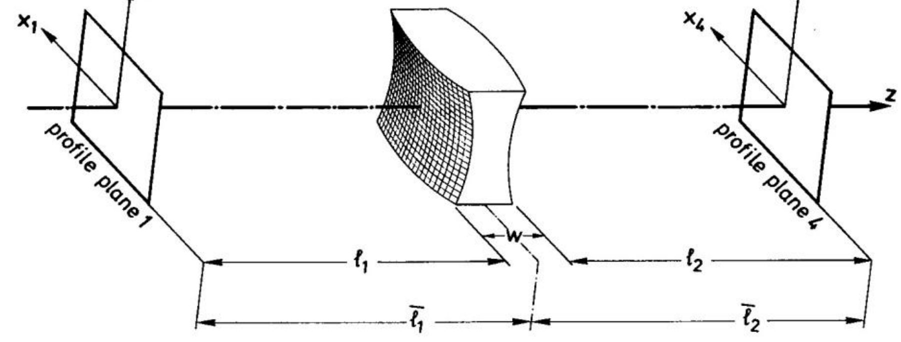
Fig. 9.15. Pole arrangements of magnetic quadrupoles, hexapoles, and octopoles are indicated. Also shown is a circle of radius r_0 along which the magnetic flux density is constant, and its direction varies as indicated. Finally, strings of zeros indicate lines along which B_y the y component of the magnetic flux density vanishes. These lines separate regions in which B_y is parallel or antiparallel to the y axis.

Horizontally defocusing quadrupole for ions along – z axis into the drawing plane. See Forces ⇑⇐⇓⇒ in direction v x B

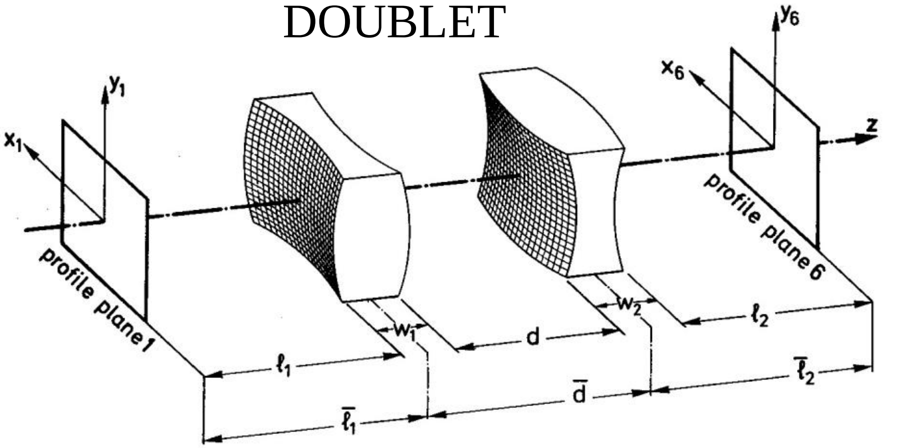
A focusing quadrupole is obtained by a 90º rotation around the z axis

Ion optics of a quadrupole SINGLET & DOUBLET

SINGLET



DOUBLET



POINT TO POINT FOCUS WITH DOUBLET

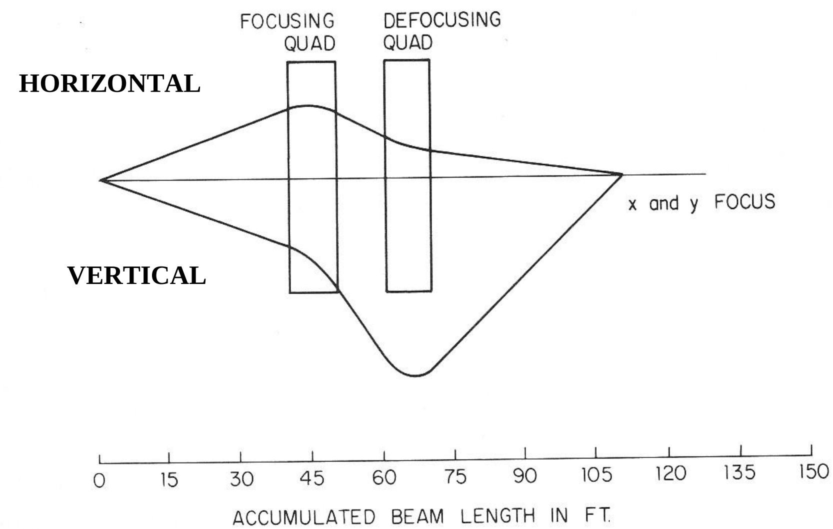
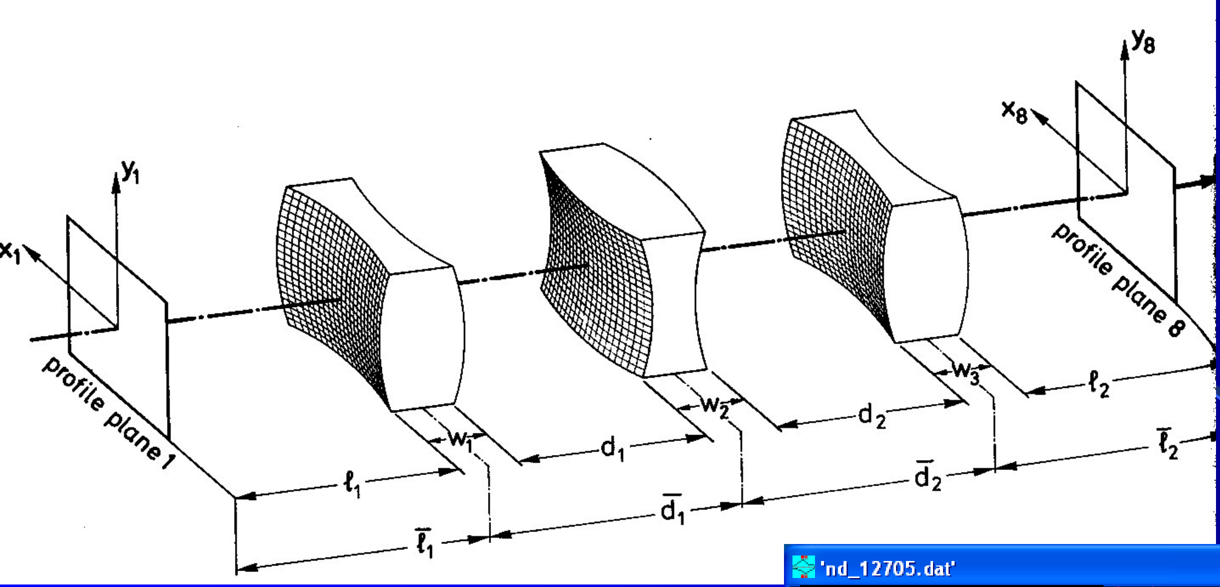


Figure 1.9 Point-to-point focusing with a quadrupole doublet. The two trajectories shown are in the horizontal and vertical planes respectively.

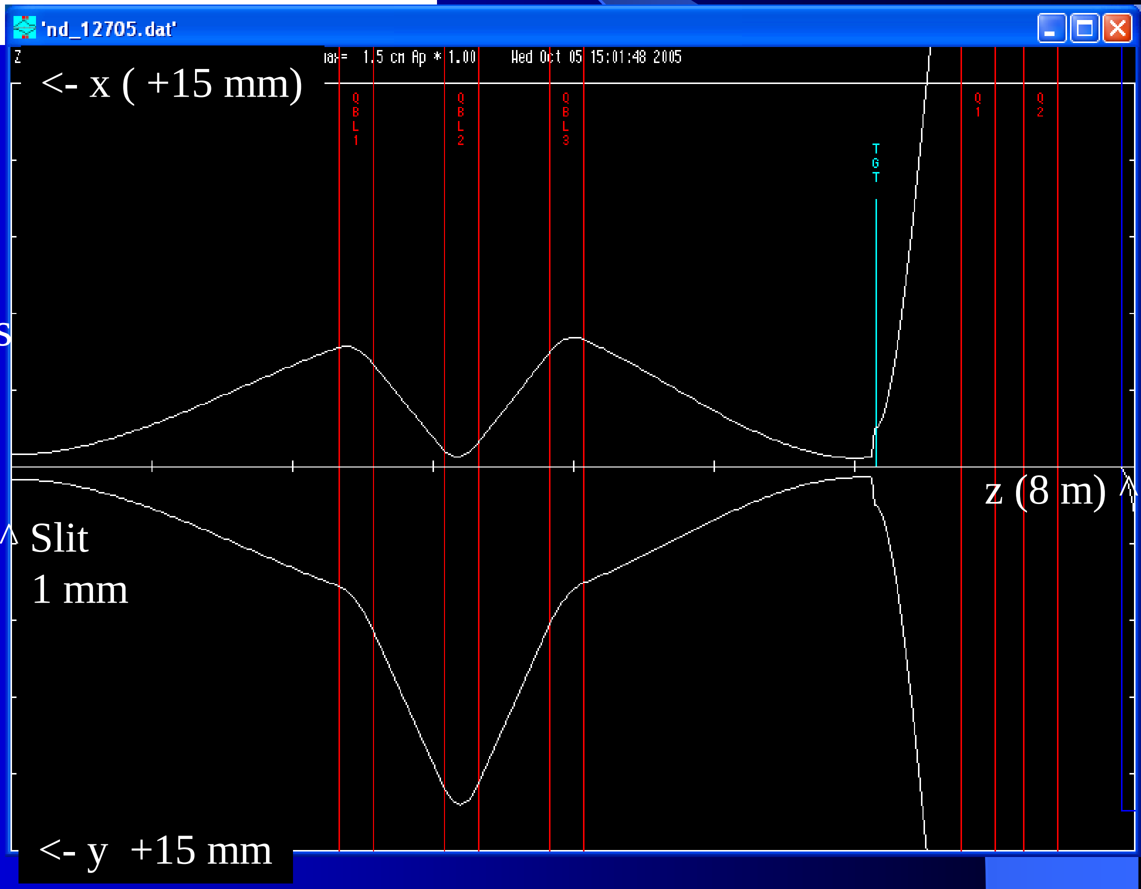
Focusing with a quadrupole TRIPLET



Screen shot of TRANSPORT design calculation of **Quadrupole Triplet** upstream of St. George target. Shown are the horiz. (x) and vert. (y) envelopes of the phase ellipse. Design good-field-Region for full transmission

Note beam at Slit has +/- 2 mrad and at target TGT +/- 45 mrad angle Opening, acceptance of St. GEORGE.

This symmetical triplet 1/2F-D-1/2F corresponds to an optical focusing lens.



K600 Spectrometer

Bending radius ρ_0 = 2.0 m
B_{max} = 1.7 T
Gap = 5 cm (D 1), 6cm (D2)
Weight = ~ 30 tons (D1)
~ 45 tons (D2)

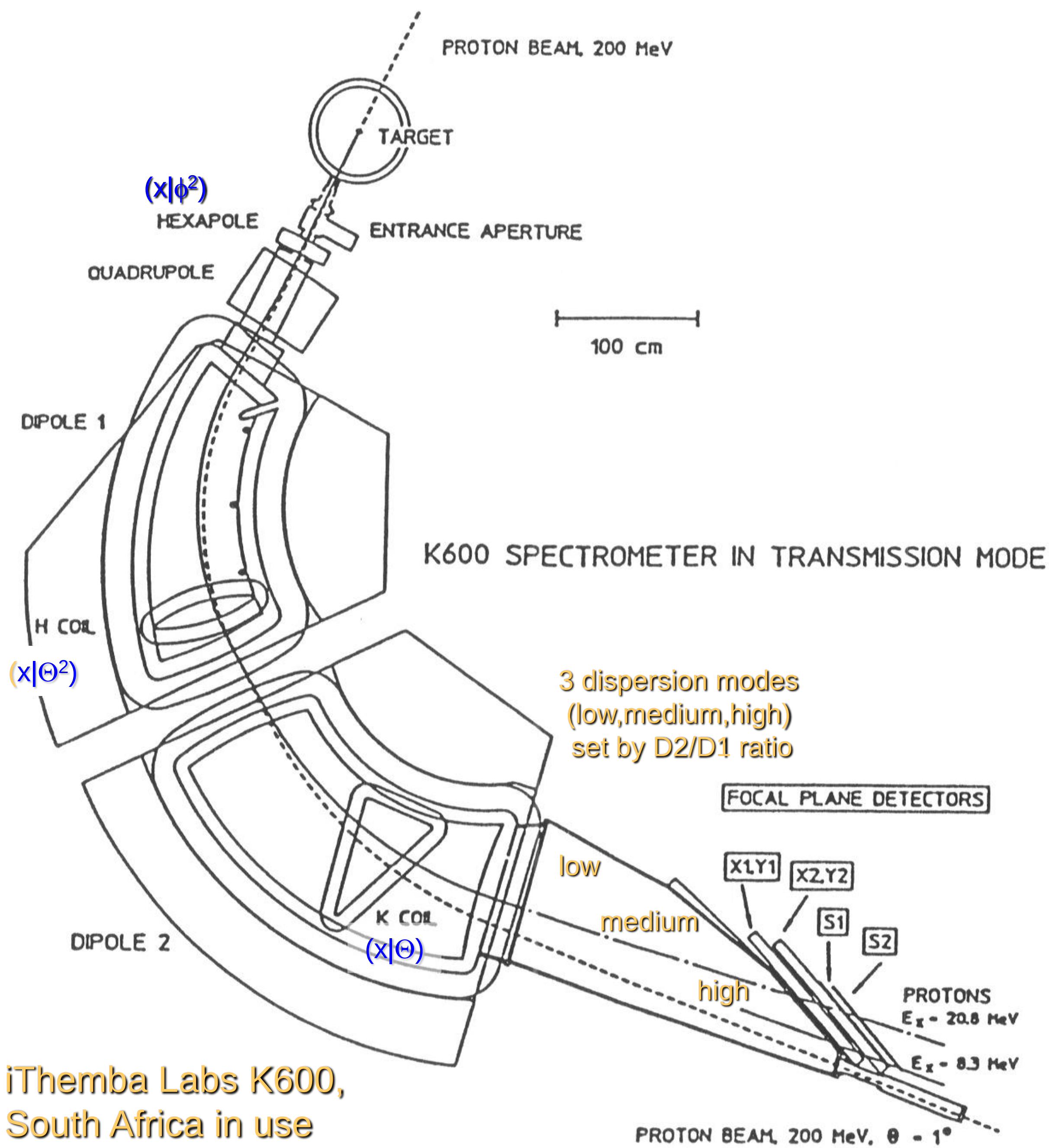
Medium Dispersion: B(D1)= B(D2)
Resolving power: $p/\Delta p$ = 20000
Dispersion = 12 cm/% (= 12 m)
Magnification M_x = 0.41
Large range: E_{min}/E_{max} = 1.14

Kinematic correction: K coil
Hexapole correction: H coil

The K600 is shown in
0^o Transmission mode

High Dispersion Plane
B(D1) > B(D2)

IUCF K600,decommissioned
In 1999, now in WS line RCNP



iThemba Labs K600,
South Africa in use

BIG KARL Spectrometer (Juelich, KFZ)

Bending radius ρ_0 = 1.98 m
B_{max} = 1.7 T
Gap = 6cm
Weight = ~ 50 tons (D1)
~ 70 tons (D2)

Resolv. power: p/Δp = 0 - 20600
Dispersion = -2.0 to 26 cm/%
Magnification M_x = 0.63 – 1.26
Magnification M_y = 25.4 – 1.94
Large range: E_{min} /E_{max} = 1.14
Solid angle: < 12.5 msr

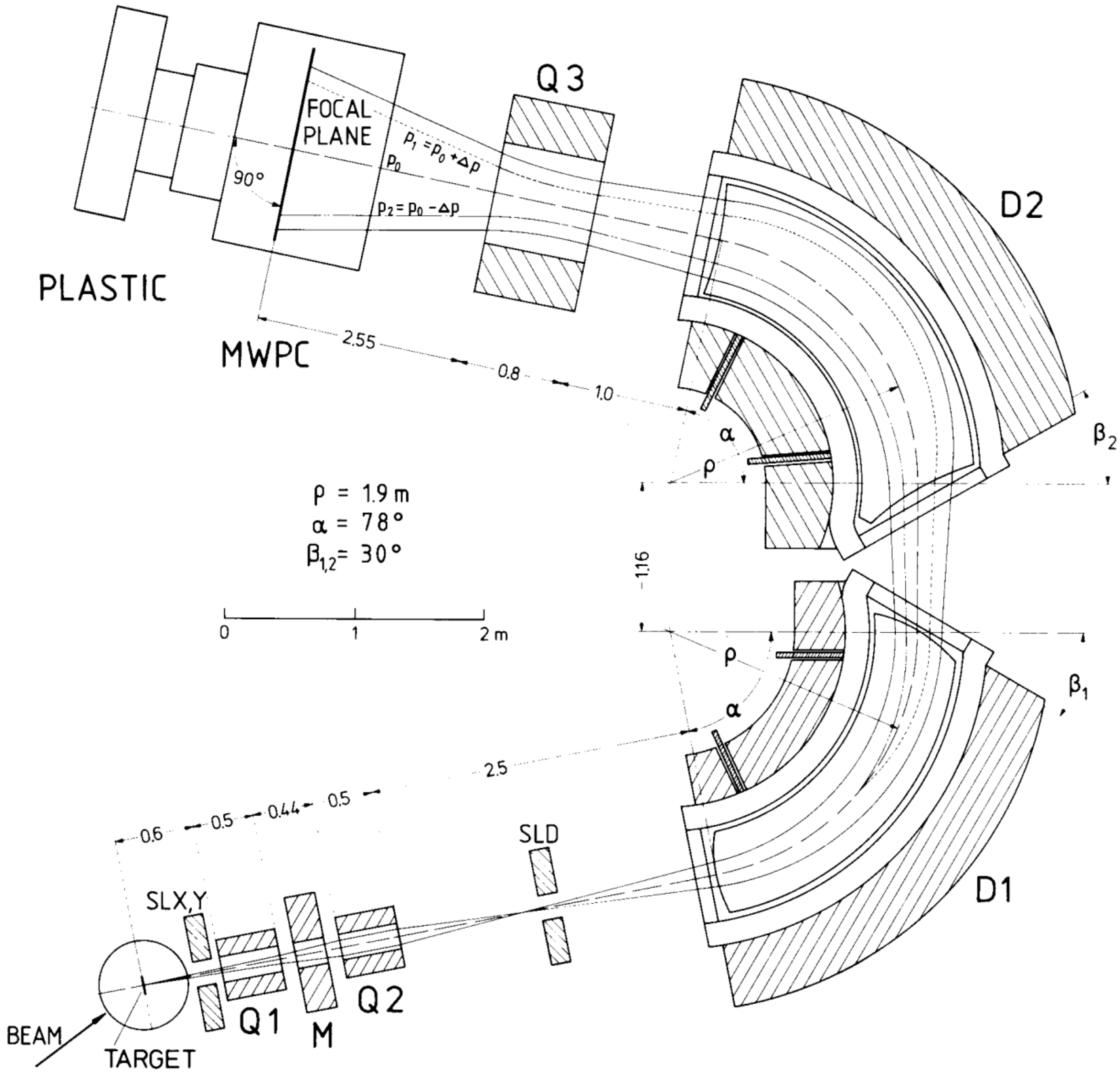


Fig. 9. Arrangement of the magnetic elements of the QQDDQ spectrometer BIG KARL. The central ray (optical axis) is shown as dashed curve. The outermost rays with the extreme radial distances are drawn as full lines. Four channels in the inner yokes allow NMR probes to be moved into the gaps of the dipoles for radial field measurements. The multipole element between Q1 and Q2 allows the correlation of vertical aberration.

BIG KARL Sample Spectra

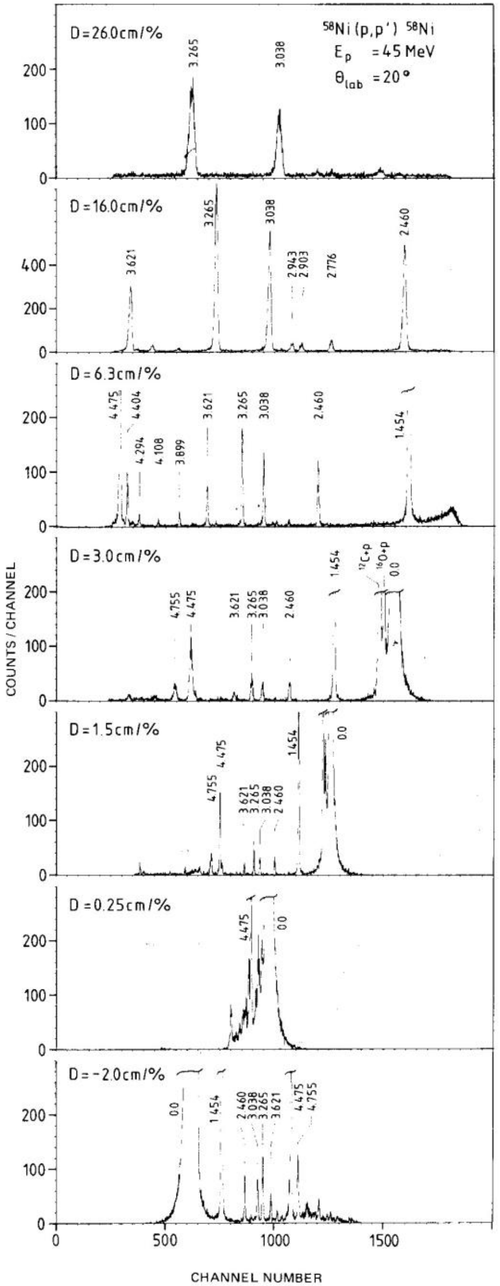


Fig. 4. Spectra of ^{58}Ni(p, p′) measured for different dispersions $D = 26$, 16, 6.3, 3, 1.5, ~ 0.25 and −2 cm/%. The spectrograph was optimized for $D = 16$ cm/%.

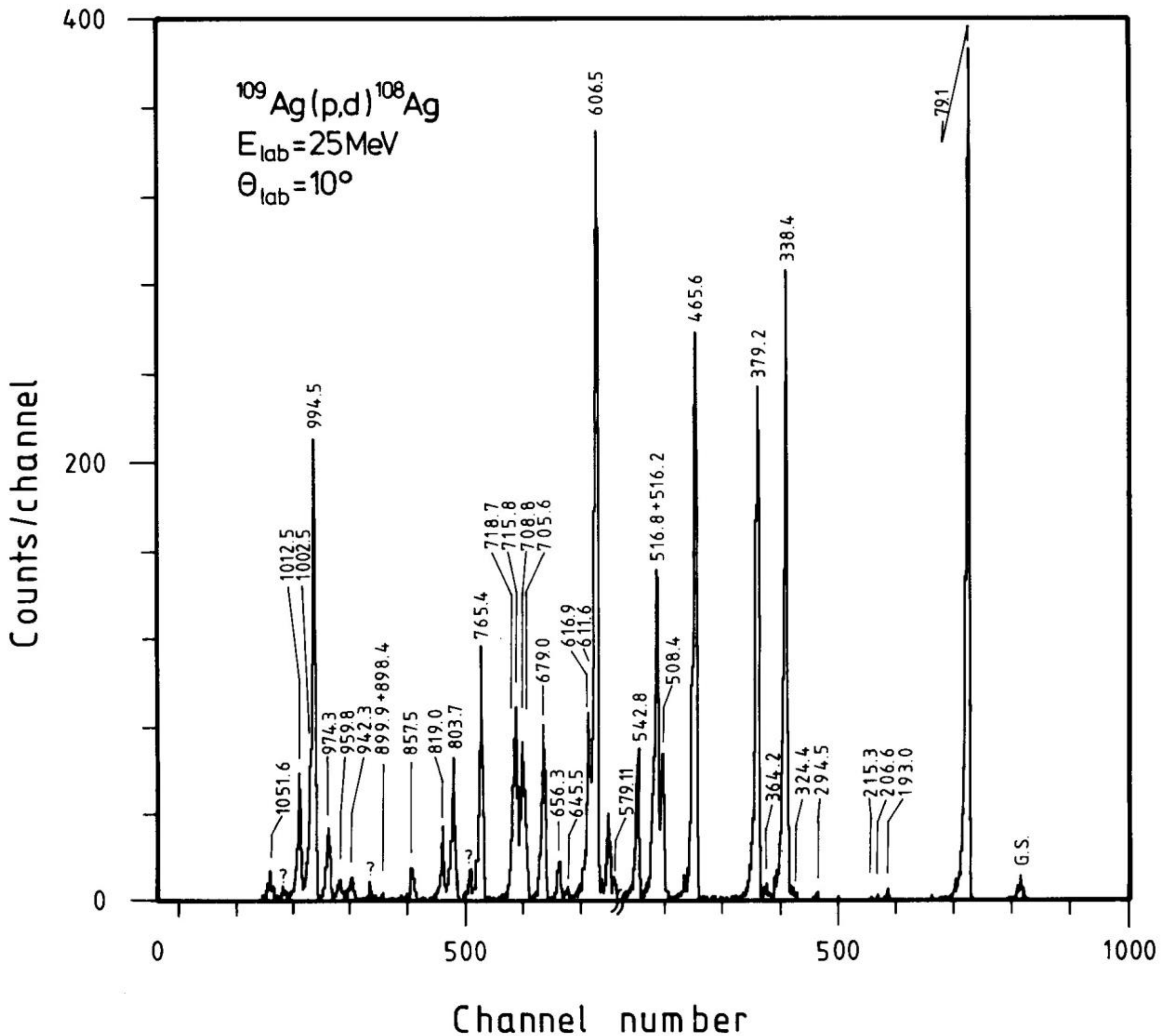
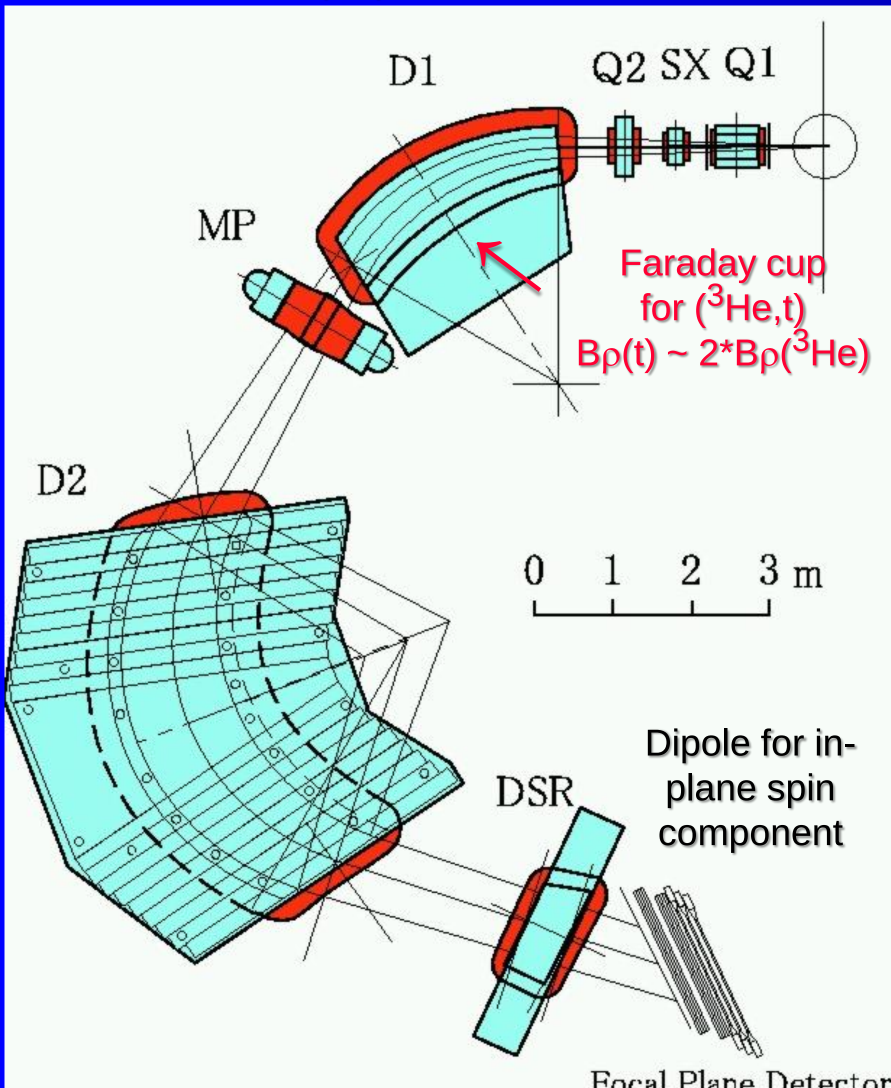


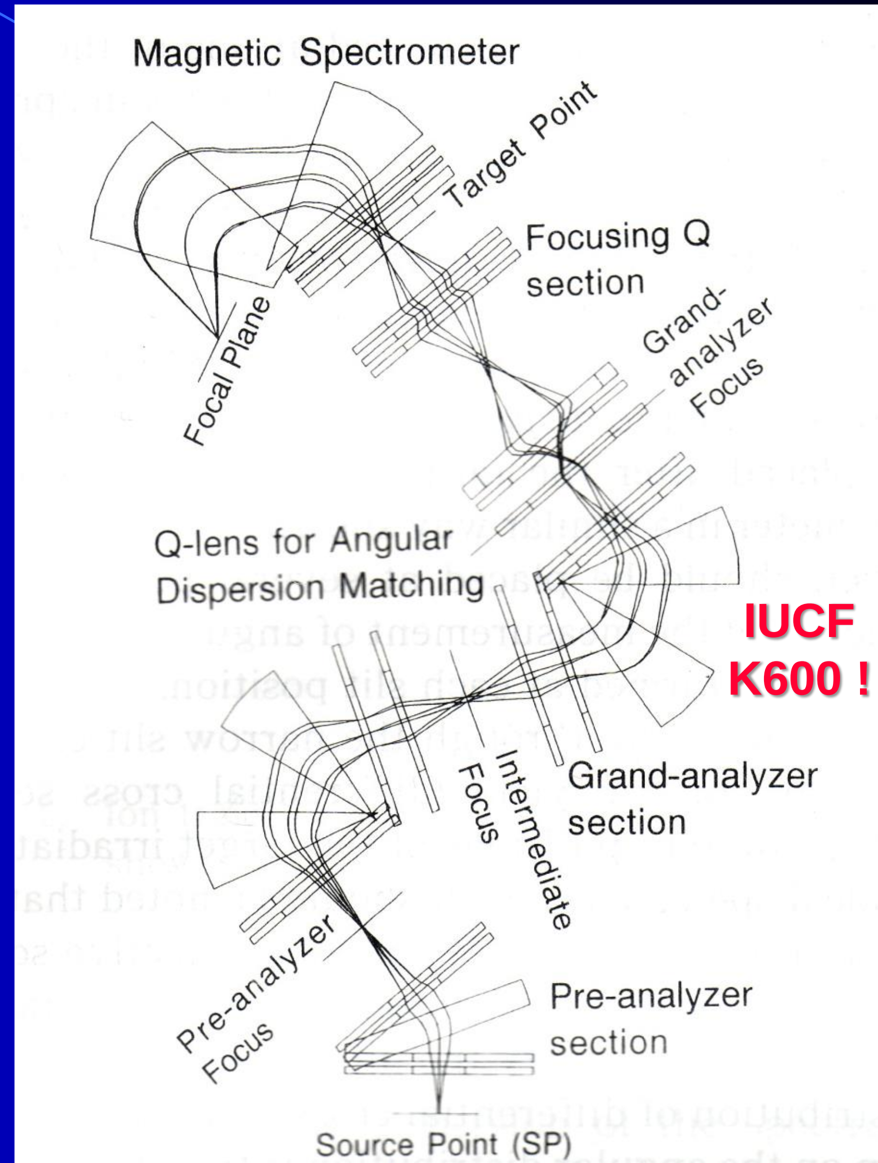
Fig. 19. High resolution spectrum of the (p, d) neutron pick up reaction on ^{109}Ag at 25 MeV incident energy and a solid angle of 1.2 msr. The resolution was 4 keV.

Grand Raiden High Resolution Spectrometer

Max. Magn. Rigidity: 5.1 Tm
Bending Radius ρ_0: 3.0 m
Solid Angle: 3 msr
Resolv. Power p/dp 37000

Beam Line/Spectrometer fully matched





IUCF K600 !

Spacial and Angular Dispersion Matching

Resolution is limited by Momentum of beam

Angular dispersion matched

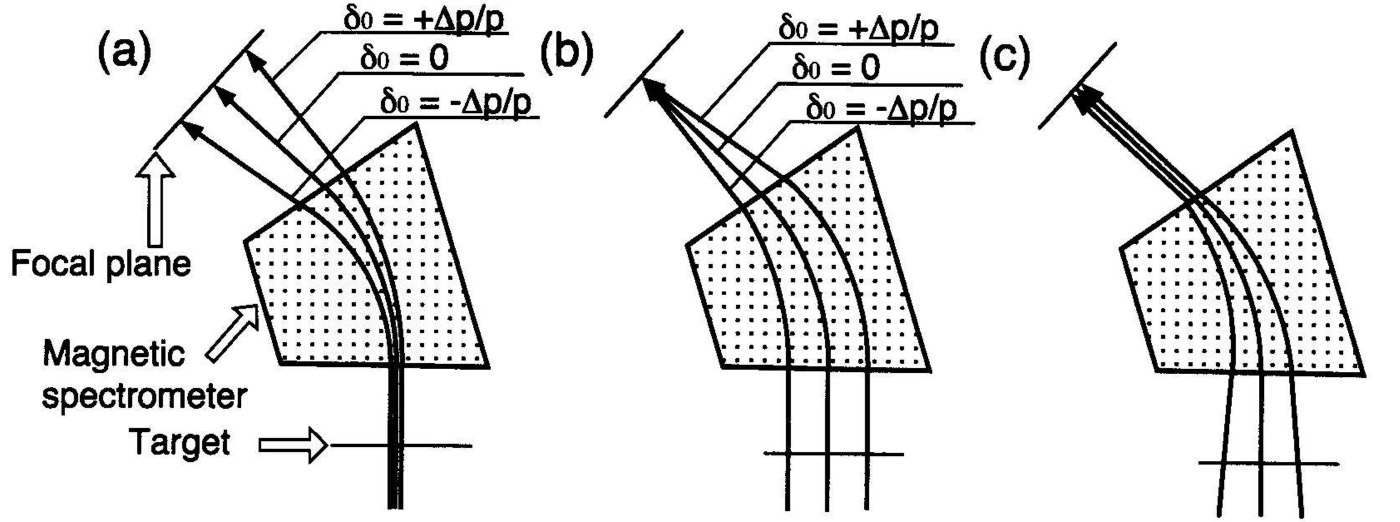


Figure 2.2: Schematic ion trajectories under different matching conditions of a beam line

Angular ambiguity

Spectrometer Transfer Matrix S

Spectrometer Design

(1^{st} Order Resolving Power)

Dispersion: $S_{16} = dx/(dp/p) \quad = D$

Magnification: $S_{11} = dx(f.p.) / dx(tgt) \quad = M$

Beam size: $2x_0$ (target, dispersive direction, monochromatic)

Resolving Power: $R_p = \dfrac{p}{\Delta p} = \dfrac{D}{M * 2x_0}$ **(16)**

Note: R_p depends on x_0, if not given here $x_0 = 1$ mm

Note: **Resolving Power** is the "best possible 1^{st} order resolution a spectrometer can provide, disregarding higher order aberrations.

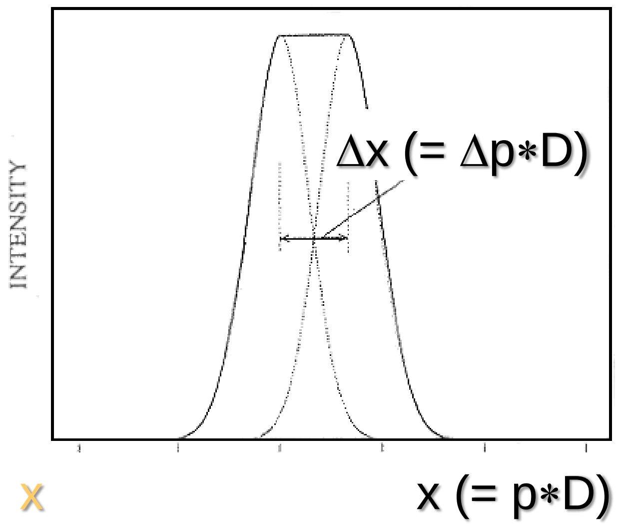
Resolution is what is measured in the Focal Plane.

Resolution is also affected (deteriorated) by:

Spectrometer aberrations, beam properties, target effects, detector resolution

Note: **"Resolution"** in Energy $R_E = \dfrac{E}{\Delta E} = 0.5 * R_p$

because $E = p^2/m$ (non-relativistic)



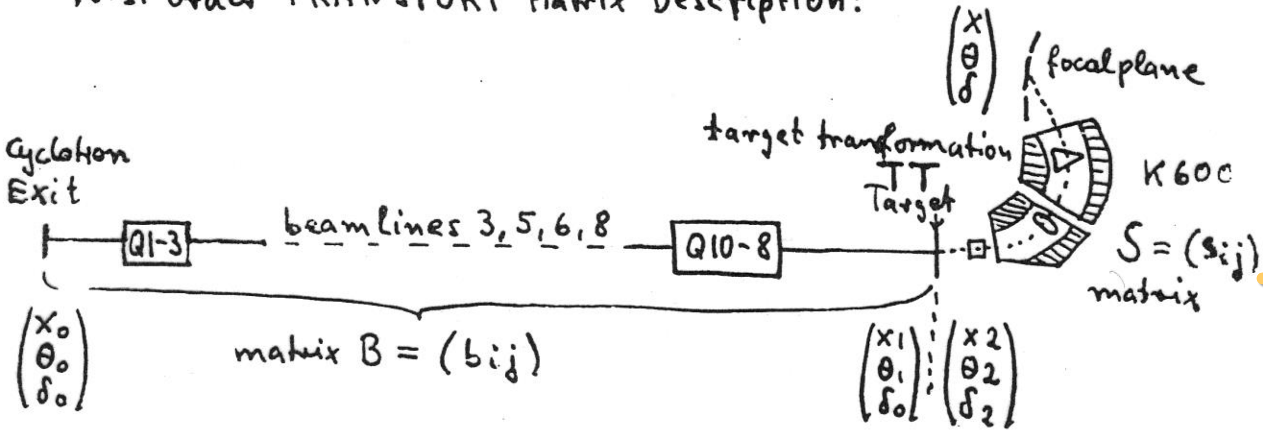
→ Peaks are "resolved" when Δx = FWHM

Dispersion Matching

- High resolution experiments
- Secondary beam (large dp/p)

Matching between beam line and spectrometer

First Order TRANSPORT Matrix Description:



$$B = \begin{pmatrix} b_{11} & b_{12} & b_{16} \\ b_{21} & b_{22} & b_{26} \\ 0 & 0 & 1 \end{pmatrix}$$

$$S = \begin{pmatrix} s_{11} & s_{12} & s_{16} \\ s_{21} & s_{22} & s_{26} \\ 0 & 0 & 1 \end{pmatrix}$$

TT:

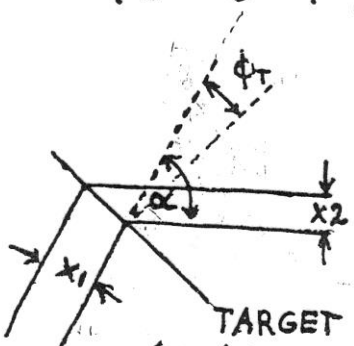
$$x_2 = T\,x_1$$

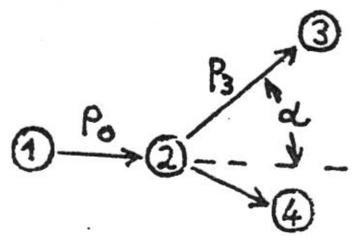
$\theta_2 = \Theta + \theta_1$

$\theta_2 =$ ~~random~~ angle within acceptance of spectrom.

$$\delta_2 = K\,\underbrace{(\theta_2 - \theta_1)}_{\Theta \text{ is random}} + C\,\delta_0$$

Reaction Kinematics





$$P_3 = P_3(P_0, \alpha, Q)$$

$$T = \frac{\cos(\alpha - \phi_T)}{\cos\phi_T}$$

$$K = \frac{\partial P_3}{\partial \alpha}\,\frac{1}{P_3} \qquad C = \frac{\partial P_3}{\partial P_0}\,\frac{P_0}{P_3}$$

$$\begin{pmatrix} x \\ \theta \\ \delta \end{pmatrix} = S \cdot TT \cdot B \cdot \begin{pmatrix} x_0 \\ \theta_0 \\ \delta_0 \end{pmatrix}$$

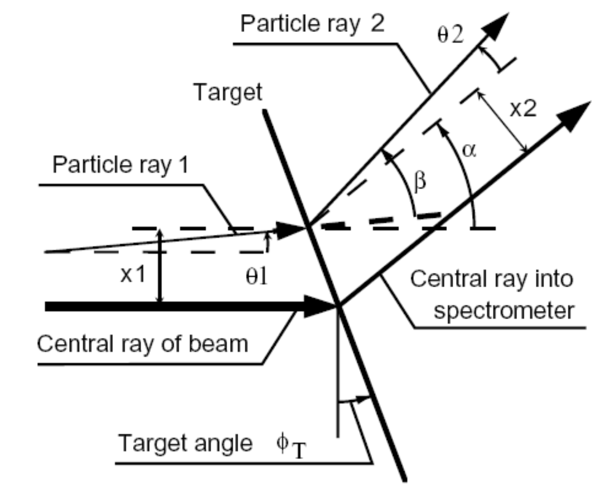


Fig. 1. Schematic layout of the incident particle 1 and the outgoing particle 2 relative to the beam and spectrometer.

Solution of first order Transport and Complete Matching

The transformation (without assuming $s_{12} = - s_{16} K$) in the bending plane from the cyclotron exit to the focal plane is given as:

$$
\begin{aligned}
x_{f.p.} = \; & x_0 \, (s_{11} b_{11} T + s_{12} b_{21}) \\
& \theta_0 \, (s_{11} b_{12} T + s_{12} b_{22}) \rightarrow \text{kin. defoc.} \quad \text{equ. (1)} \quad \textbf{(17)} \\
& \delta_0 \, (s_{11} b_{16} T + s_{12} b_{26} + s_{16} C) \rightarrow \text{disp. matching} \\
& \Theta \, (s_{12} + s_{16} K) \rightarrow \text{kin. correction (kin. displac.)}
\end{aligned}
$$

$$
\begin{aligned}
\theta_{f.p.} = \; & x_0 \, (s_{21} b_{11} T + s_{22} b_{21}) \\
& \theta_0 \, (s_{21} b_{12} T + s_{22} b_{22}) \quad \text{equ. (2)} \quad \textbf{(18)} \\
& \delta_0 \, (s_{21} b_{16} T + s_{22} b_{26} + s_{26} C) \rightarrow \text{angular disp. matching} \\
& \Theta \, (s_{22} + s_{26} K)
\end{aligned}
$$

$$\delta_{f.p.} = K \cdot \Theta + C \, \delta_0$$

For details see: Y. Fujita et al., NIM B 126 (1997) 274

Complete Matching

For best **Resolution** in the focal plane, minimize the coefficients of all terms in the expression of **x** **f.p.**

For best **Angle Resolution** Minimize Coefficients of δ_0 in expression of Y f.p.

Note: Also the beam focus **b12** on target is important (**b12** = 0 for kinem. k = 0)

Spacial Dispersion Matching:
D.L. Hendrie In: J. Cerny, Editor, *Nuclear Spectroscopy and Reactions, Part A*, Academic Press, New York (1974), p. 365.

(17')

Hendrie, Dispersion Matching $b_{16} = - \frac{D}{M} * \frac{C}{T}$

D = s_{16} = Spectrometer dispersion

M = s_{11} = Spectrometer magnification

Spacial and Angular Dispersion Matching

Solutions for b_{16} and b_{26} under conditions that both δ_0-coefficients = 0 in **(17)** and **(18)**

$$s_{11}\, b_{16}\, T + s_{12}\, b_{26} + s_{16}\, C \;=\; 0$$

$$s_{21}\, b_{16}\, T + s_{22}\, b_{26} + s_{26}\, C \;=\; 0$$

Solutions:

$$b_{16} \;=\; -\frac{s_{16}}{s_{11}}(1 + s_{11}\, s_{26}\, K \;-\; s_{21}\, s_{16} K)\;\frac{C}{T} \qquad \textbf{(19)}$$

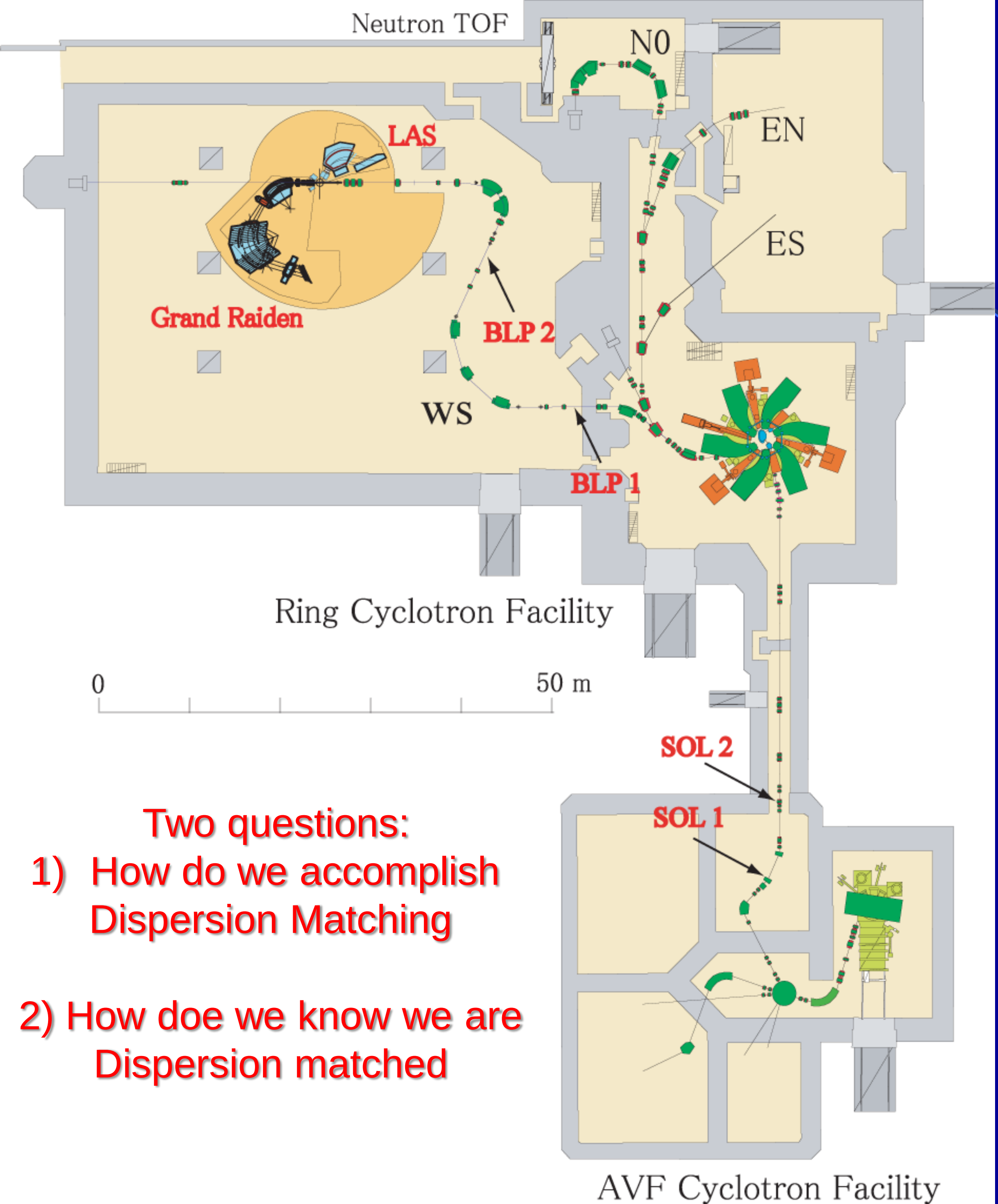
Spacial Dispersion Matching

$$b_{26} \;=\; (s_{21}\, s_{16} \;-\; s_{11}\, s_{26})\; C \qquad \textbf{(20)}$$

Angular Dispersion Matching

$$b_{12} \;=\; -\,\frac{s_{12}\, b_{22}}{s_{11}\, T} \;=\; \frac{s_{16}\, b_{22}\, K}{s_{11}\, T} \qquad \textbf{(21)}$$

Focusing Condition



RCNP Facility Layout Osaka, Japan

$D = S_{16} = 17$ cm/% = 17 m

$M = S_{11} \sim -0.45$

Dispersion on target:
$B_{16} = D/M = -37$ m

Resolving power:
$2x_0 = 1$ mm
$R = p/\Delta p = 37000$

Two questions:

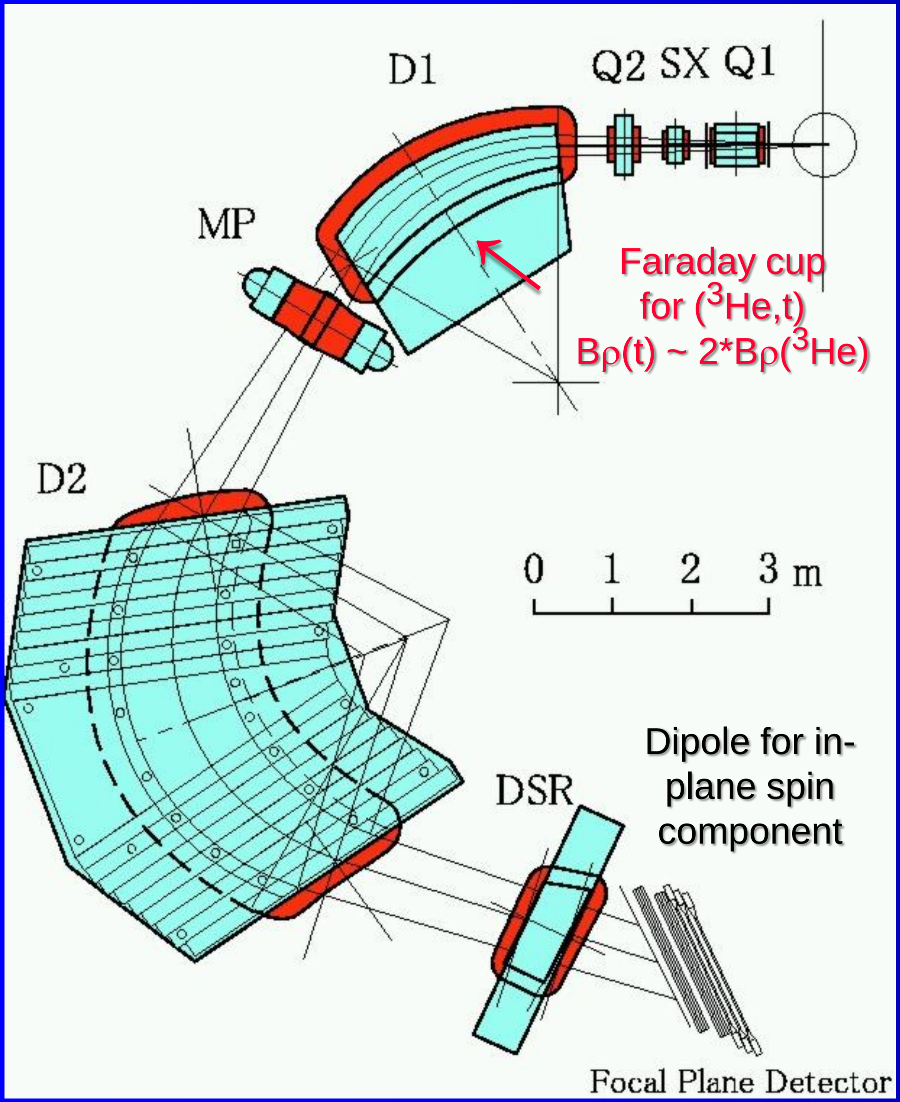
1) How do we accomplish Dispersion Matching

2) How doe we know we are Dispersion matched

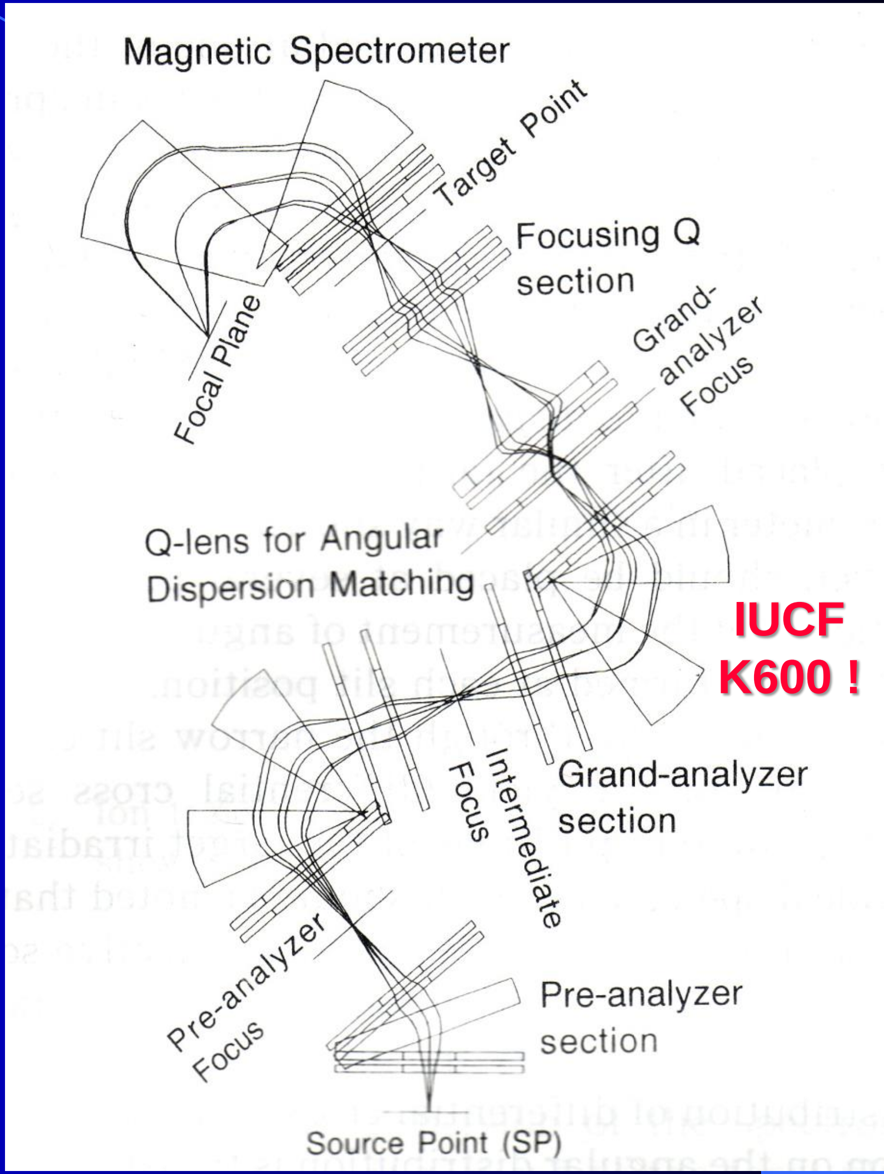
Dispersion matched beam line WS to the high resolution spectrometer Grand Raiden

Grand Raiden High Resolution Spectrometer

Max. Magn. Rigidity:	5.1 Tm
Bending Radius ρ_0:	3.0 m
Solid Angle:	3 msr
Resolv. Power p/dp	37000

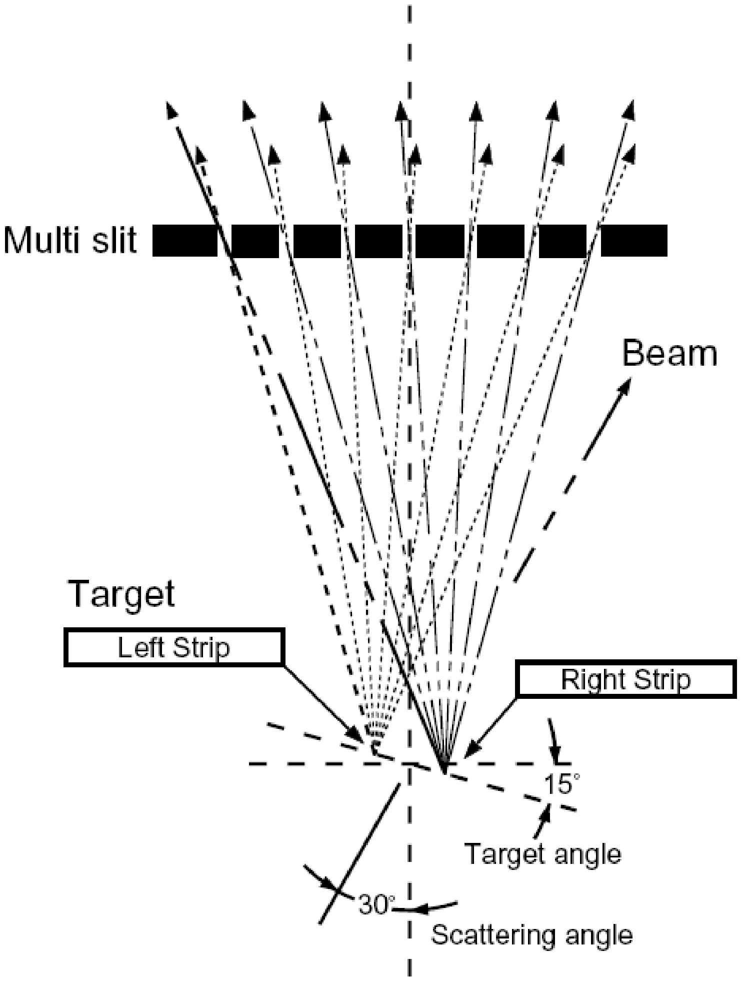


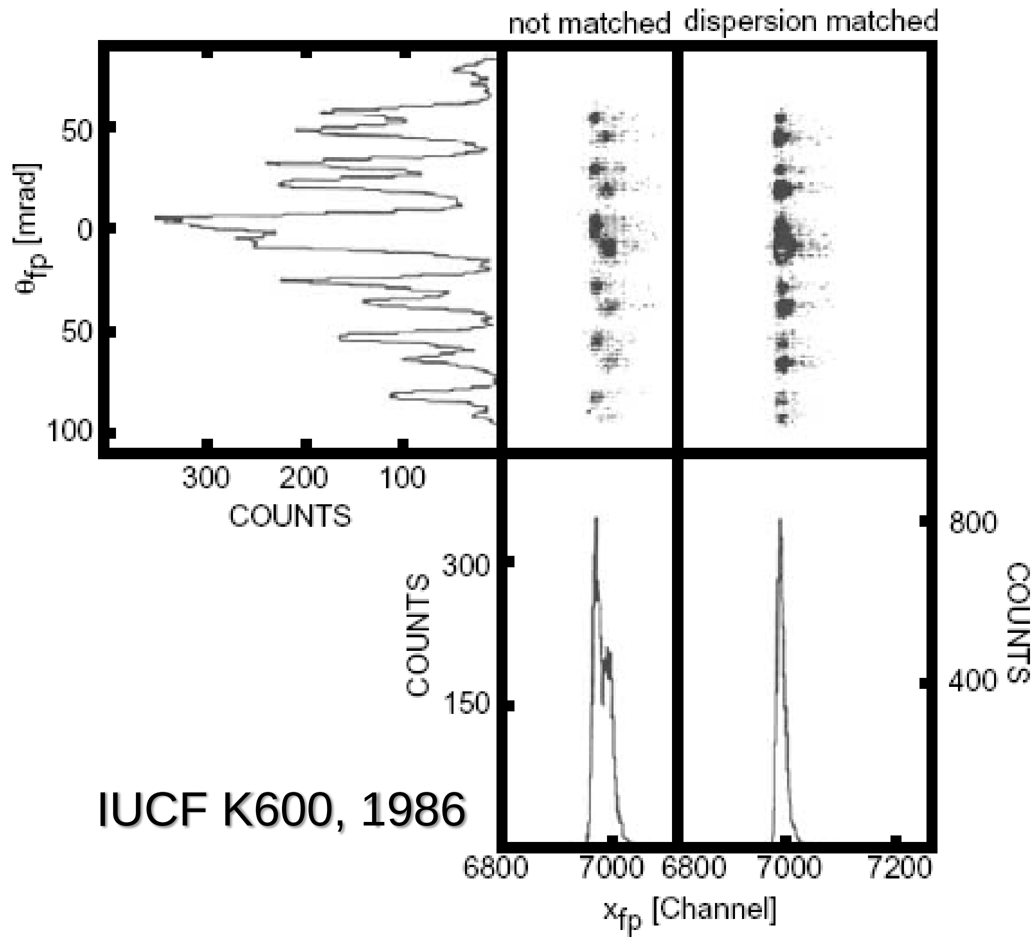
Beam Line/Spectrometer fully matched



Diagnostic of Dispersion Matching

of beam line & spectrometer using
a double strip target & multi slit





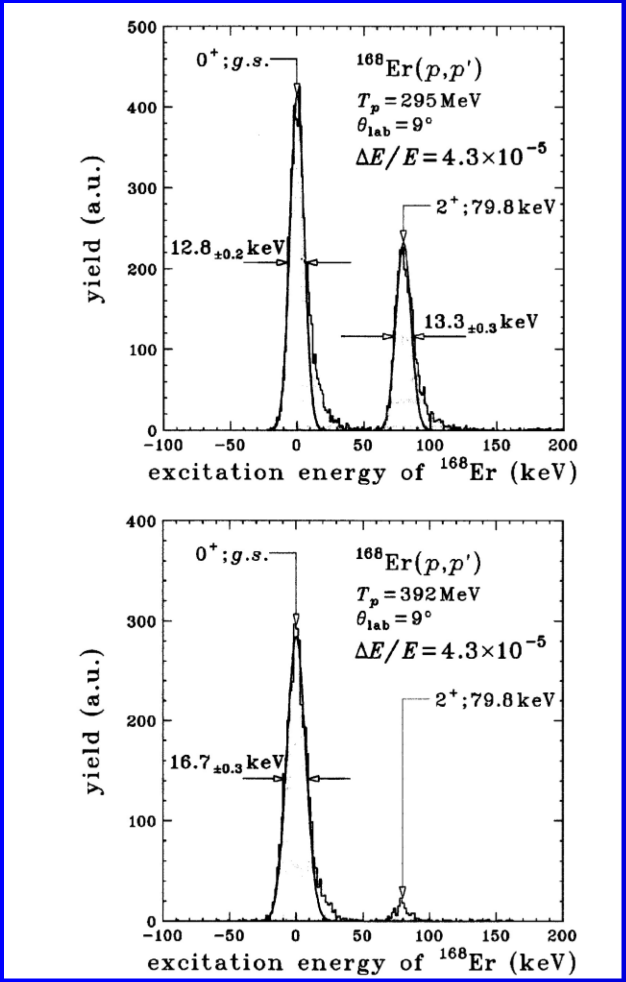
IUCF K600, 1986

Fig. 4. Scatterplots of horizontal position x_{fp} versus angle θ_{fp} and projections measured in the focal plane of the K600 using the "multi-slit system". For details, see text.

Momentum and Angular Resolution

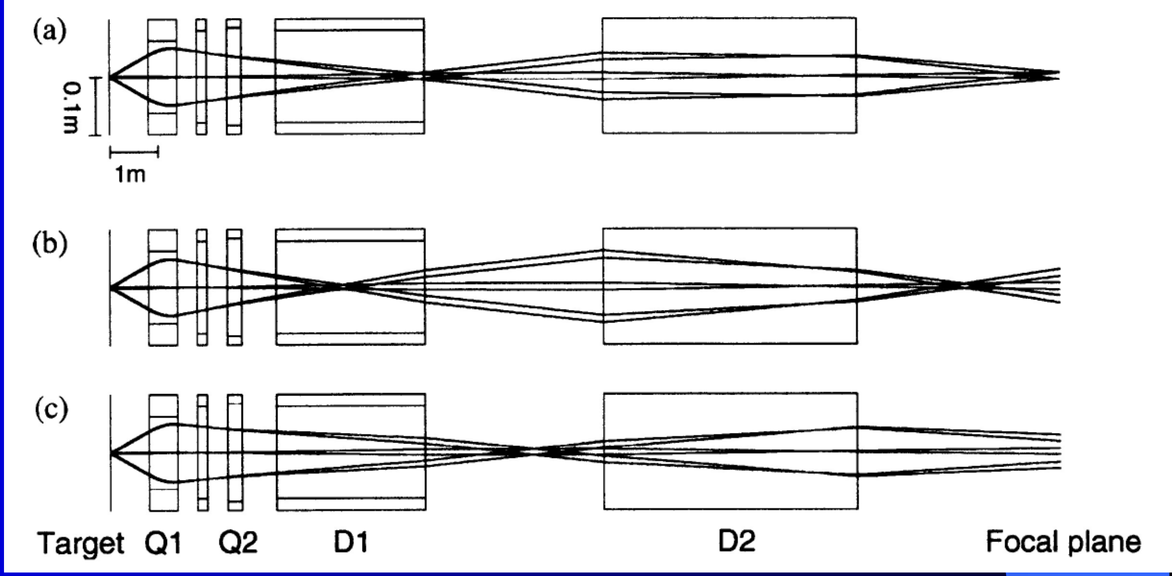
Spacial & Angular Dispersion Matching & Focus Condition allows

Energy Resolution: $E/\Delta E$=23000, $\Delta p/p$ = 40000, despite beam spread: $E/\Delta E$ = 1700 - 2500



Angular resolution: ΔY_{scatt} = SQRT($\Delta Y^2_{hor}+\Delta\Phi^2$) = 4 - 8 msr

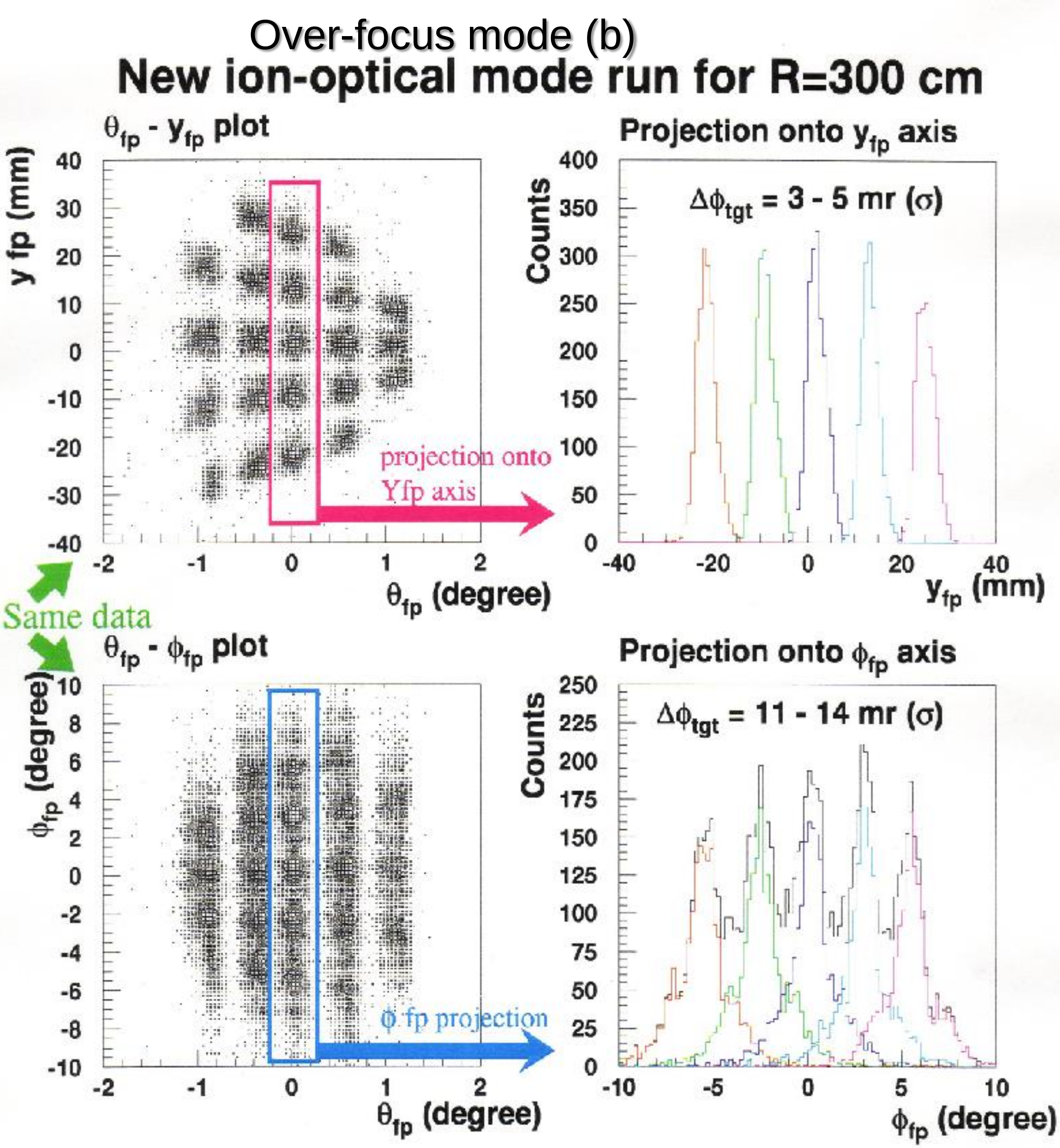
At angles close to beam (e.g. 0 deg) vert. angle component is needed → Overfocus mode, small target dimension, because (y|y) is large, Limitation: multiple scattering in detector



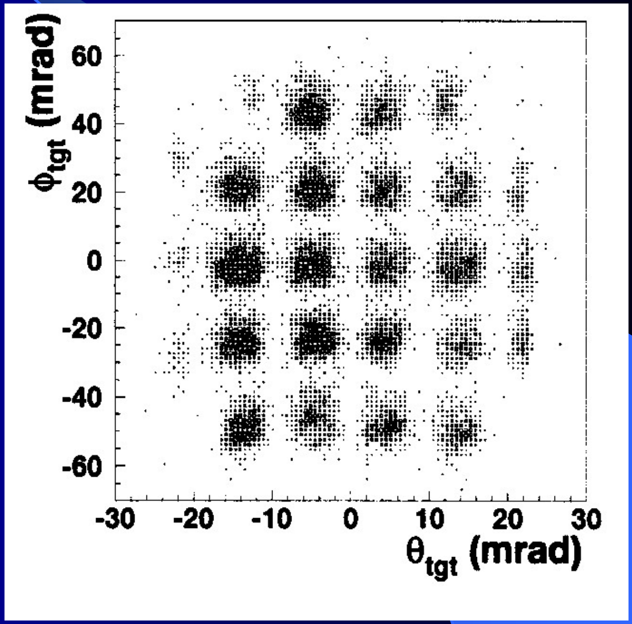
Refs.: Y.Fujita et al, NIM B126(1997)274, H.Fujita et al. NIM A 469(2001)55, T.Wakasa et al, NIM A482(2002)79

Grand Raiden Angle Calibration

Data suggest: Use y_{fp} not Φ_{fp} to calibrate angle!



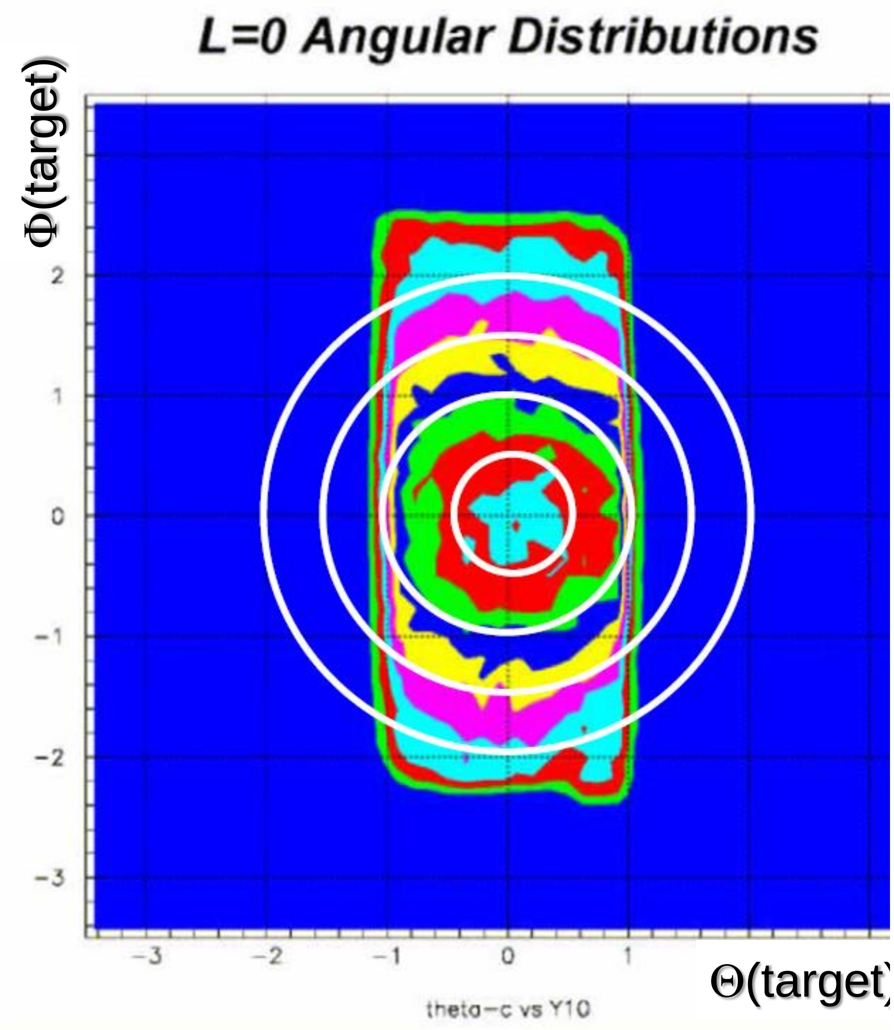
Calibrated!



Scattering Angle

reconstructed from focal plane measurements
using complete dispersion matching techniques

$E(^3He) = 420$ MeV



Scatt. Angle reconstruction near 0 deg using Overfocus Mode

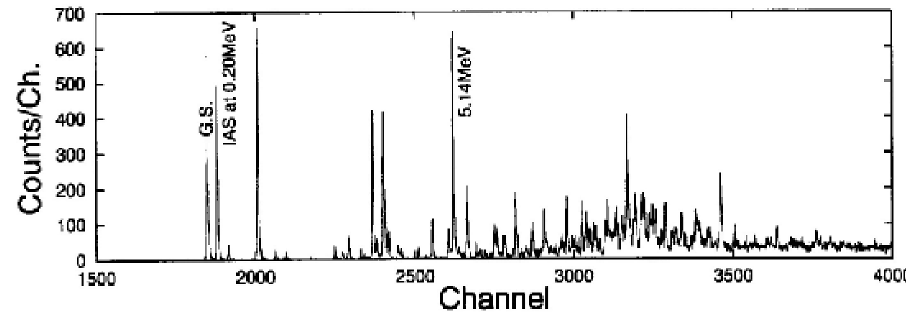


Figure 4.4: Spectrum of ^{58}Ni(^{3}He,t) reaction. The *lateral* and *angular dispersion matching* technique and *over-focus mode* were applied in this experiment for high energy and scattering angle resolution. Energy resolution of about 30 keV (FWHM) was realized.

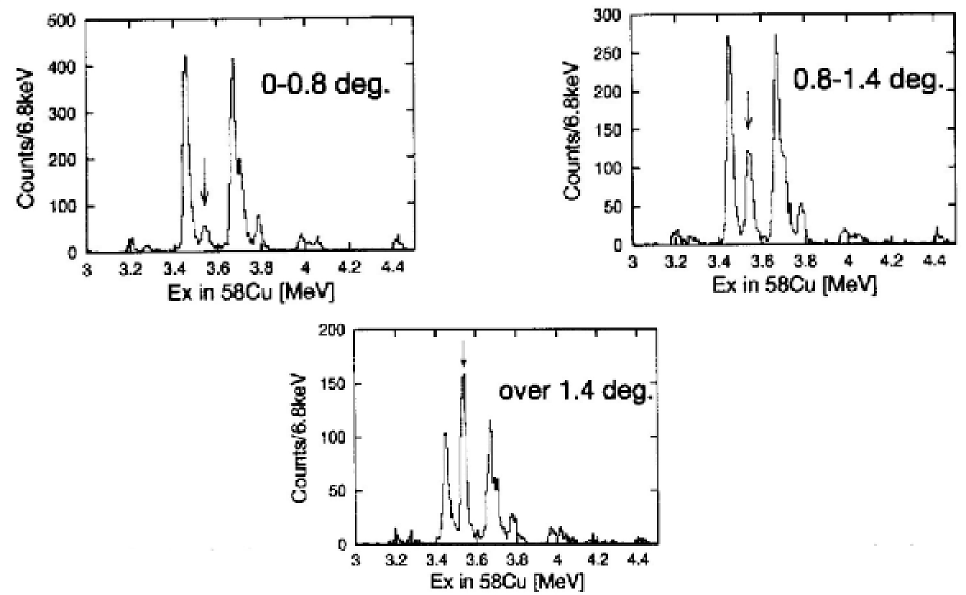


Figure 4.5: Example of angle dependence in the ^{58}Ni(^{3}He,t) spectra near 0°. Three spectra are shown for the angle ranges 0-0.8° (left), 0.8-1.4° (middle) and over 1.4° (right), respectively. The 3.54 MeV state show clearly different angular distribution from the adjacent 1^+ states which are dominated at forward angle.

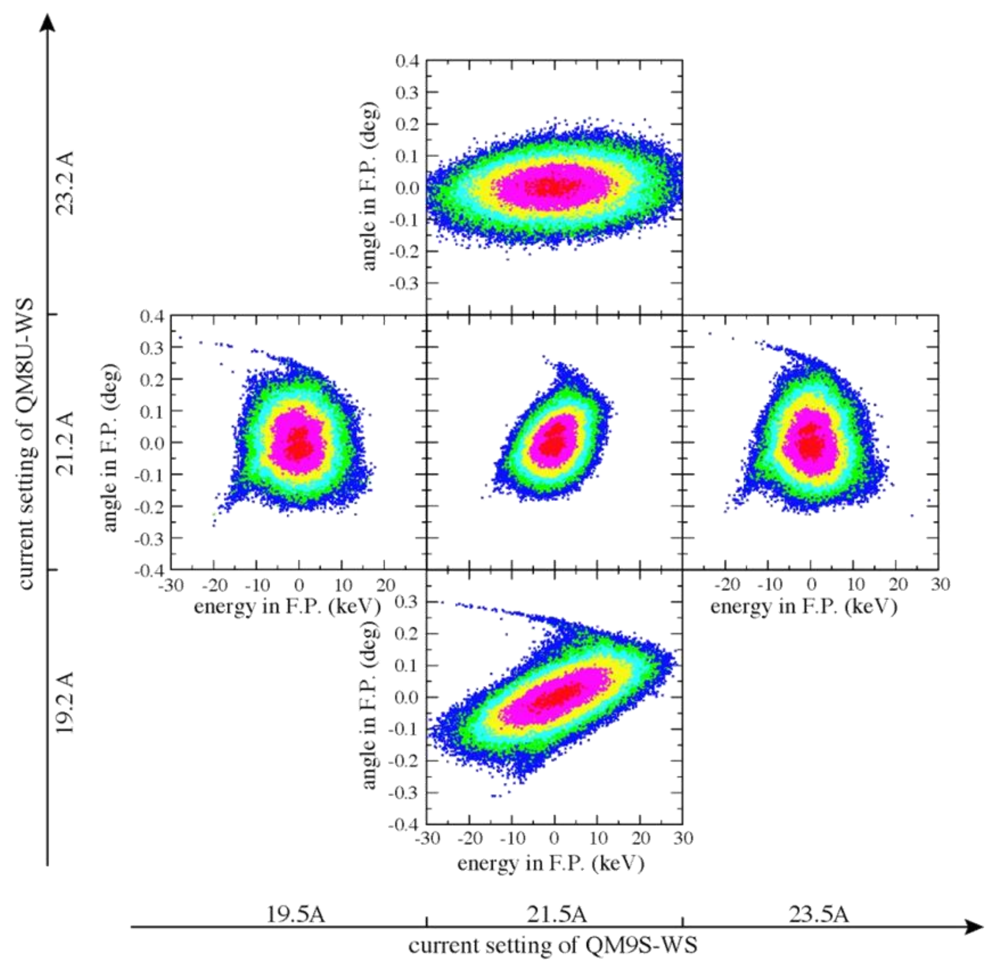
Horizontal Beam Profiles in the Focal Plane of Grand Raiden

- QM8U
  →Control lateral
  dispersion
- QM9S
  →Control angular
  dispersion
- Lateral and angular dispersions can be controlled independently
- References

  H. Fujita et al., NIMA

  T. Wakasa et al., NIMA

Dispersion matching for K = 0 with faint beam



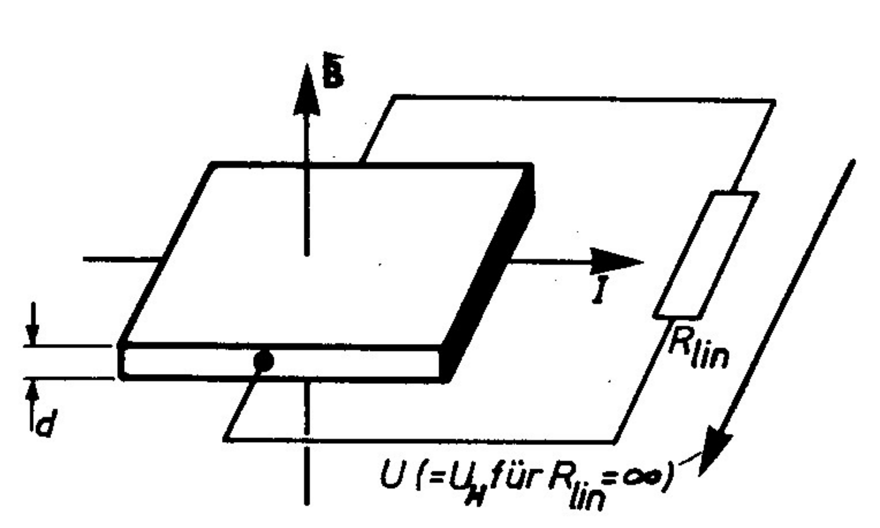
Discussion of Diagnostic Elements

Some problems:

- Range < 1 to $> 10^{12}$ particles/s
- Secondary beam intensities typically up to 10^6 part./s
- Interference with beam, notably at low energies
- Cost can be very high
- Signal may not represent beam properties (e.g. blind viewer spot)

Some solutions:

- Viewers, scintillators, quartz with CCD readout
- Slits (movable) Faraday cups (current readout)
- Harps, electronic readout, semi- transparent
- Film (permanent record, dosimetry, e.g. in Proton Therapy)
- Wire chambers (Spectrometer, secondary beams)
- Faint beam $10^{12} \longrightarrow 10^3$ (Cyclotrons: MSU, RCNP, iThemba)



Hall Probe

Hall Effect: $U_H = \dfrac{R_H}{d} BI$ **(23)**

Lorentz force $e v \times B$ on electrons with velocity v that constitute the current I

R_H = Hall constant, material property

Remarks:

- Precision down to ~ $2\ 10^{-4}$
- Needs temperature calibration
- Probe area down to 1 mm by 1 mm
- Average signal in gradient field (good for quadrupole and fringe field measurement)

NMR Probe

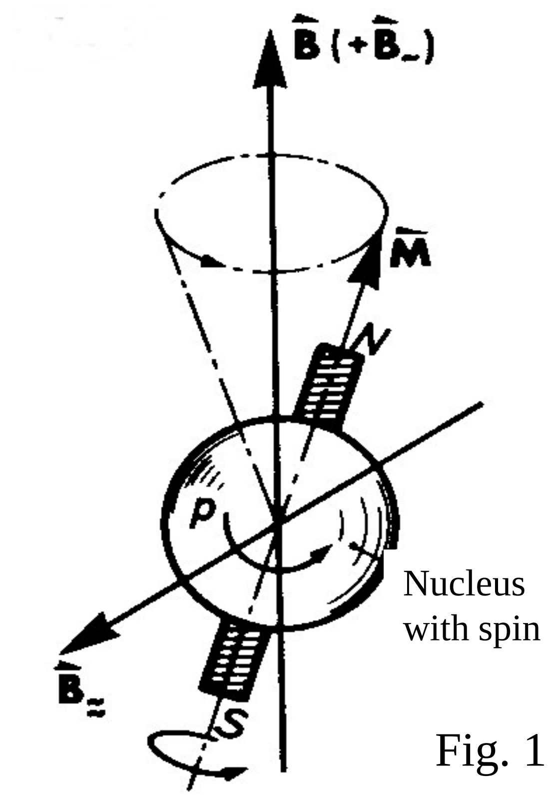


Fig. 1

Nuclear spin precesses
in external field B
With Larmor frequency

$$f_L = \frac{2\mu}{h} B \qquad \textbf{(24)}$$

μ = p, d magn. Moment
h = Planck constant

f_L (proton)/B = 42.58 MHz/T

f_L (deuteron)/B = 6.538 MHz/T

Principle of measurement:
Small (e.g. water probe), low frequency wobble coil $B + B_{\sim}$, tuneable HF field $B_{\approx}$ (Fig. 1) with frequency f_t , observe Larmor resonance on Oscilloscope (Fig. 2). When signal a & b coincide the tuneable frequence $f_t = f_L$

- Precision ~ 10^{-5}
- Temperature independent
- Needs constant B in probe (5 x 5 mm) to see signal!

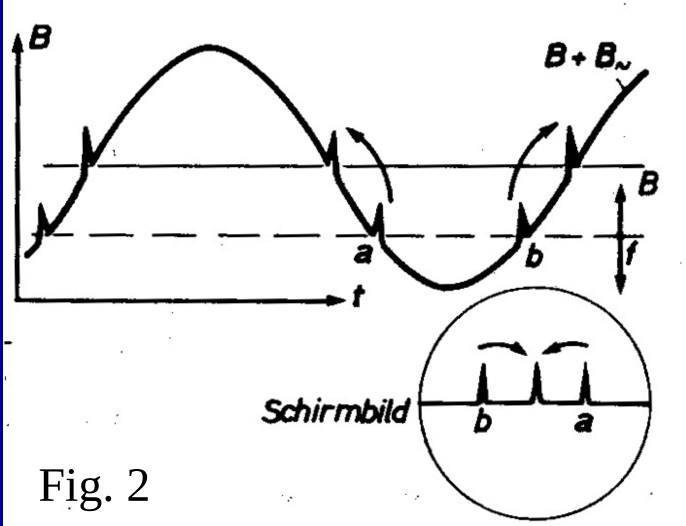
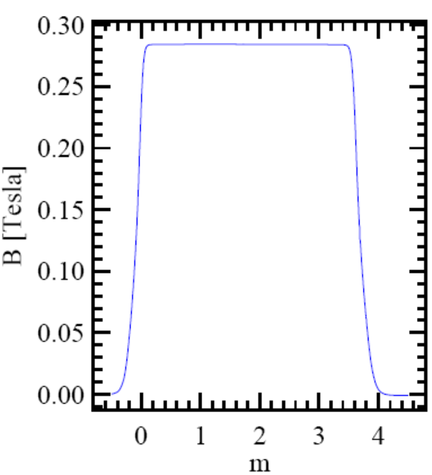


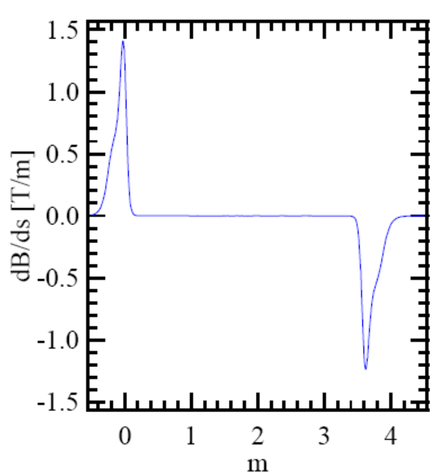
Fig. 2

Search Coil used in field map of S800, MSU

Induced Voltage
in search coil
$U = - dB/dt$

Coil is moved from outside
$B = 0$ T to inside B_{max}
(calibrated with NMR probe)
B at location (x,y) is obtained
by intergration





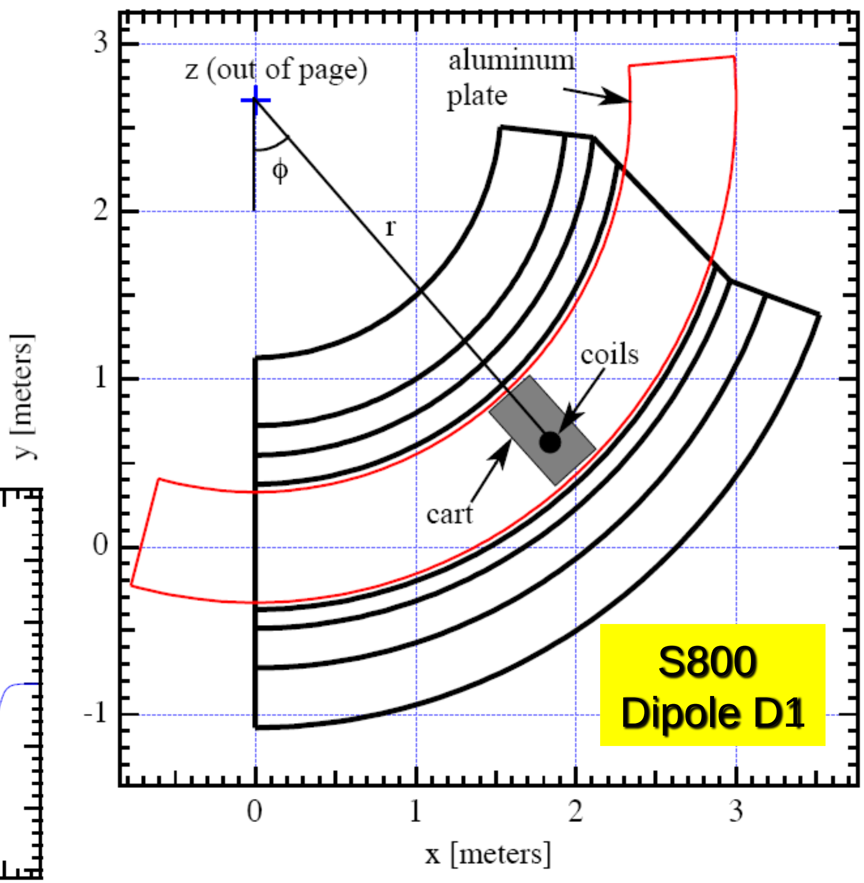


Figure 3.3: Schematic of the mapper showing the coordinate system used. The thick black lines are the magnet steel, the thin lines are the mapper plate. The gray box represents the cart that moves up and down along the plate. The three cylindrical coordinates are shown originating from a typical coil position on the cart (z is out of the page).

Ref: J. A. Caggiano, Dissertation, NSCL MSU, 1999,
Spectroscopy of Exotic Nuclei with the S800 Spectrometer

Field map of D2 of S800, MSU

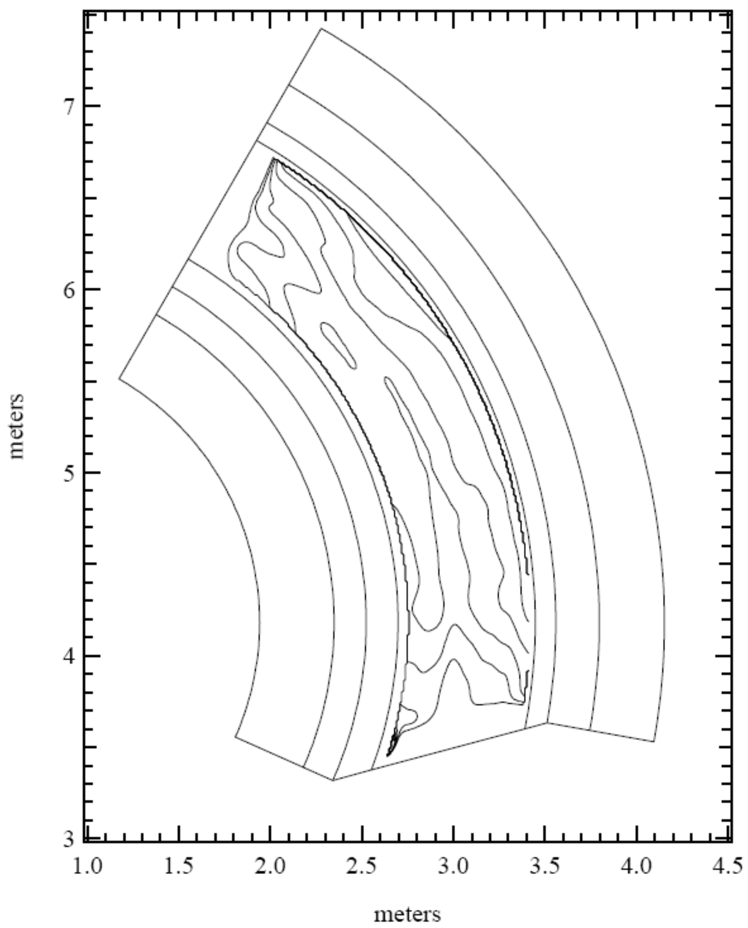


Figure 3.12: A D2 field map at 0.28 Tesla, plotted above 99% of the field strength. The contours are separated by one Gauss, indicating a very flat central field.

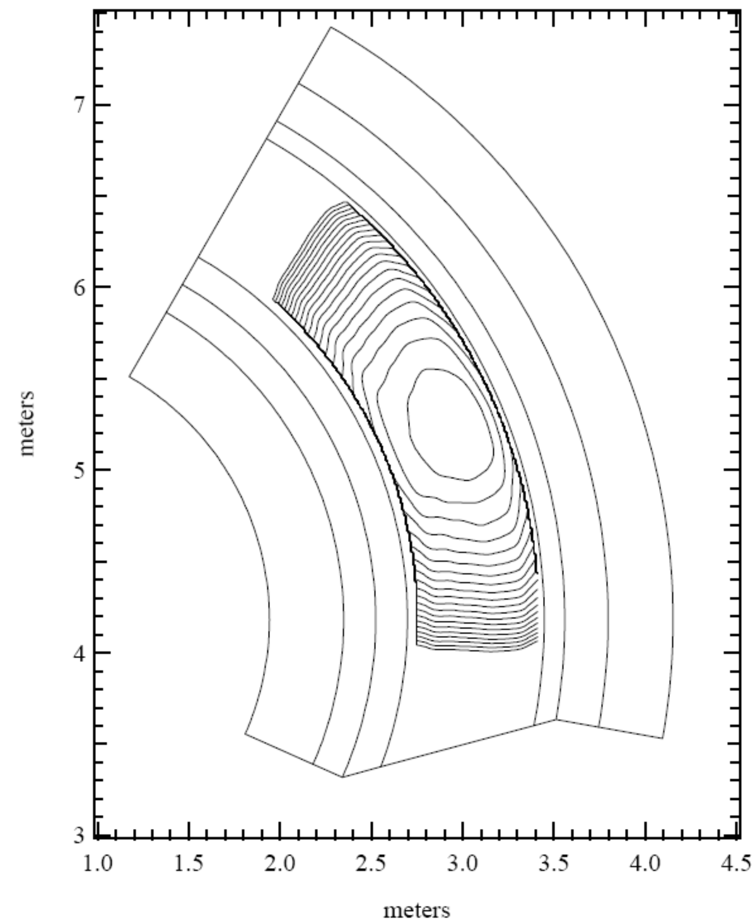


Figure 3.13: Field measurements for D2 at 1.6 Tesla. All contours are separated by 5 Gauss. In sharp contrast to Figure 3.12, the field exhibits a bowing behavior characteristic of saturation effects.

Ref: J. A. Caggiano, Dissertation, NSCL MSU, 1999, Spectroscopy of Exotic Nuclei with the S800 Spectrometer

End Lecture 2